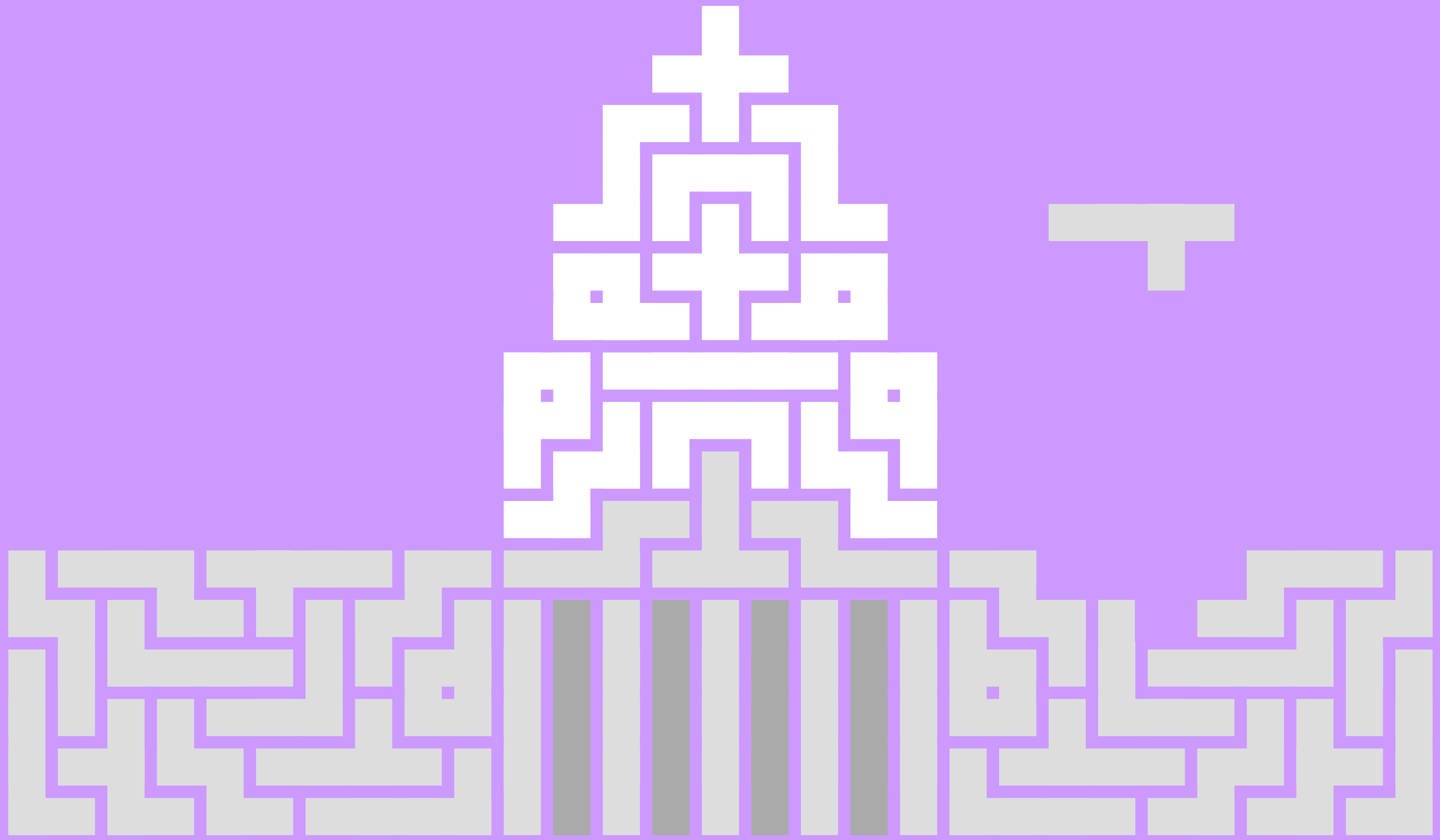

Algorithms For Democratic Decision-Making

Jamie Tucker-Foltz • Yale University • Spring 2026

Lecture 1: **Course Preview**

About me

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- Finite model theory and descriptive complexity

Other non-research interests:

- Juggling
- Unicycling
- Juggling while unicycling

Course Description

[Follow along in the syllabus, handouts at the front.]

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We will consider various models and settings with individuals who have heterogeneous preferences. How do we combine them into *societal preferences* to make public decisions?

Prerequisites and Enrollment

Class is open to all grad students, undergrads by permission only.

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This material is interdisciplinary, drawing on:

- Economics - models of preferences, axiomatic approach, reasoning about incentives, game-theoretic foundations
- Computer science - computational tractability and hardness, approximation ratios, tools for proving theoretical guarantees
- Political science - models of institutions, ideas about fairness
- Mathematics - topological existence theorems, geometric insights
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By Wednesday evening, please fill out pre-course survey linked on Canvas!

Learning Objectives and Course Structure

Learning objectives:

- To build theoretical/mathematical research skills.

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No written homework. Instead, there will be in-class problem-solving. You are expected to actively participate!

Grading

20% Attendance and participation.

20% Paper presentation 1

20% Paper presentation 2

30% Final project report (also submit a brief proposal and mid-term update)

10% Project presentation

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Assignments will be submitted via Canvas. Slides for all student presentations are due at 11:59pm Anywhere on Earth on the day before the presentation.

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Project group size: 1-2 people. I may consider allowing groups of size 3 in exceptional circumstances.

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The project must have some theoretical component.

The best case scenario is that the project report becomes the seed of an eventual publication in an area related to your own research interests."

Course Calendar and Materials

https://jamie.tuckerfoltz.com/Teaching/AFDDM26/

Collaboration/AI Policies

For both presentations and projects: "Students are all expected to contribute, and will each individually submit a detailed explanation of what they contributed to the final report, in terms of both ideas and writing."

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3. Do not let AI replace you as a writer. For instance, if you write a paragraph in your introduction motivating why your problem is interesting, it should be written in your own words using your own ideas. I do not want to read about what AI thinks about a problem. I want to hear what **you** think."

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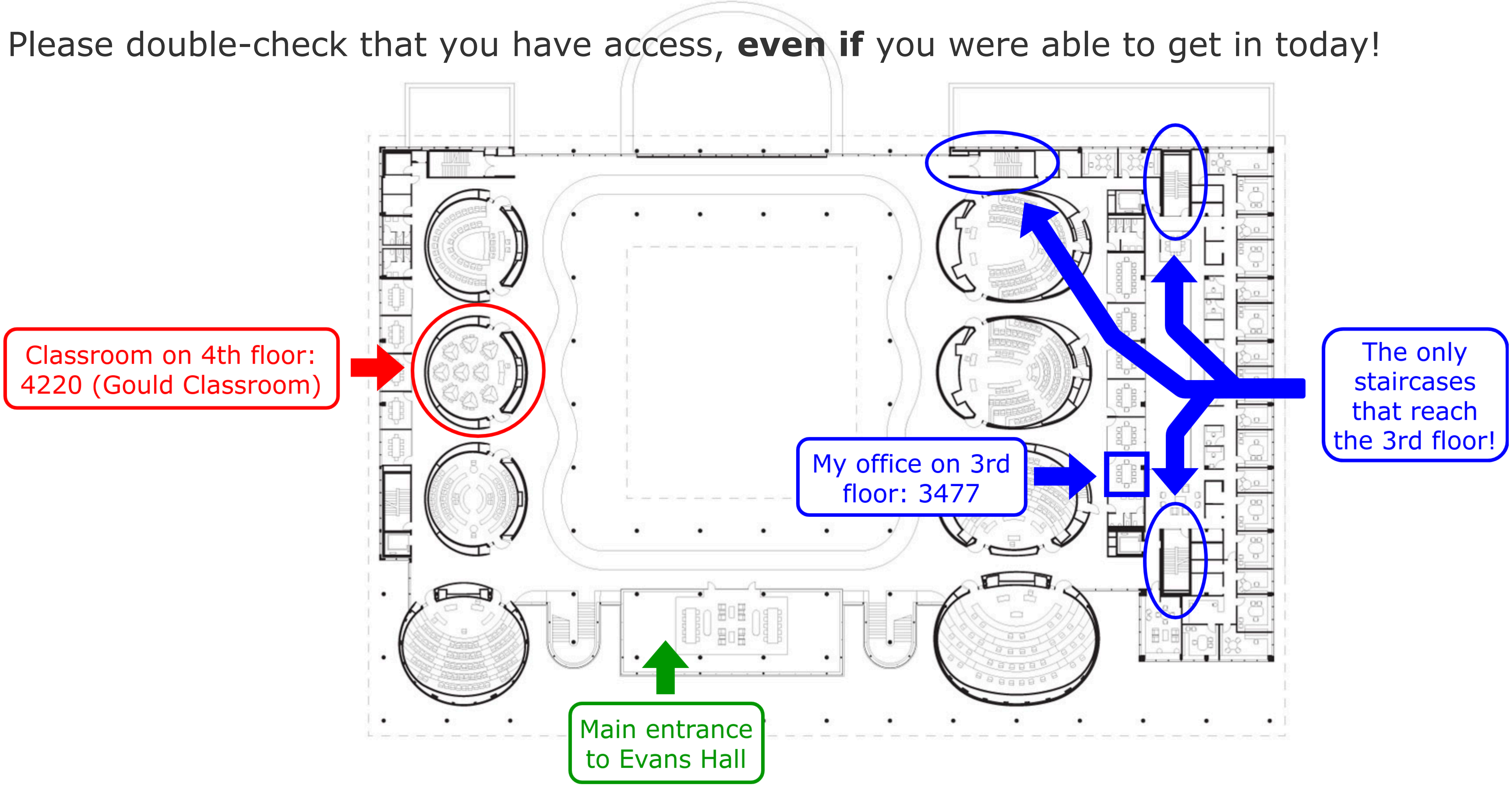

Lecture 2 - Voting Systems

Introduction to *Social Choice Theory* - the study of preference aggregation.

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Num voters:	33	16	3	8	18	22
First choice:	a	b	c	c	d	e
	b	d	d	e	e	c
...	c	c	b	b	c	b
	d	e	a	d	b	d
Last choice:	e	a	e	a	a	a

Who should win?

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Who should win?
> We'll see a bunch of different answers and various justifications

Theorem (Arrow, 1951)

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Can we circumvent impossibilities via computational complexity?

A topological impossibility theorem (Chichilnisky, 1981)

Lecture 4 - Restricted Preferences

Geometric way of thinking about voting over m candidates: Input is a point in the (m! - 1)-dimensional simplex. What if not all preferences are possible?

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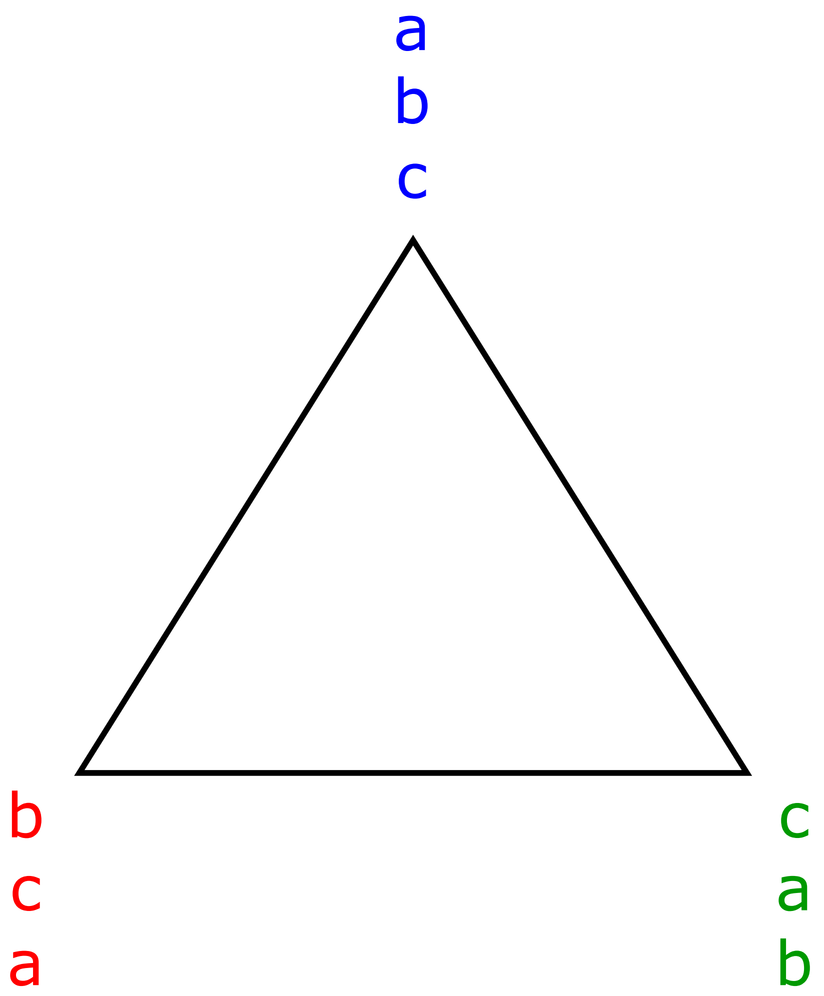

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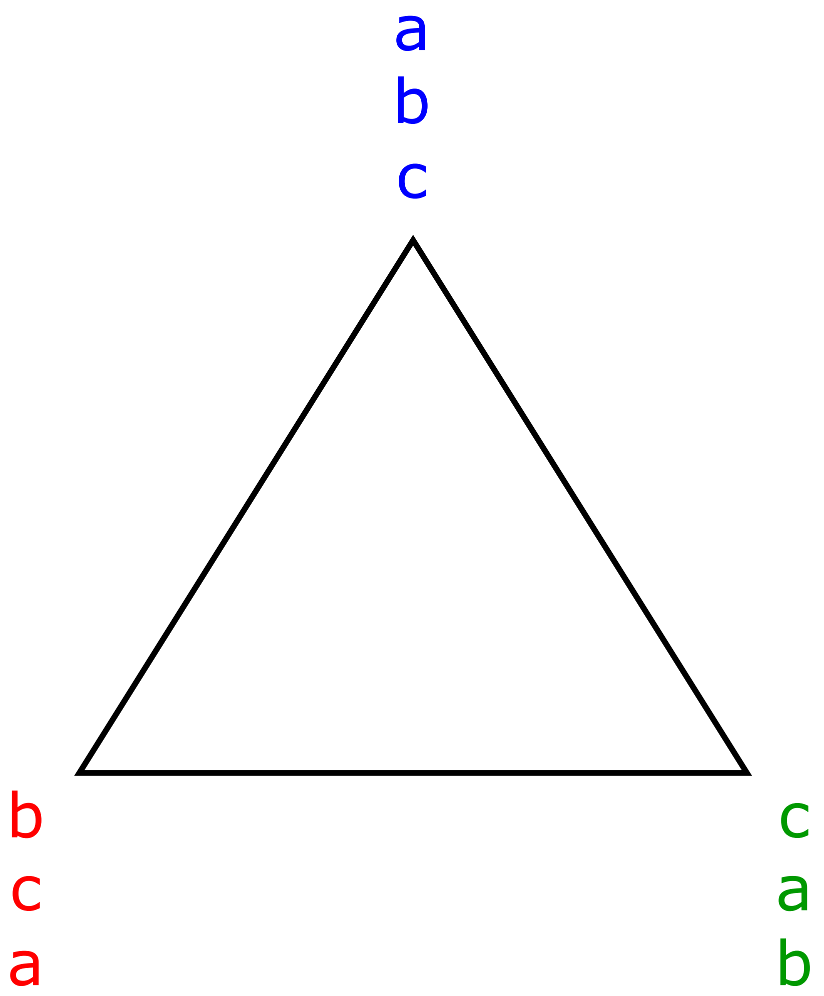

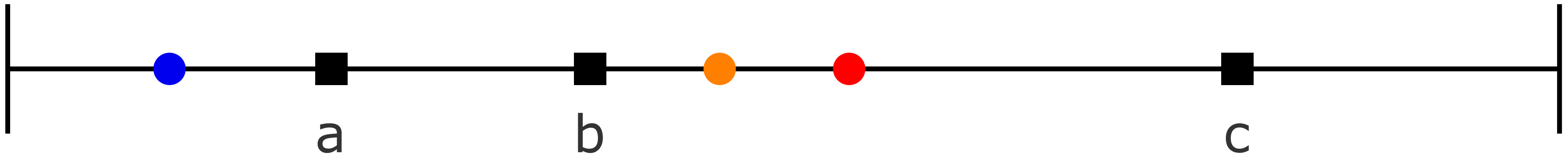

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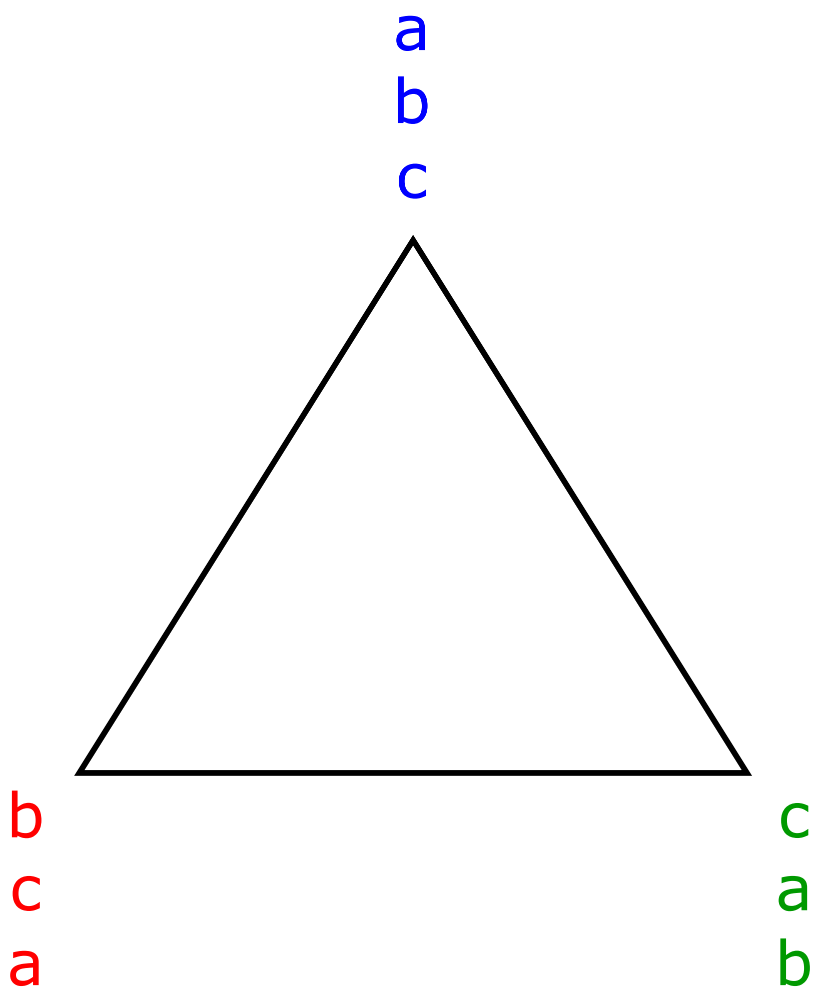

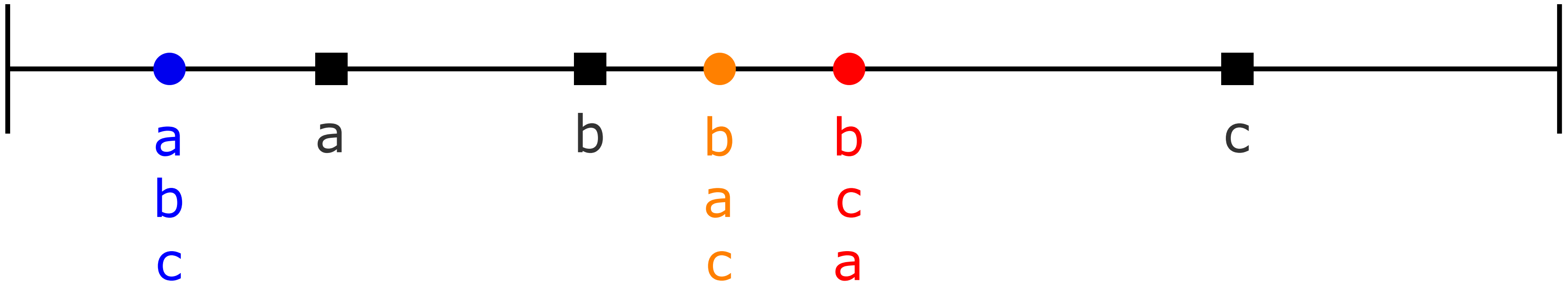

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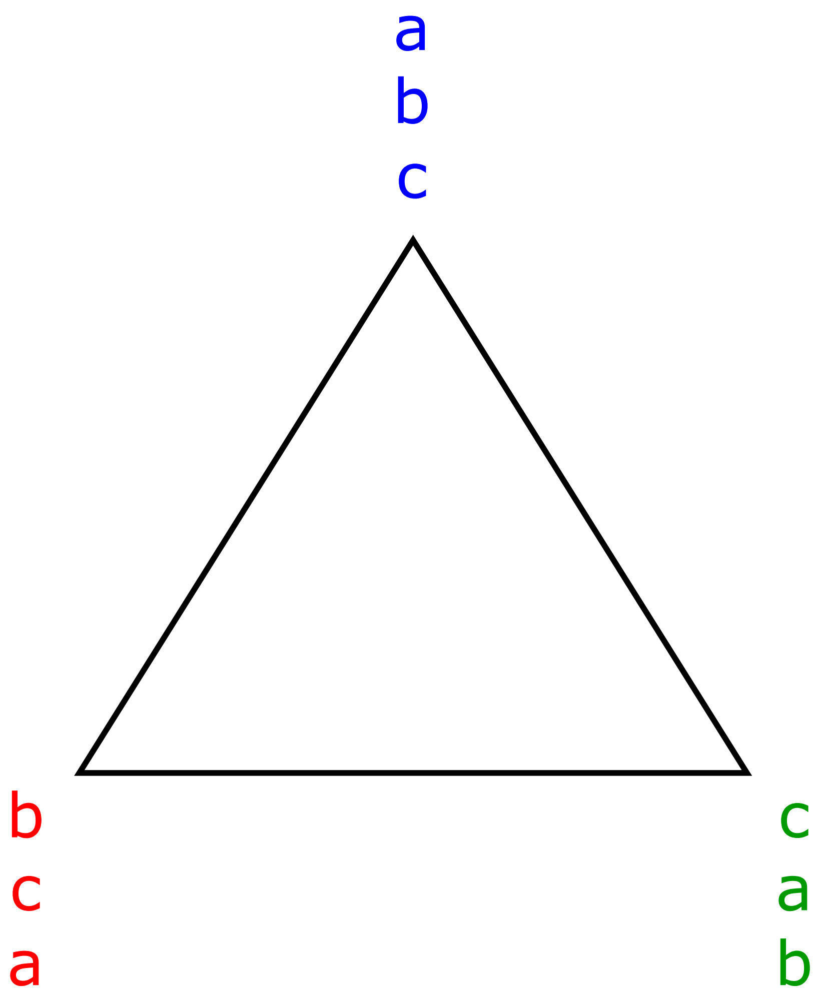

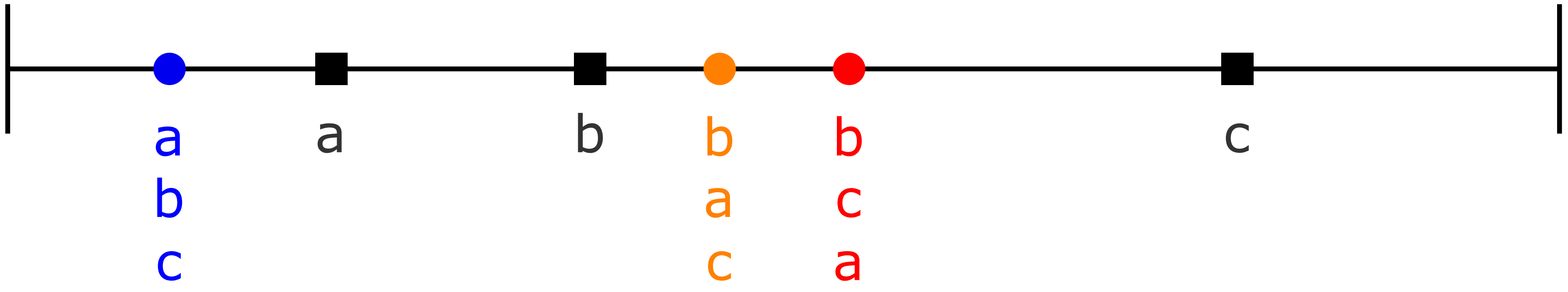

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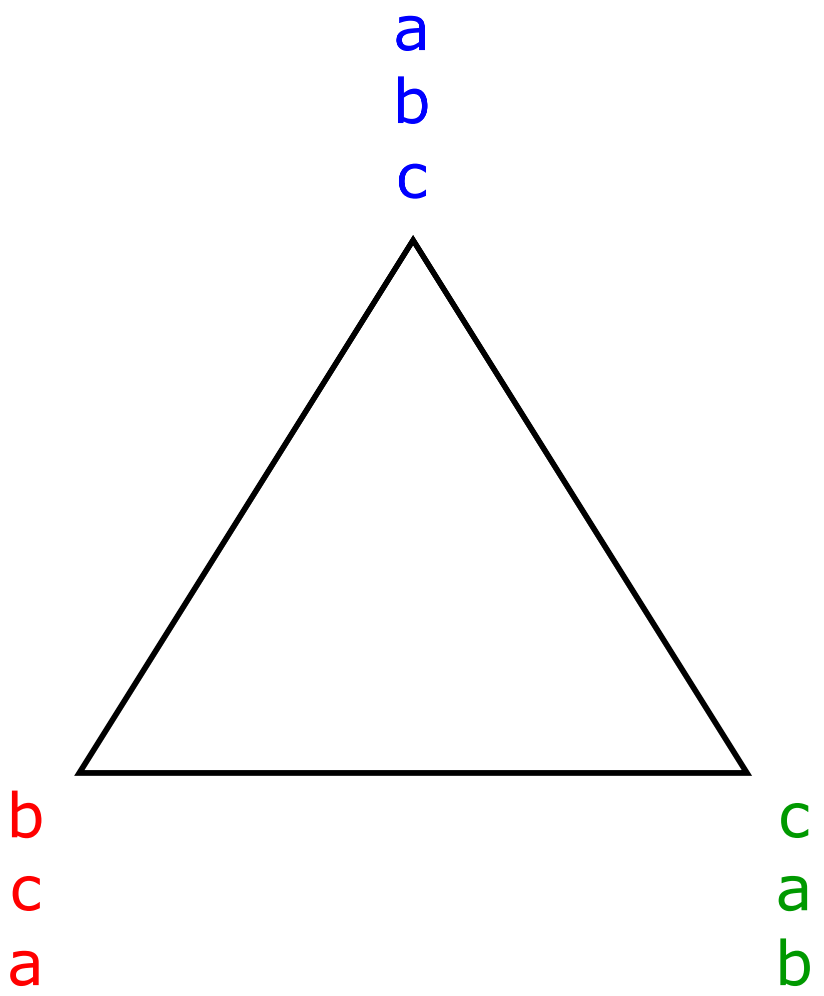

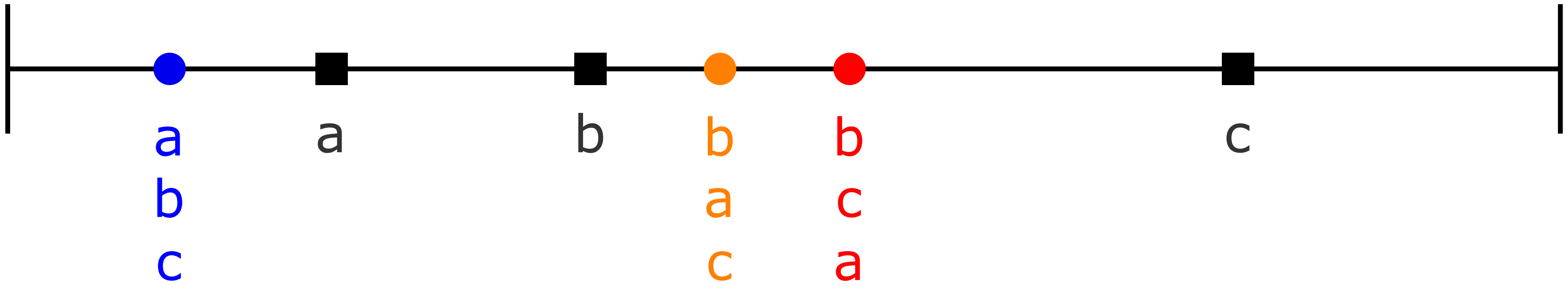

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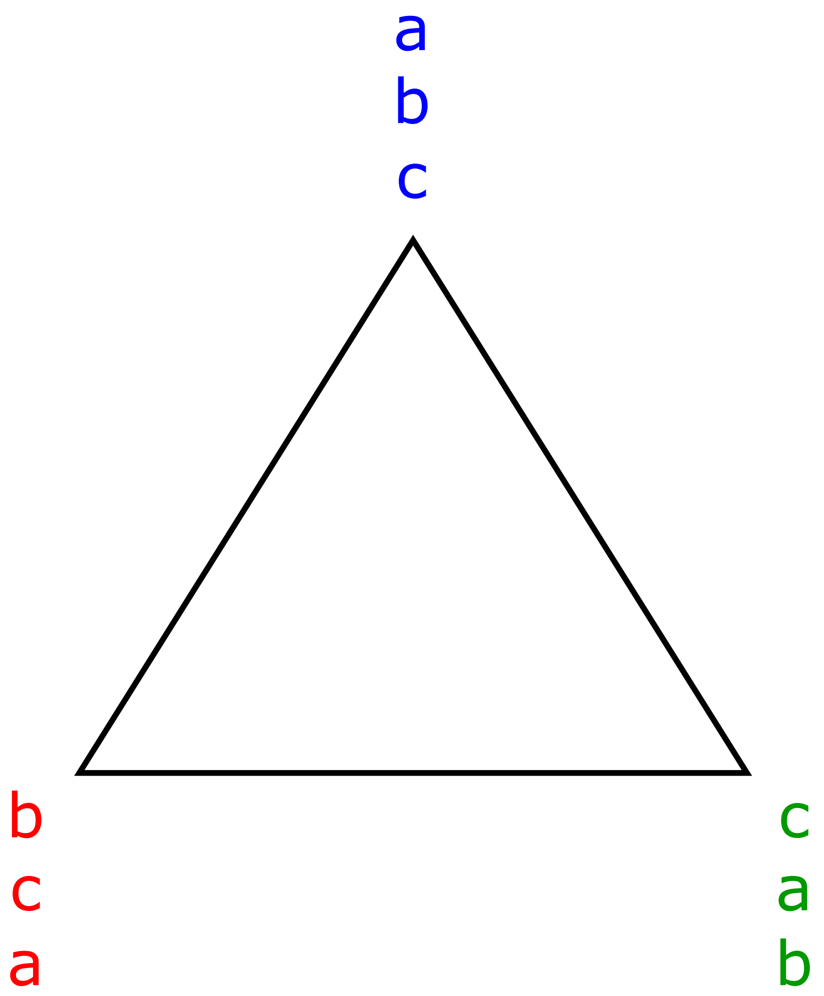

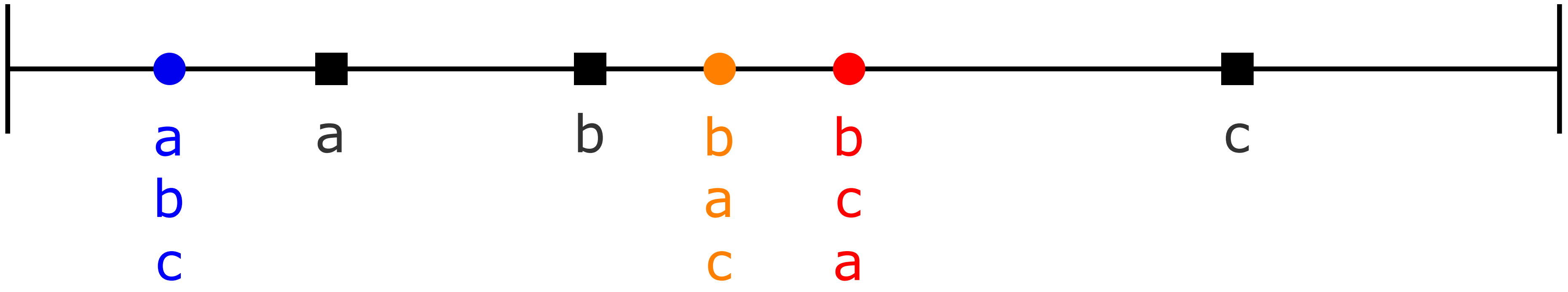

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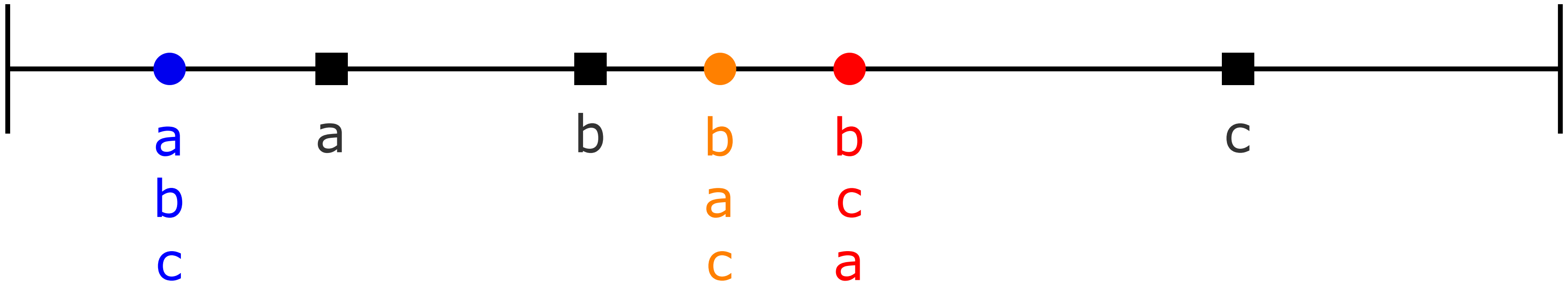
Phantom characterization (Moulin, 1980)

Lecture 5 - Metric Distortion



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Also allowing randomization: Better constant known, huge open question

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●	●	●
a	b	b
b	a	c
c	c	a

Without any assumptions: No constant factor approximation

Assuming voters and candidates lie in a *metric space*: Constant factor, solved problem

Also allowing randomization: Better constant known, huge open question

Breaking the randomized barrier (Charikar, Ramakrishnan, Wang, Wu, 2024)

Lecture 6 - The Epistemic Approach

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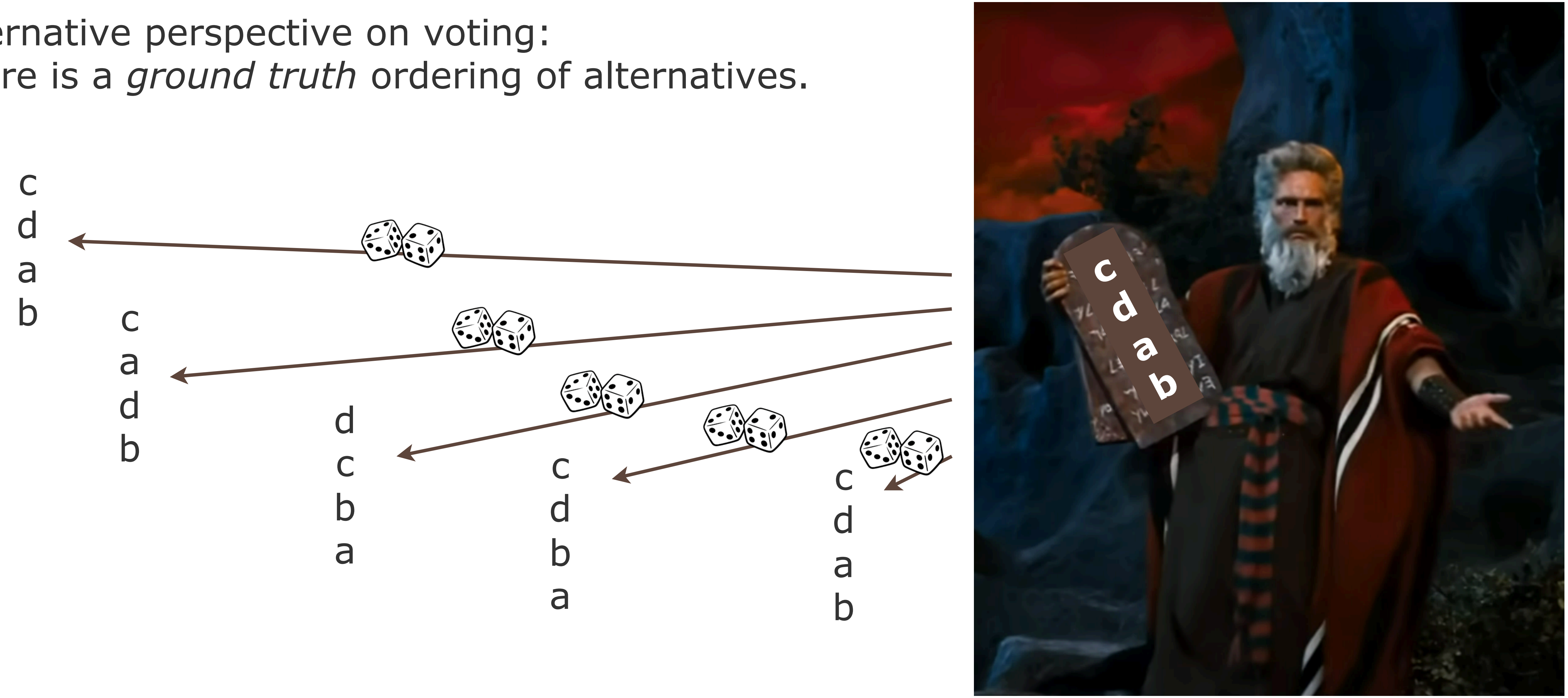

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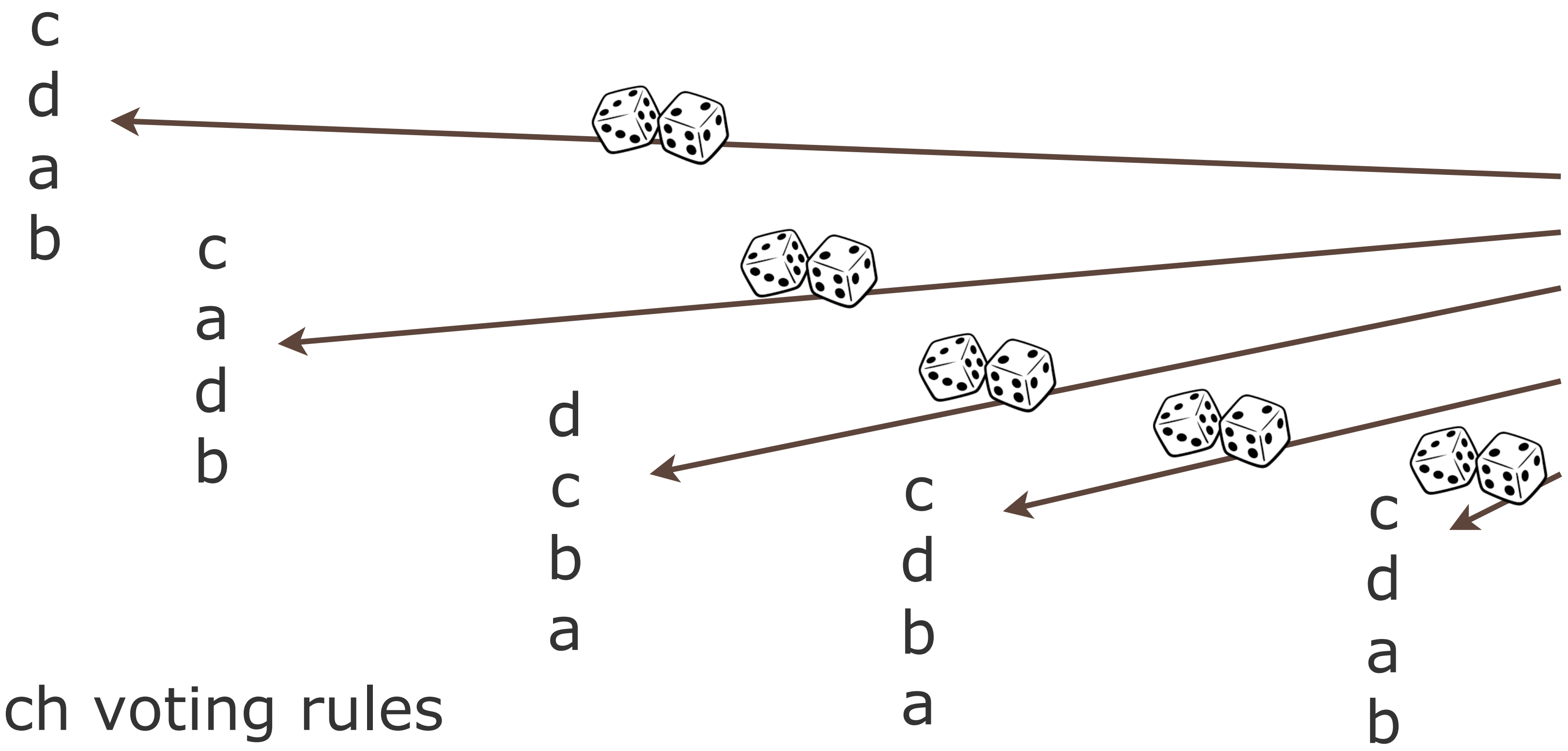

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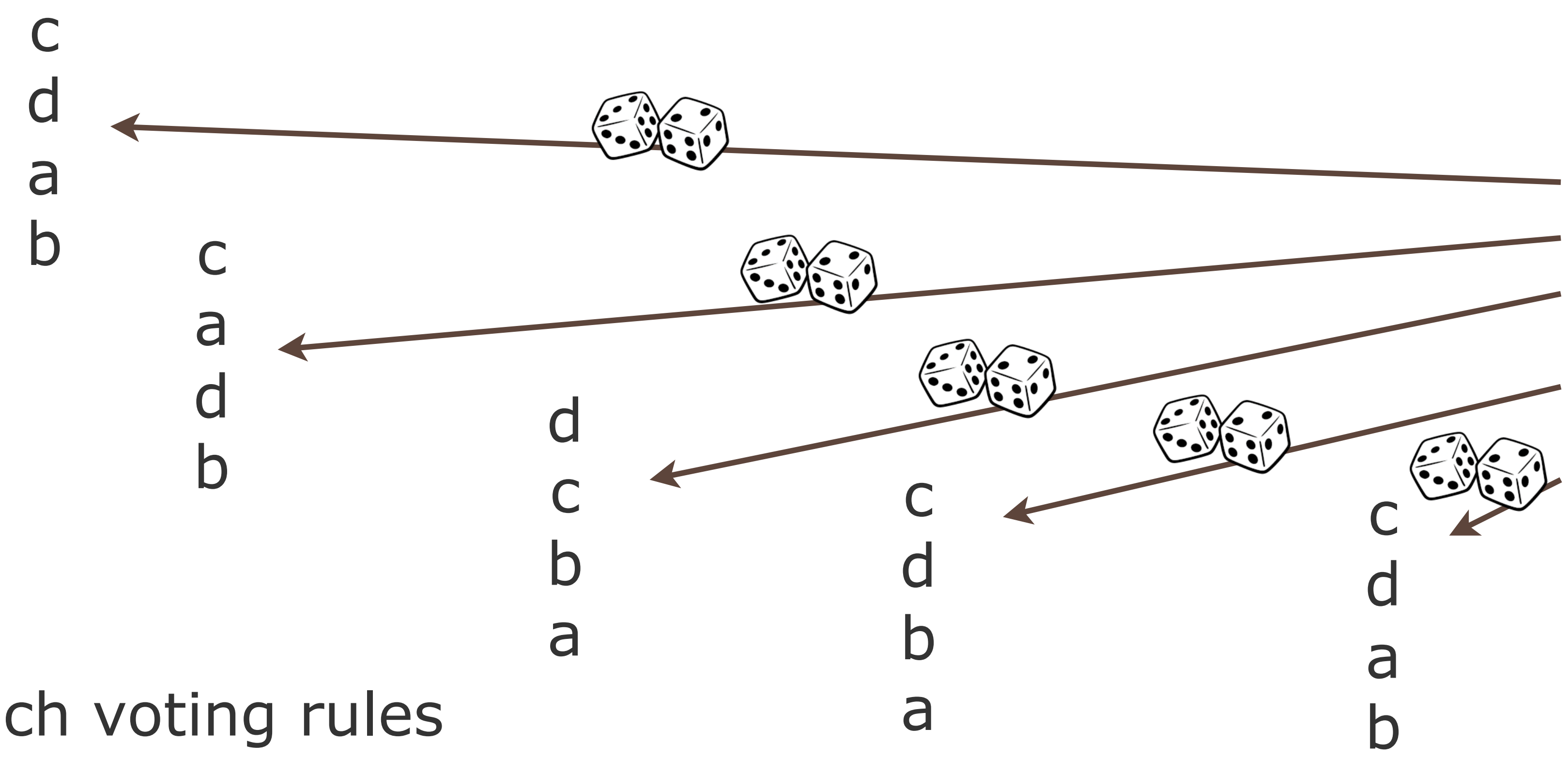

Which voting rules
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[Insincere voting with 2 alternatives](#) (Austen-Smith, Banks, 1996)

More common ballot format for many candidates: *approval preferences*.

	Voter 1	Voter 2	Voter 3	Voter 4	Voter 5	Voter 6	Voter 7	Voter 8
Cand. 1	✔	✘	✘	✘	✔	✔	✔	✔
Cand. 2	✔	✔	✔	✔	✘	✘	✘	✘
Cand. 3	✘	✘	✔	✔	✔	✔	✔	✔
Cand. 4	✘	✘	✔	✔	✔	✔	✔	✔
Cand. 5	✘	✘	✔	✘	✔	✔	✔	✔
Cand. 6	✘	✔	✘	✘	✔	✔	✔	✔
Cand. 7	✔	✔	✔	✔	✘	✘	✘	✘
Cand. 8	✘	✘	✔	✔	✔	✔	✔	✔
Cand. 9	✘	✘	✔	✔	✔	✔	✔	✔
Cand. 10	✘	✘	✘	✔	✔	✔	✔	✔

Lecture 7 - Multi-Winner Voting

More common ballot format for many candidates: *approval preferences*.

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Cand. 1	✓	✗	✗	✗	✓	✓	✓	✓
Cand. 2	✓	✓	✓	✓	✗	✗	✗	✗
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Cand. 8	✗	✗	✓	✓	✓	✓	✓	✓
Cand. 9	✗	✗	✓	✓	✓	✓	✓	✓
Cand. 10	✗	✗	✗	✓	✓	✓	✓	✓

What if we need to select a committee of size 4?

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Cand. 7	✓	✓	✓	✓	✗	✗	✗	✗
Cand. 8	✗	✗	✓	✓	✓	✓	✓	✓
Cand. 9	✗	✗	✓	✓	✓	✓	✓	✓
Cand. 10	✗	✗	✗	✓	✓	✓	✓	✓

What if we need to select a committee of size 4? Tradeoff between utility and **fairness**.

More common ballot format for many candidates: *approval preferences*.

	Voter 1	Voter 2	Voter 3	Voter 4	Voter 5	Voter 6	Voter 7	Voter 8
Cand. 1	✔	✘	✘	✘	✔	✔	✔	✔
Cand. 2	✔	✔	✔	✔	✘	✘	✘	✘
Cand. 3	✘	✘	✔	✔	✔	✔	✔	✔
Cand. 4	✘	✘	✔	✔	✔	✔	✔	✔
Cand. 5	✘	✘	✔	✘	✔	✔	✔	✔
Cand. 6	✘	✔	✘	✘	✔	✔	✔	✔
Cand. 7	✔	✔	✔	✔	✘	✘	✘	✘
Cand. 8	✘	✘	✔	✔	✔	✔	✔	✔
Cand. 9	✘	✘	✔	✔	✔	✔	✔	✔
Cand. 10	✘	✘	✘	✔	✔	✔	✔	✔

What if we need to select a committee of size 4? Tradeoff between utility and **fairness**.

JR: If you're selecting *k* candidates, every set of *1/k* of the population that can agree on a candidate cannot all be getting zero approved candidates.

Lecture 7 - Multi-Winner Voting

More common ballot format for many candidates: *approval preferences*.

	Voter 1	Voter 2	Voter 3	Voter 4	Voter 5	Voter 6	Voter 7	Voter 8
Cand. 1	✔	✘	✘	✘	✔	✔	✔	✔
Cand. 2	✔	✔	✔	✔	✘	✘	✘	✘
Cand. 3	✘	✘	✔	✔	✔	✔	✔	✔
Cand. 4	✘	✘	✔	✔	✔	✔	✔	✔
Cand. 5	✘	✘	✔	✘	✔	✔	✔	✔
Cand. 6	✘	✔	✘	✘	✔	✔	✔	✔
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Cand. 10	✘	✘	✘	✔	✔	✔	✔	✔

What if we need to select a committee of size 4? Tradeoff between utility and **fairness**.

PSC and monotonicity (Aziz, Lederer, Peters, Peters, Ritossa, 2025)

Lecture 8 - Social Choice with Incomplete Preferences

What if there are so many candidates that you can't poll any one voter on all of them?

	Voter 1	Voter 2	Voter 3	Voter 4	Voter 5	Voter 6	Voter 7	Voter 8
Cand. 1				✗	✓	✓		
Cand. 2	✓		✓		✗		✗	
Cand. 3						✓		✓
Cand. 4		✗		✓				
Cand. 5	✗		✓	✗				✓
Cand. 6		✓				✓		✓
Cand. 7	✓			✓	✗			✗
Cand. 8			✓		✓		✓	
Cand. 9	✗			✓				✓
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Cand. 5	✗		✓	✗				✓
Cand. 6		✓				✓		✓
Cand. 7	✓			✓	✗			✗
Cand. 8			✓		✓		✓	
Cand. 9	✗			✓				✓
Cand. 10		✗			✓		✓	

Application to online platforms: https://pol.is/home2

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Cand. 9	✗			✓				✓
Cand. 10		✗			✓		✓	

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Preferences must be elicited through limited queries. Can we still guarantee fair representation with high probability?

Lecture 9 - Participatory Budgeting

Like committee selection from approval ballots, but with weighted candidates!

Project	Cost	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11
Bike path	$700	✓	✓	✓	✓	✓	✓					✓
Outdoor gym	$400	✓	✓	✓	✓	✓	✓					
New park	$250		✓		✓	✓		✓			✓	
New	$200							✓	✓	✓	✓	
Library for kids	$100							✓		✓	✓	

Example from Dominik Peters and Piotr Skowron

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Suppose the total budget is $1,100. A greedy policy would spend the entire budget on the first two projects.

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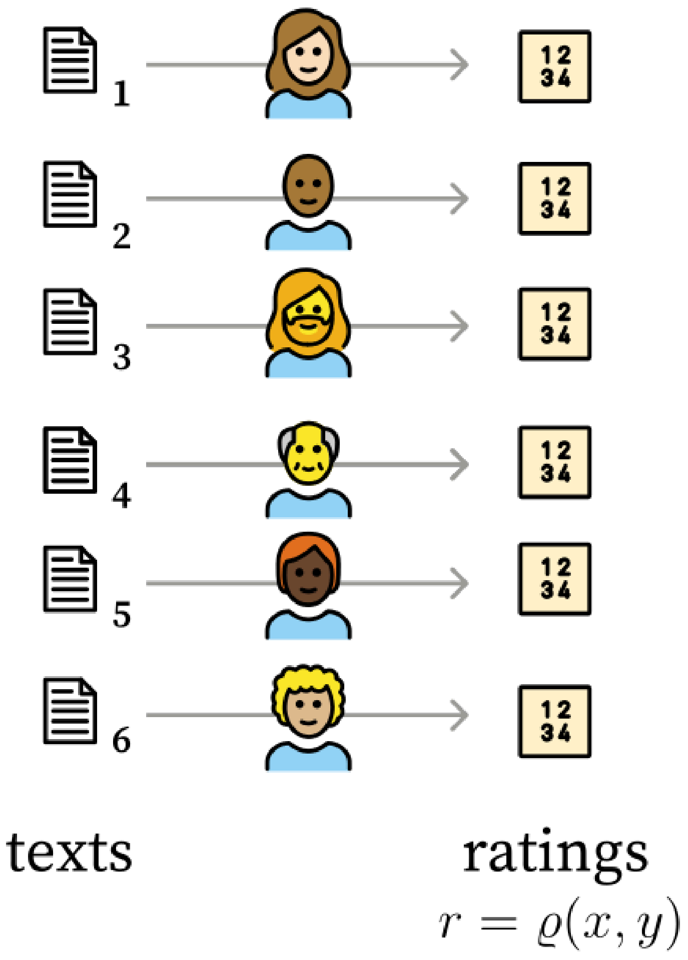
Underspending in MES (Aziz, Lederer, Peters, Peters, Ritossa, 2025)

Lecture 10 - Budget Aggregation

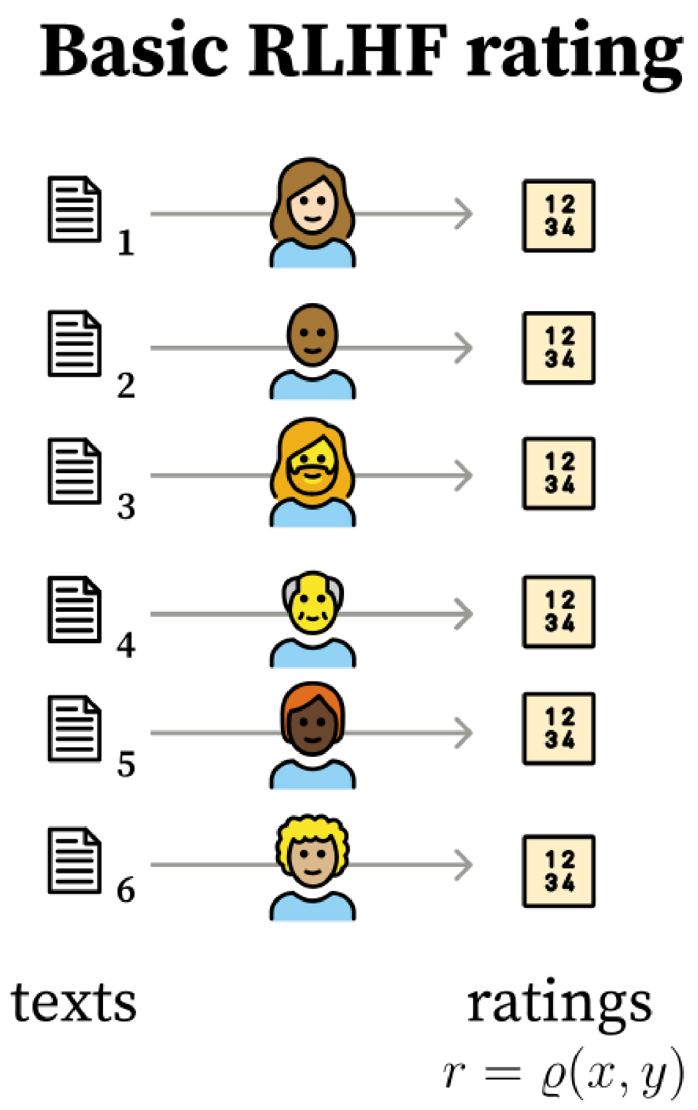
Guest lecture by Rupert Freeman
University of Virginia, Darden School of Business

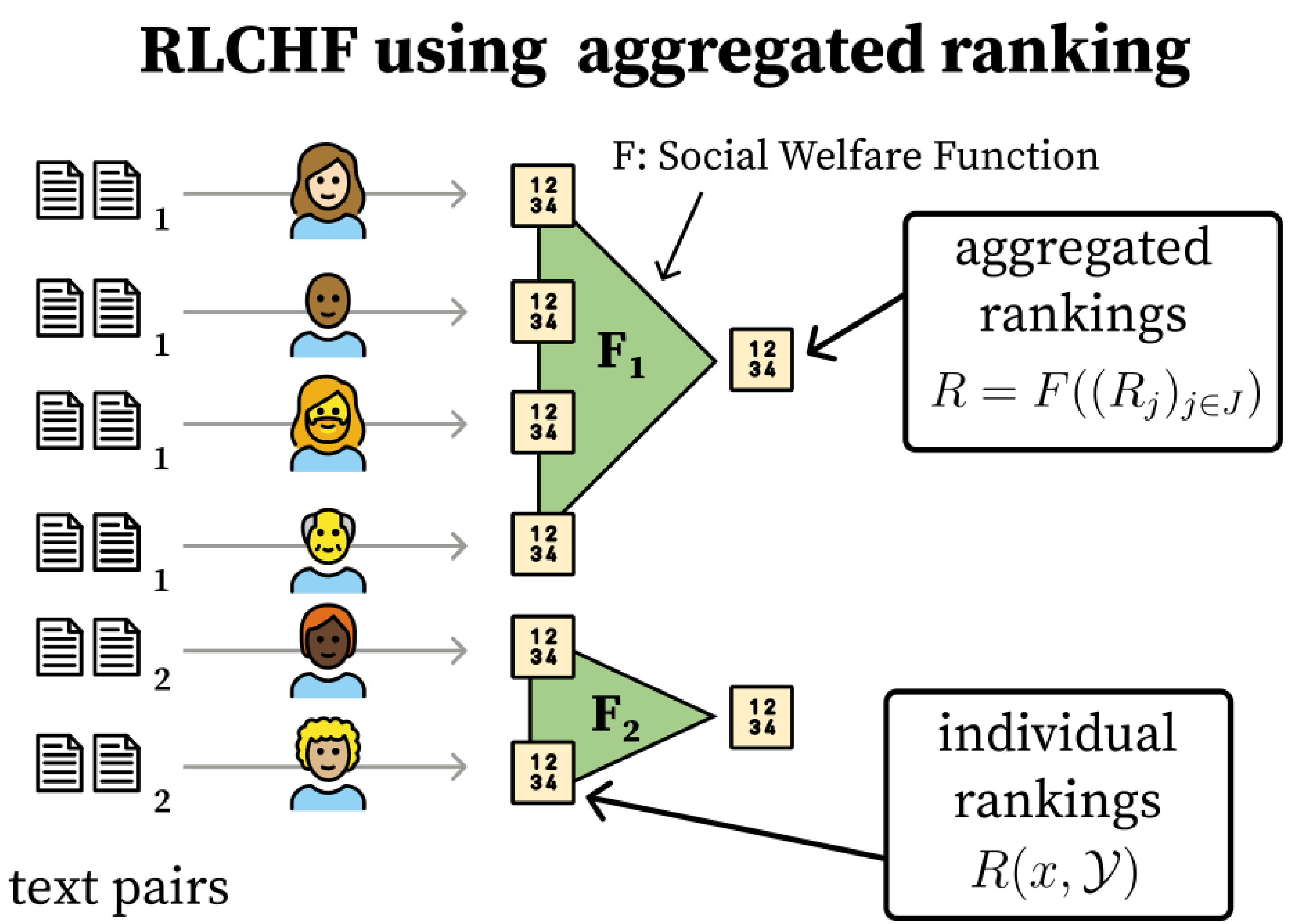
Basic RLHF rating

(Reinforcement Learning from Human Feedback)



Lecture 11 - Social Choice for AI Alignment





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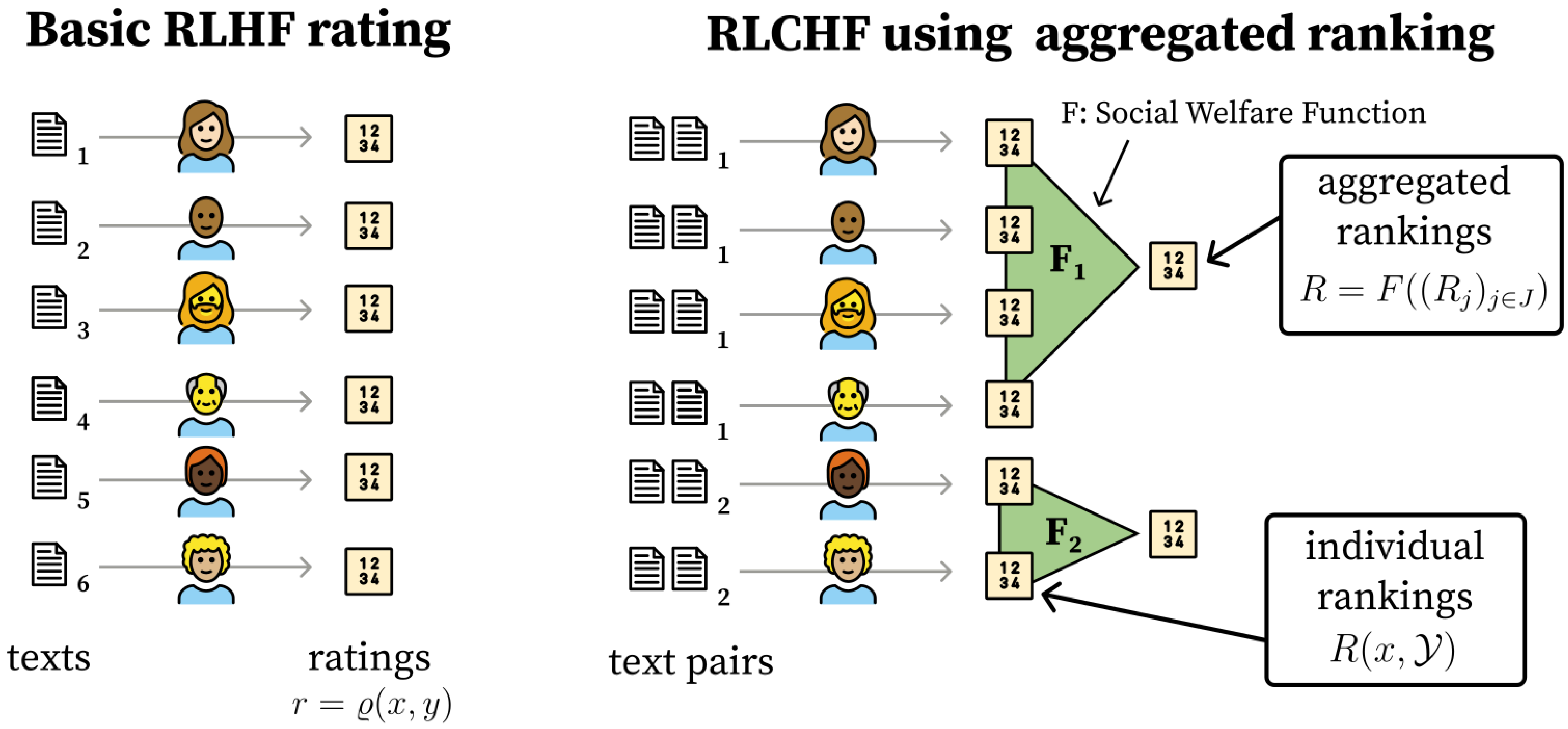

Borda is bad for RLHF (Siththaranjan, Laidlaw, Hadfield-Menell, 2024)

Lecture 12 - Content Moderation (?)

Guest lecture by Johan Ugander
Yale University, Statistics and Data Science

Lecture 13 - Fair Division 1

We have to divide *m* items are divided among *n* agents.
In an *envy-free (EF)* allocation, each agent likes their bundle the most.

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Example

	Item 1	Item 2	Item 3	Item 4	Item 5
Agent 1	7	3	2	2	6
Agent 2	7	5	5	5	7
Agent 3	20	3	3	3	3

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▸ **Is there an EF allocation?**

Respond at:
pollev.com/jtuckerfoltz255 or
bit.ly/jtfpoll or
text jtuckerfoltz255 to 37607

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An EF allocation does not exist, so we can either:

- Relax the axiom
- Consider randomized allocations.

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EFX exists for three agents (Chaudhury, Garg, Mehlhorn, 2020)

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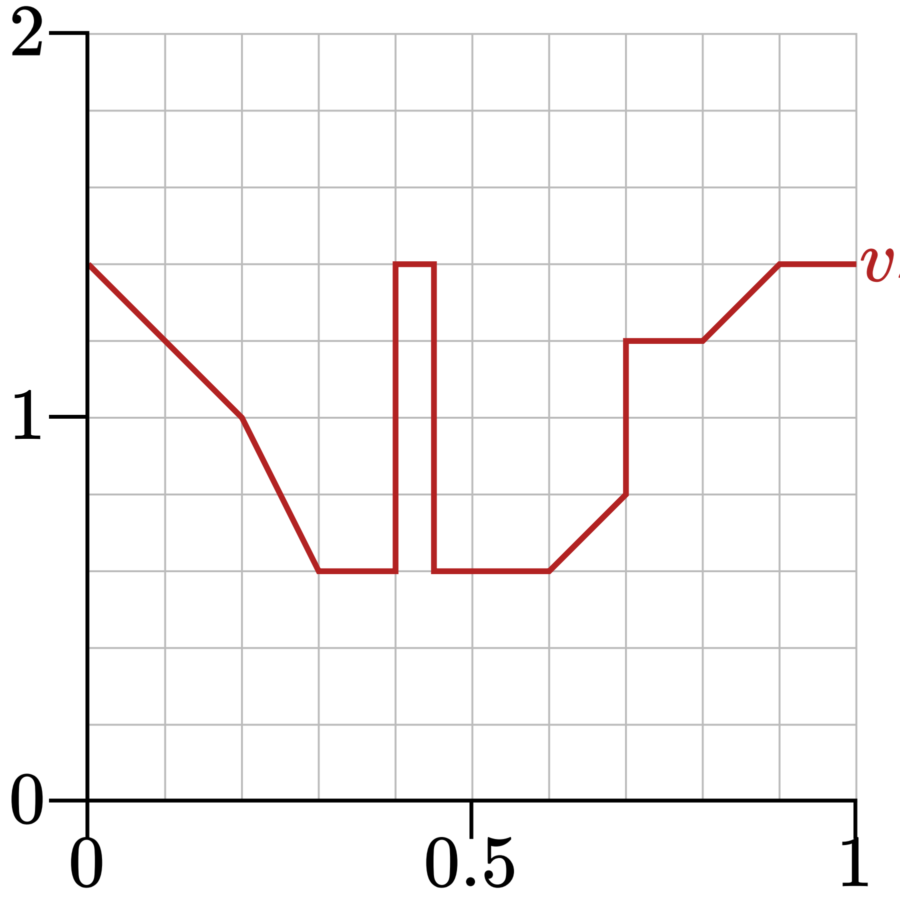
Cake-Cutting model: [0, 1] must be divided among agents with arbitrary value functions.



$$v_i(X) = \int_X f_i$$

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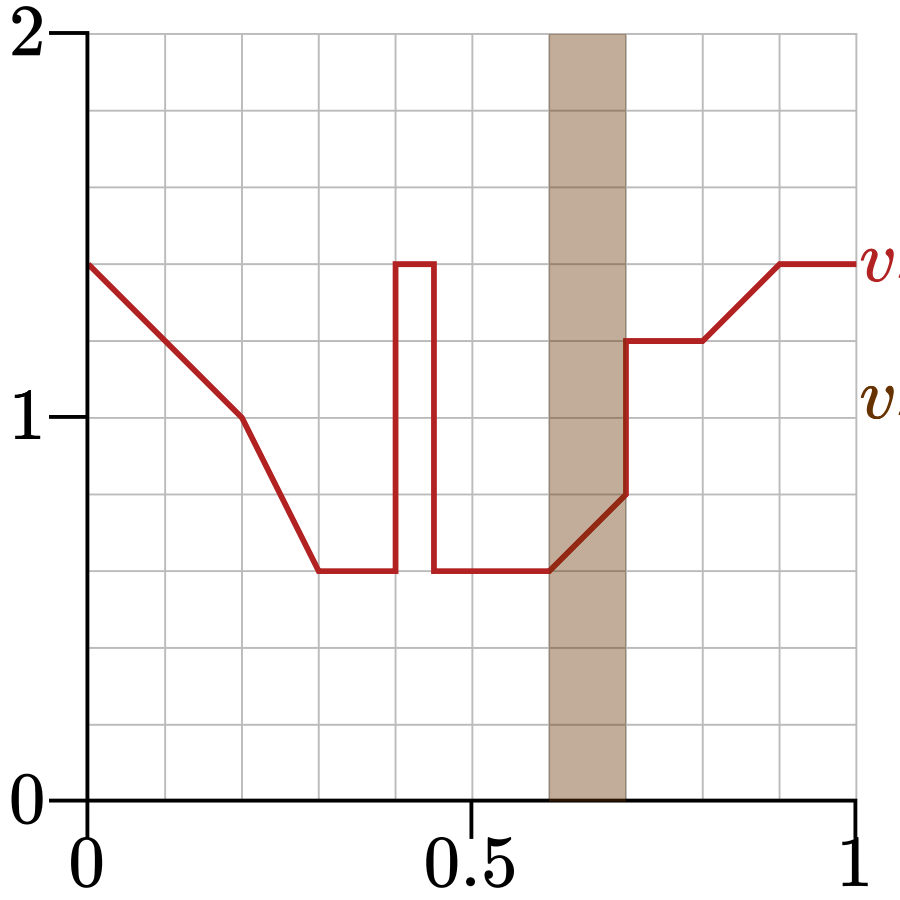

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$$v_i(X_1) = 0.07$$

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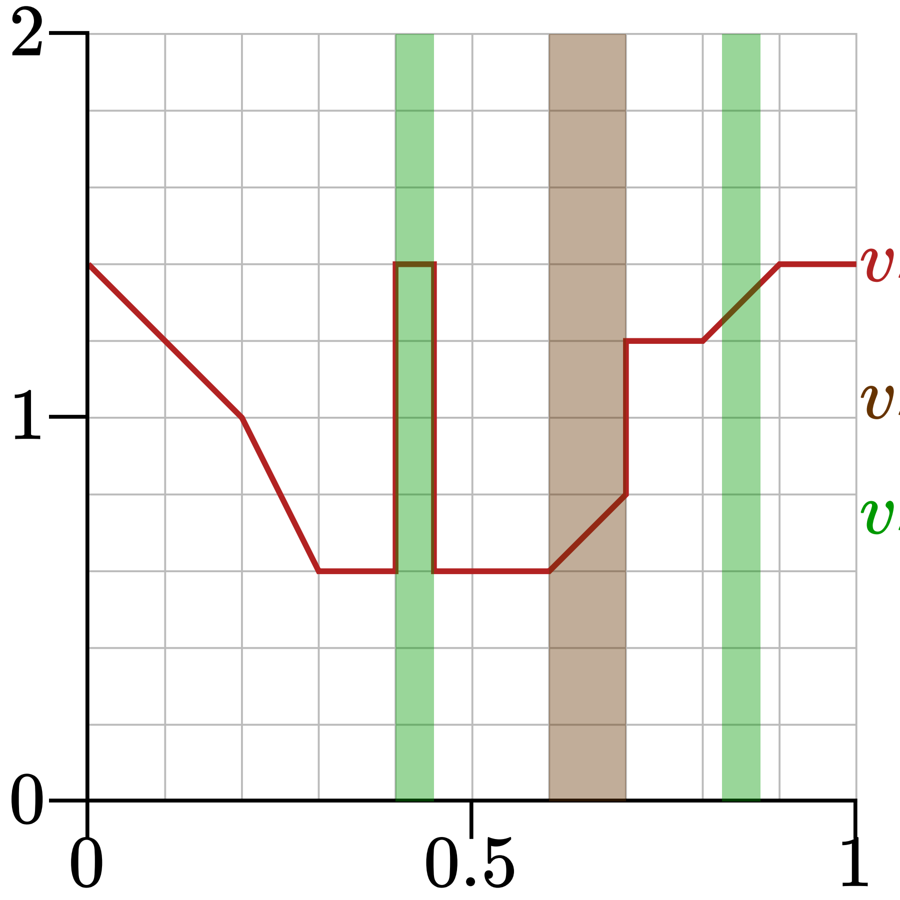

$v_i(X) = \int_X f_i$

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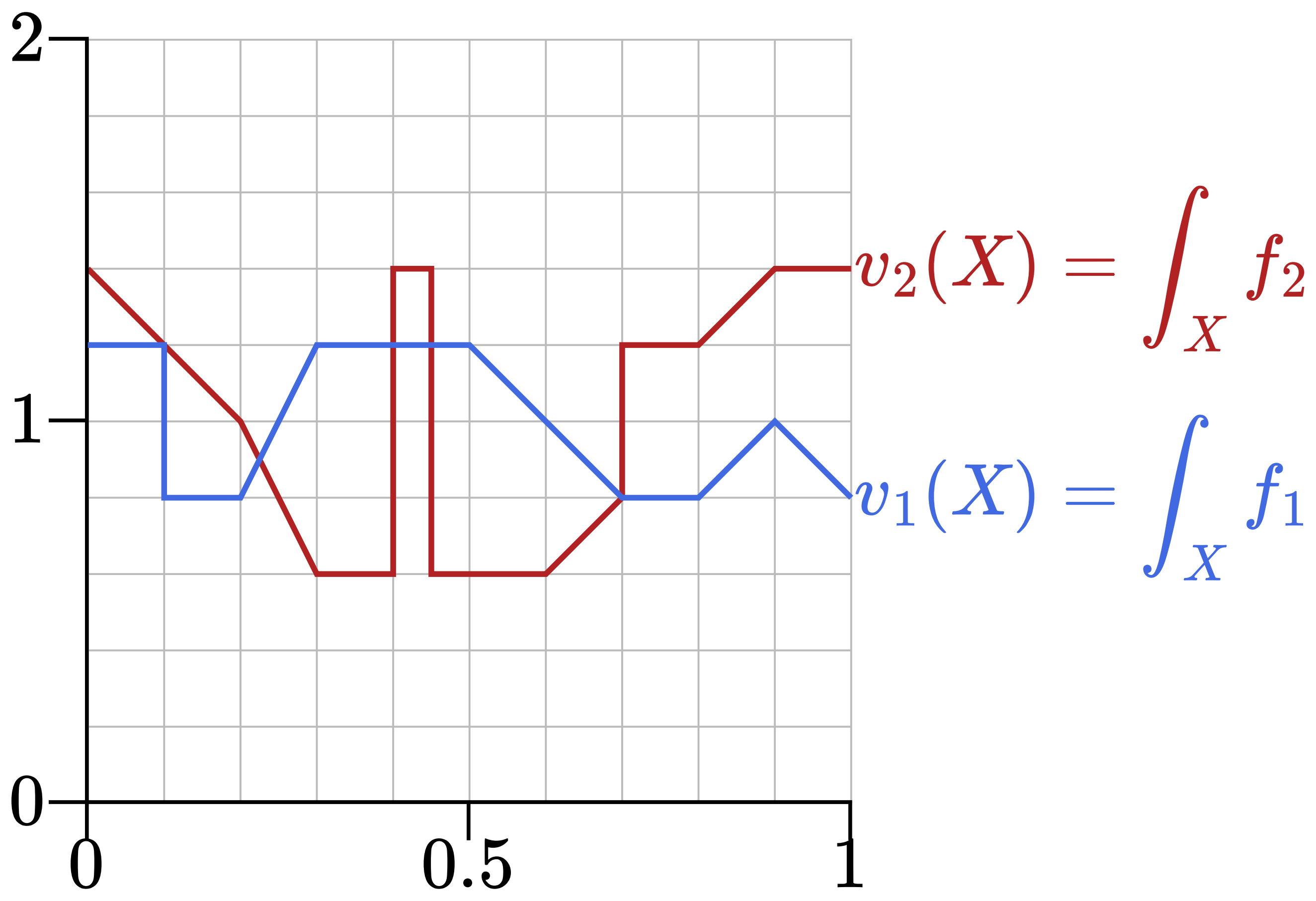

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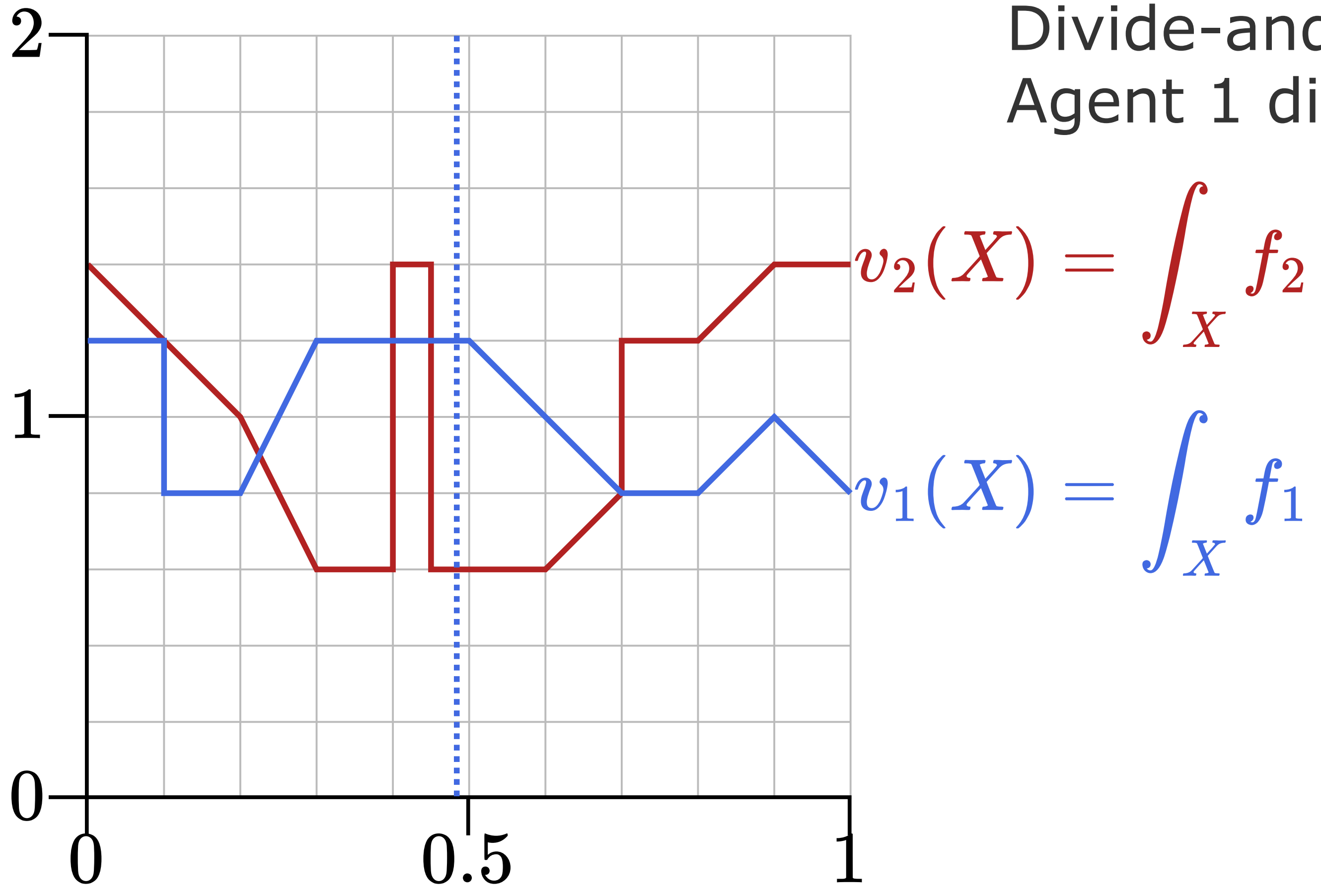

Divide-and-Choose method:
Agent 1 divides into equal pieces,

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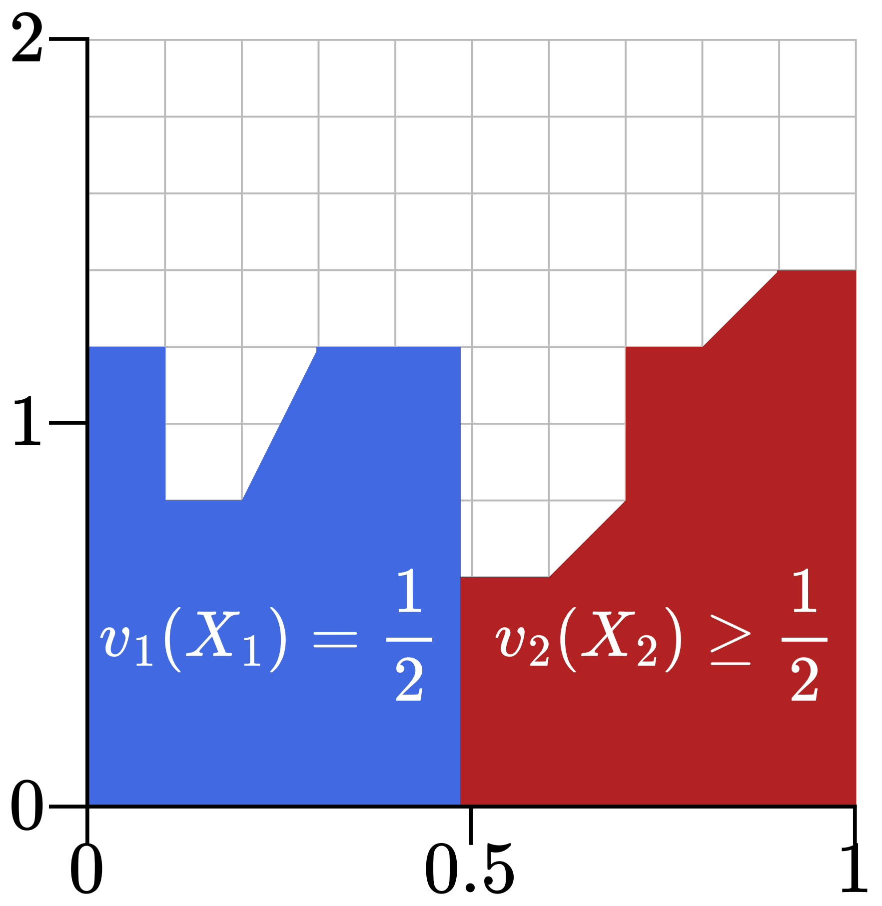

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Agent 1 divides into equal pieces,
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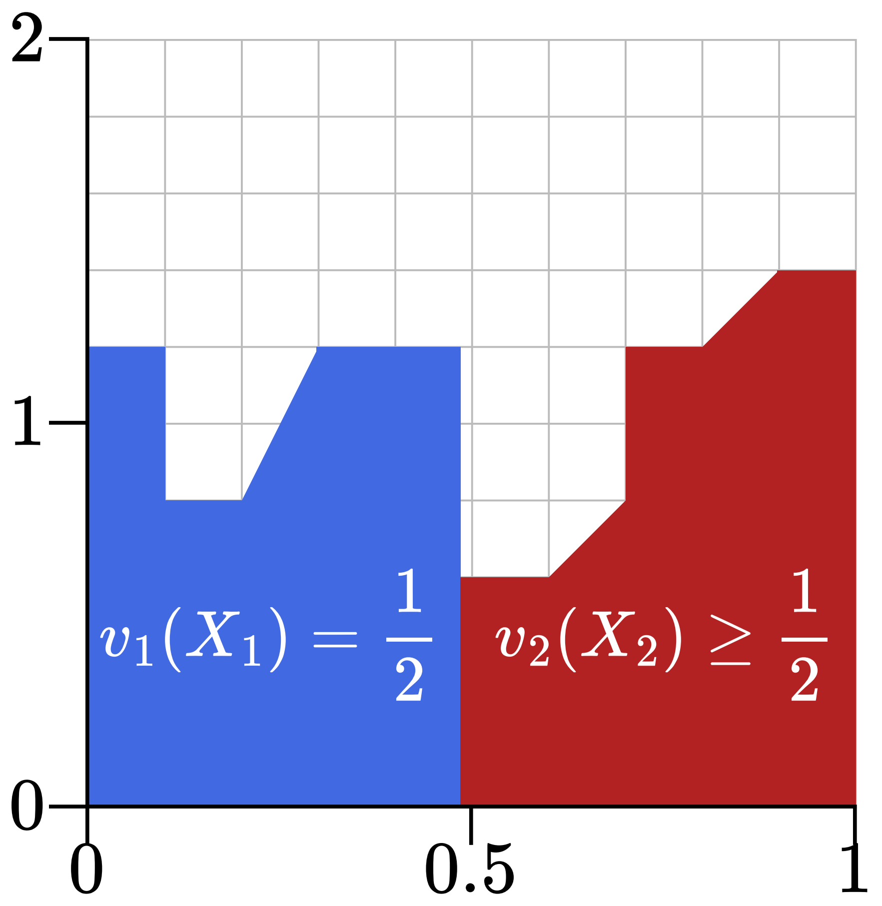

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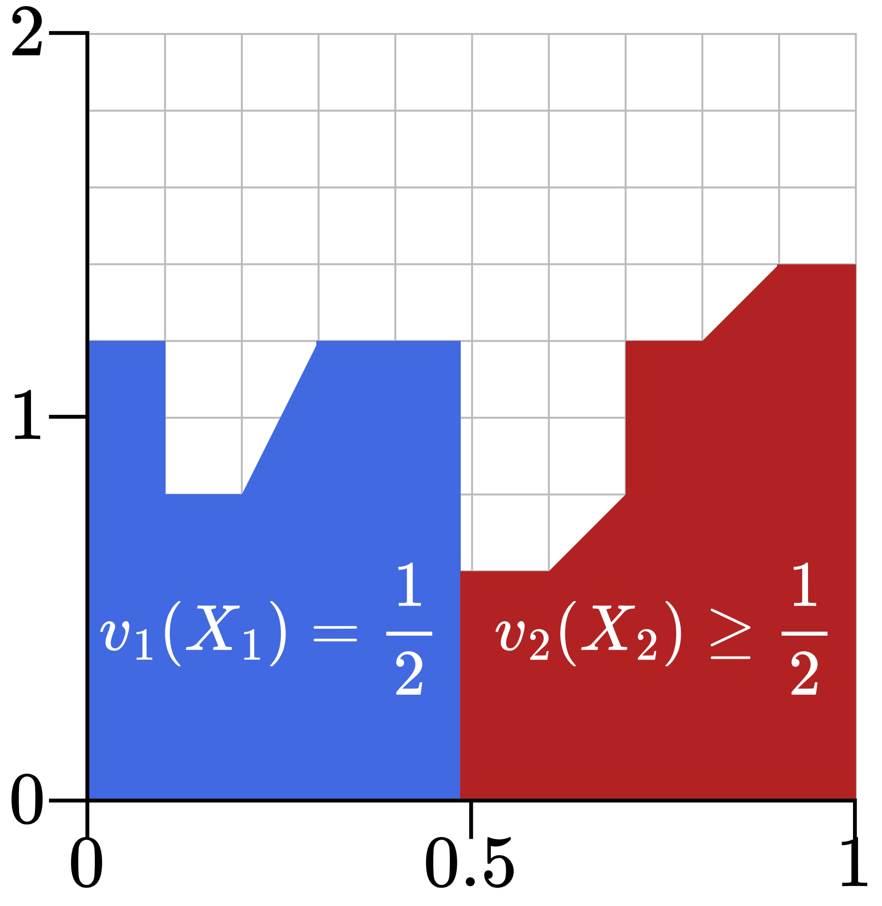

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Theorem (Borsuk-Ulam)

In any continuous function from the sphere to the plane, two antipodal points collide.

Lecture 14 - Fair Division 2

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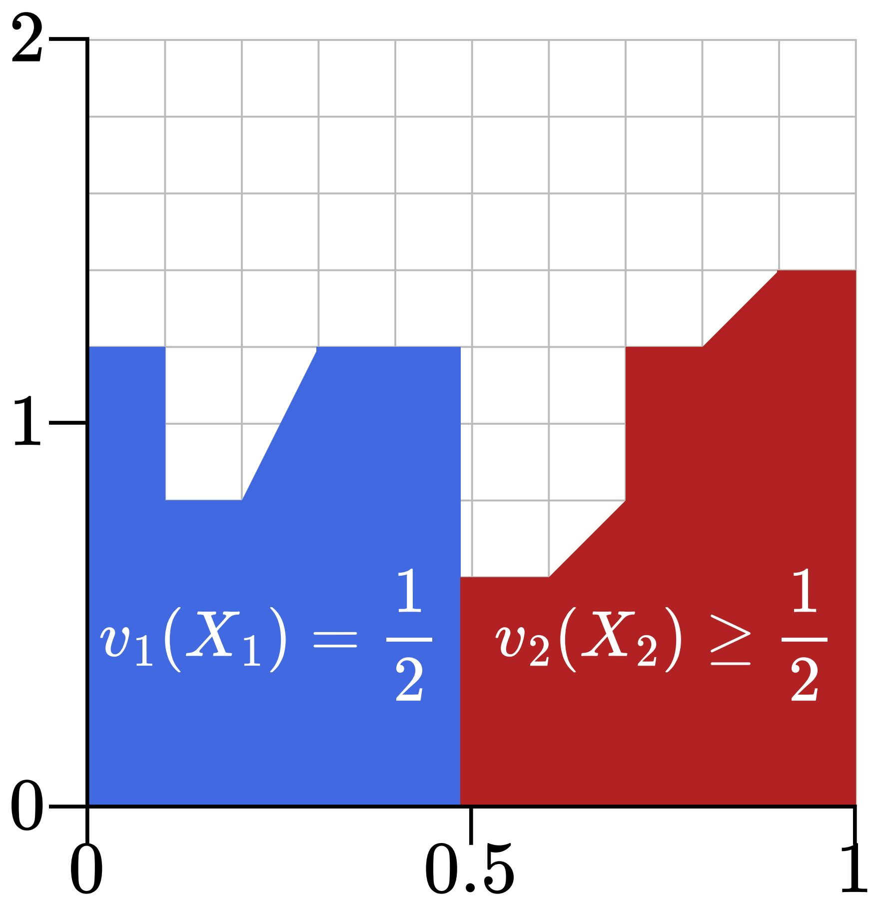

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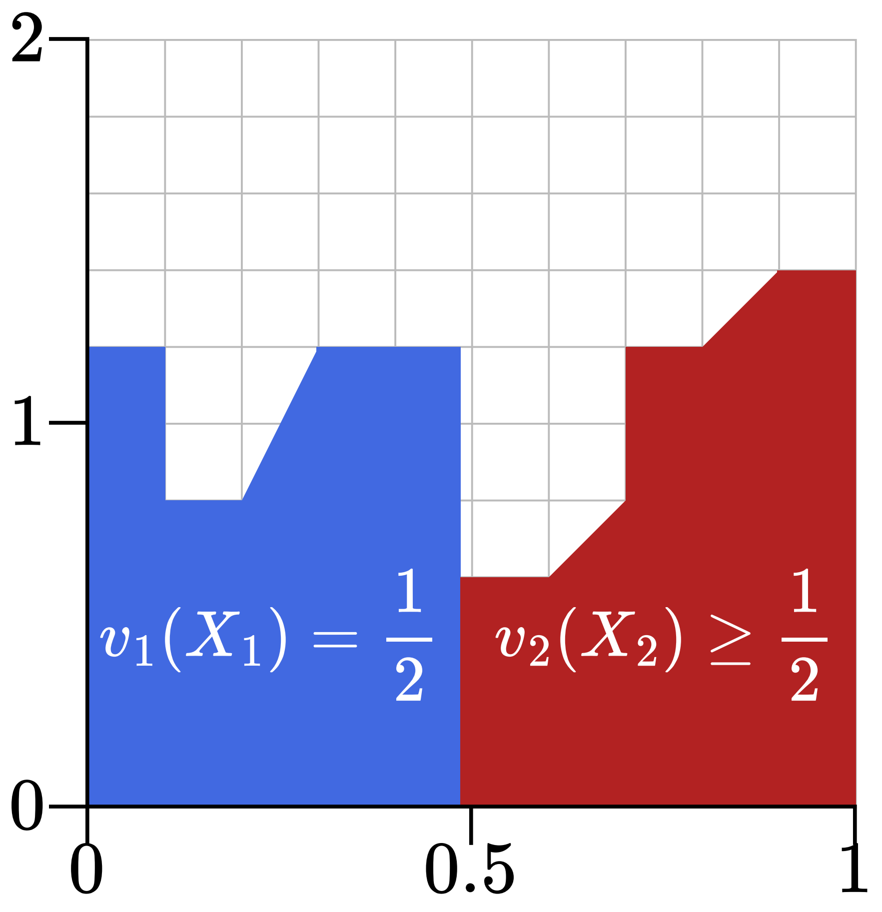

[Two sided matching fairness](#) (Freeman, Micha, Shah, 2021)

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Now Bob can win!

Lecture 16 - Liquid Democracy

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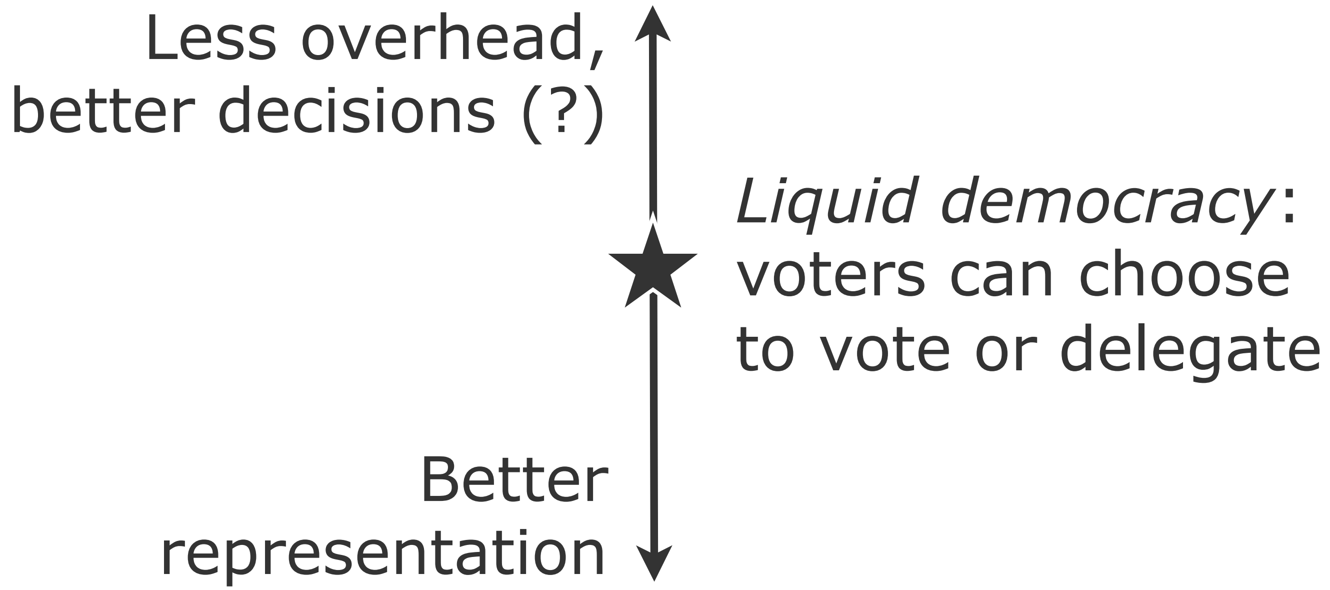

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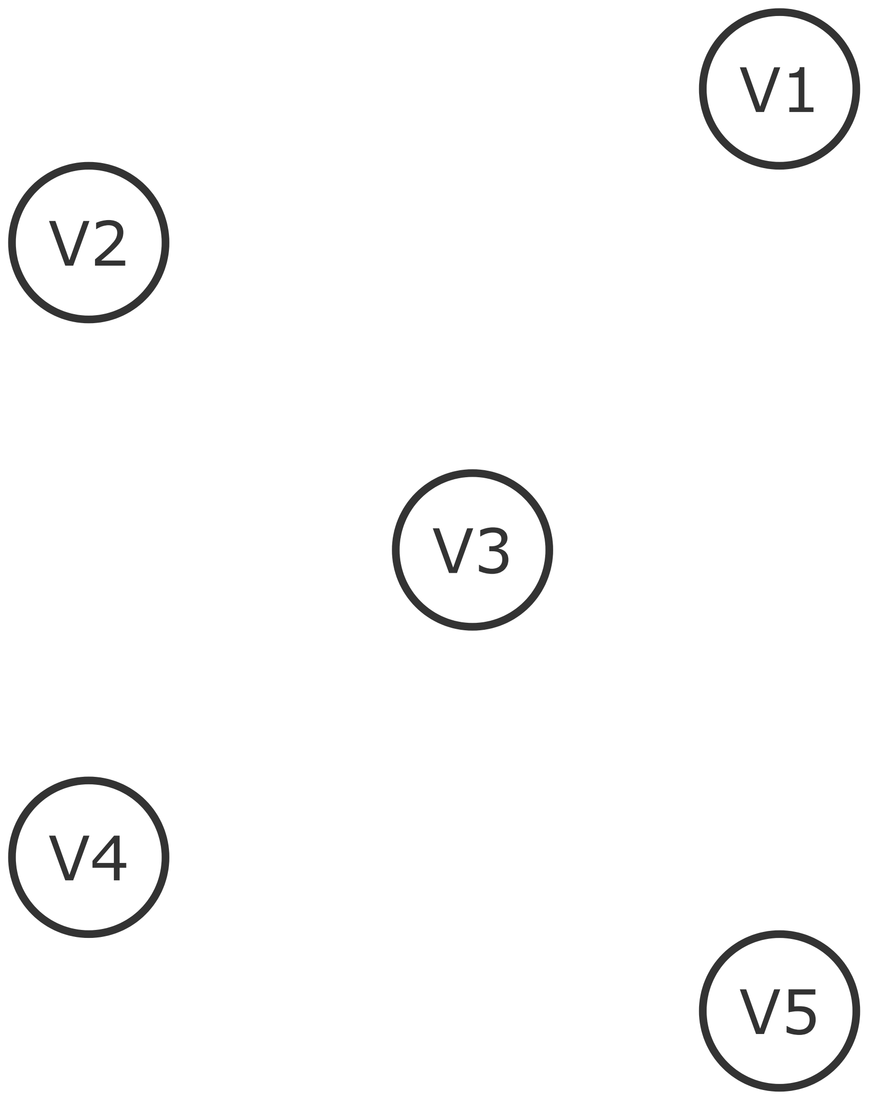

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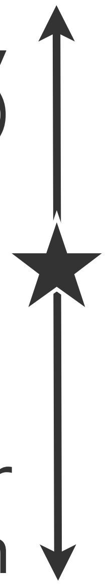

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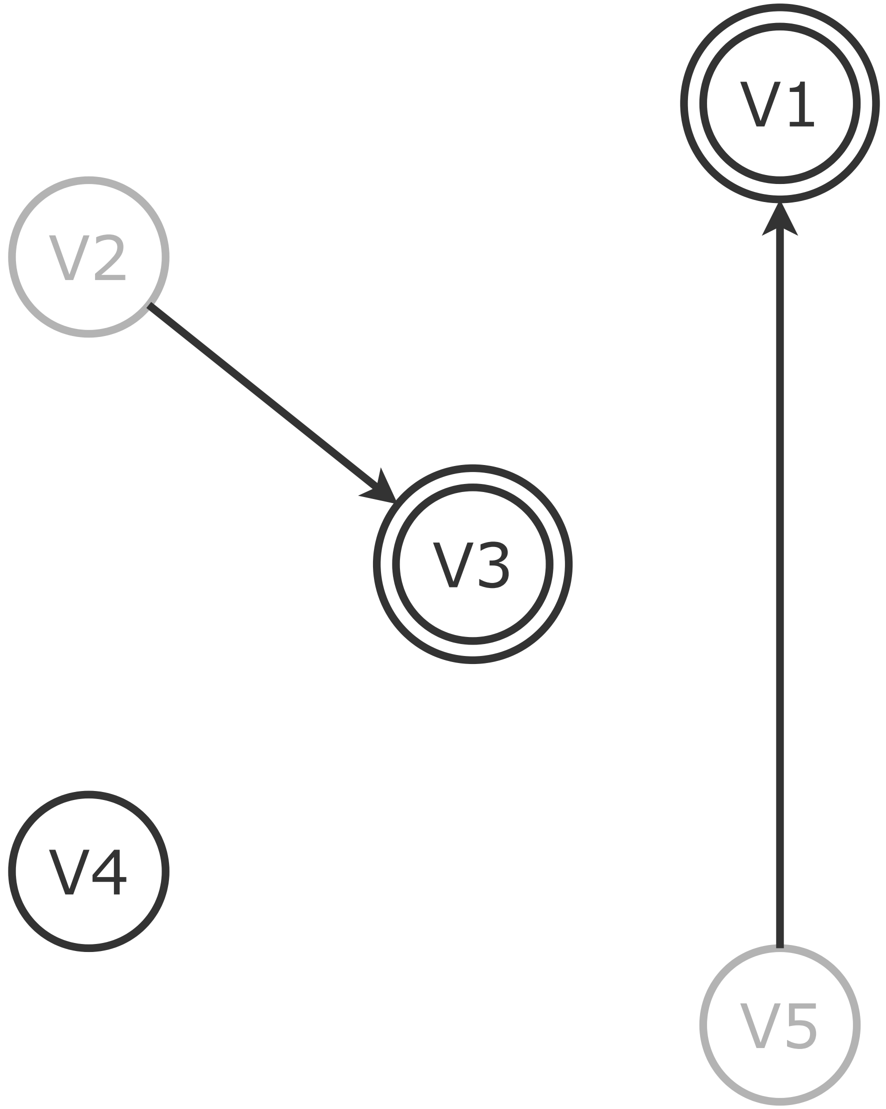

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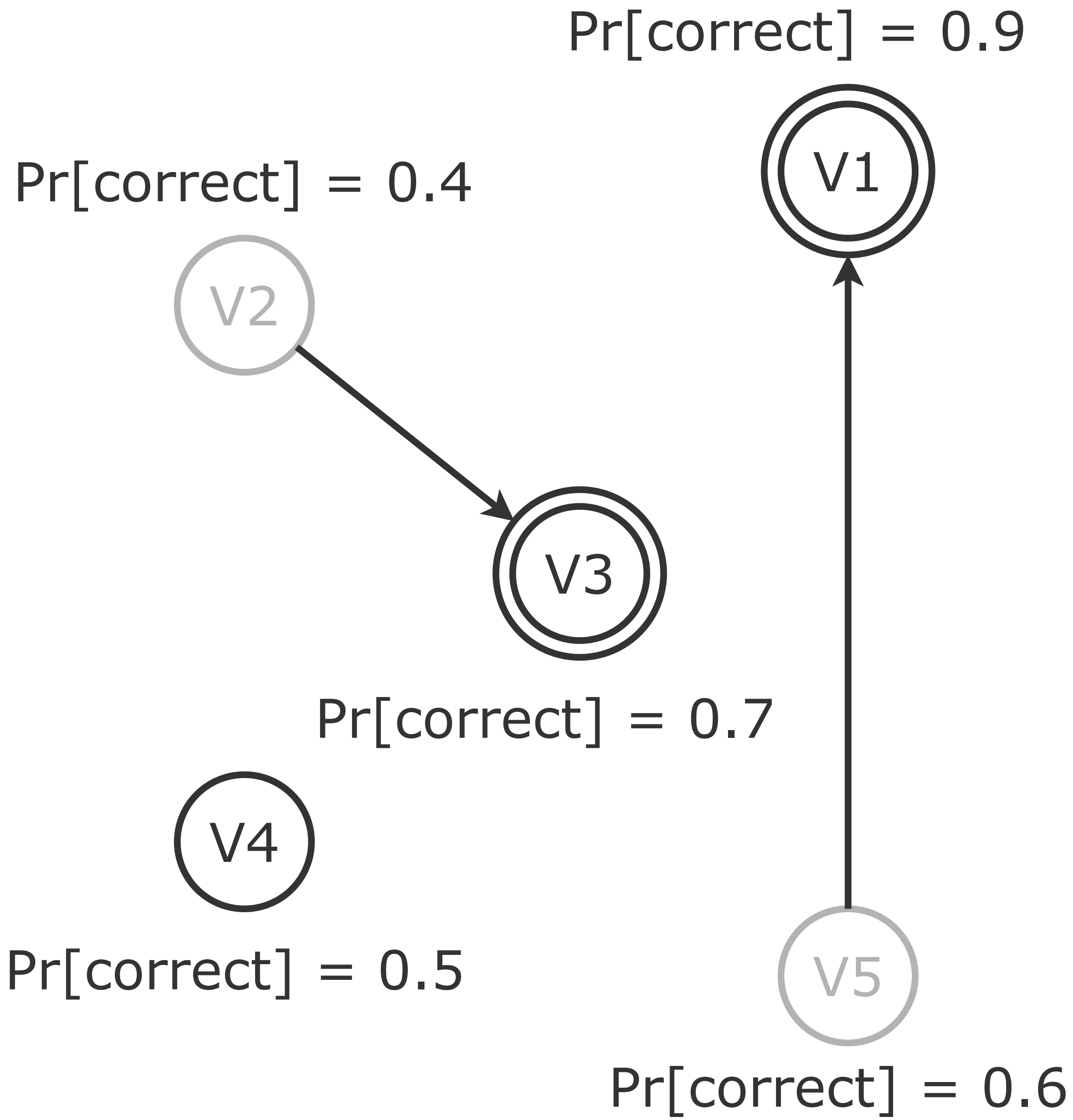

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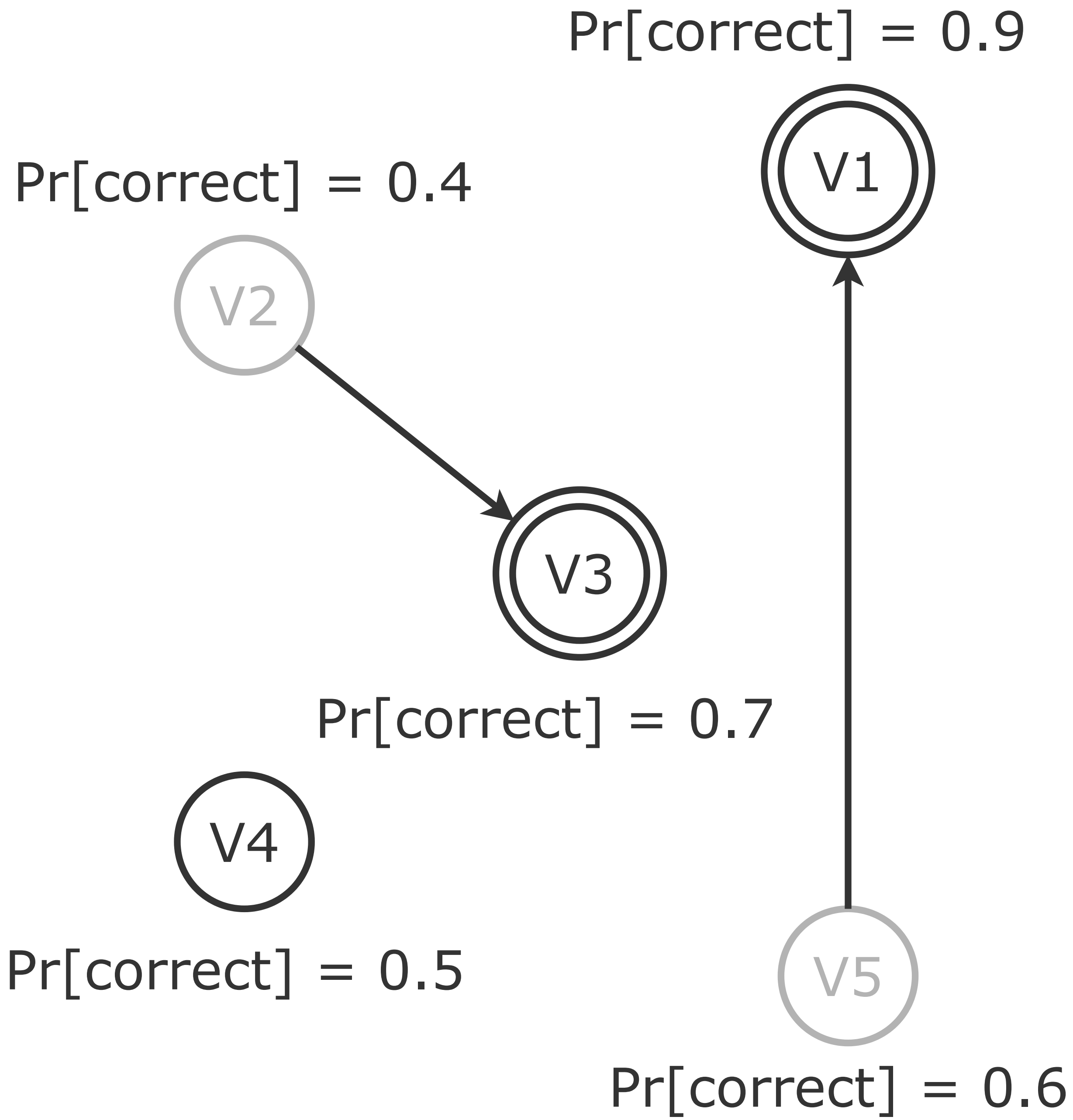


[Tracking truth with liquid democracy](#) (Berinksi et al., 2025)

Lecture 17 - Citizens Assemblies

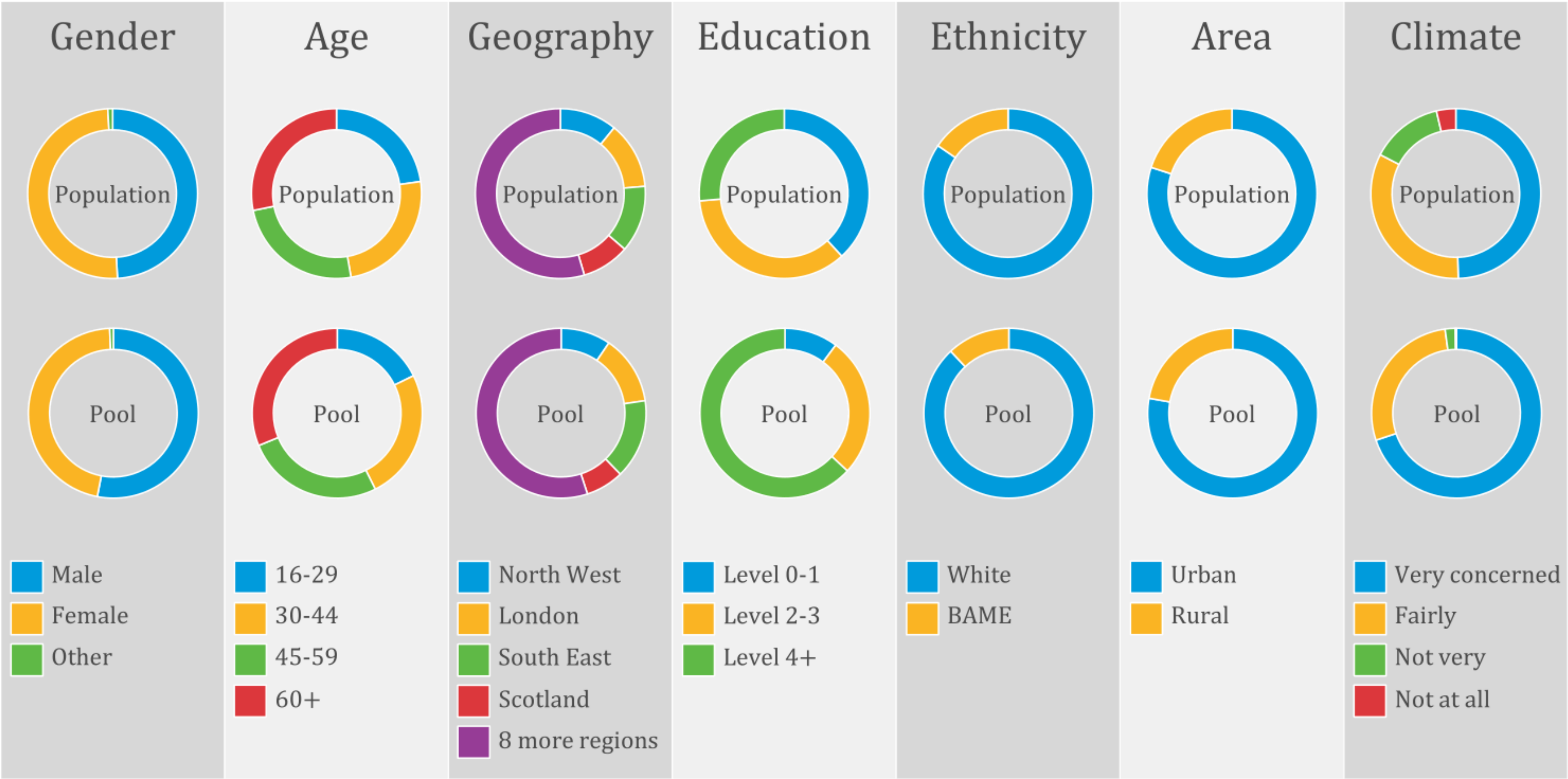
Governments and parliaments around the world are increasingly using citizens' assemblies in their work.

What is a citizens' assembly?

A citizens' assembly is a group of people who are brought together to learn about and discuss an issue or issues, and reach conclusions about what they think should happen.

The people who take part are chosen so they reflect the wider population – in terms of demographics (e.g. age, gender, ethnicity, social class) and sometimes relevant attitudes (e.g. their views on climate change).

Lecture 17 - Citizens Assemblies



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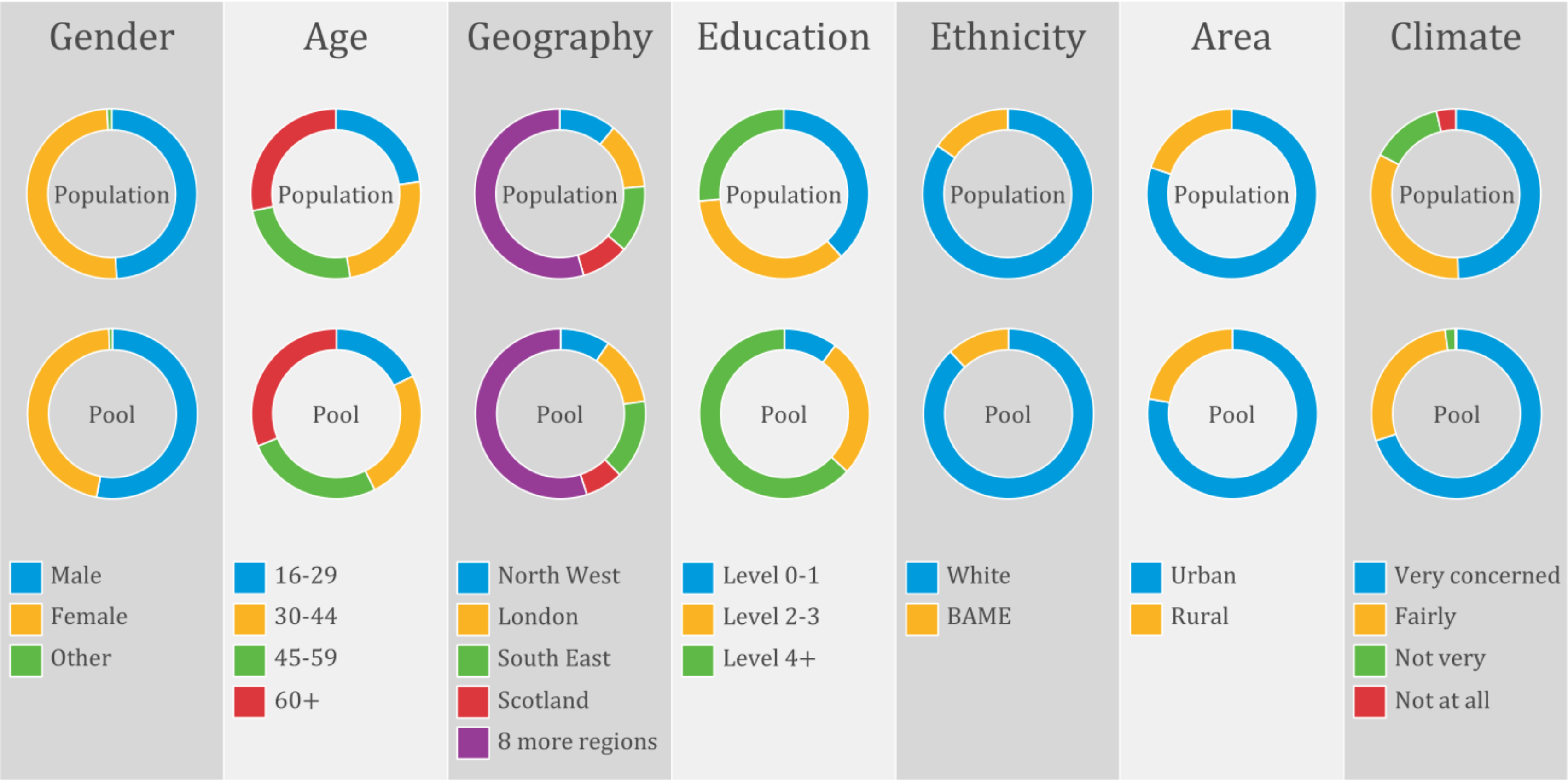

How to design the selection algorithm?
Tradeoffs between:

Governments and parliaments around the world are increasingly using citizens' assemblies in their work.

What is a citizens' assembly?

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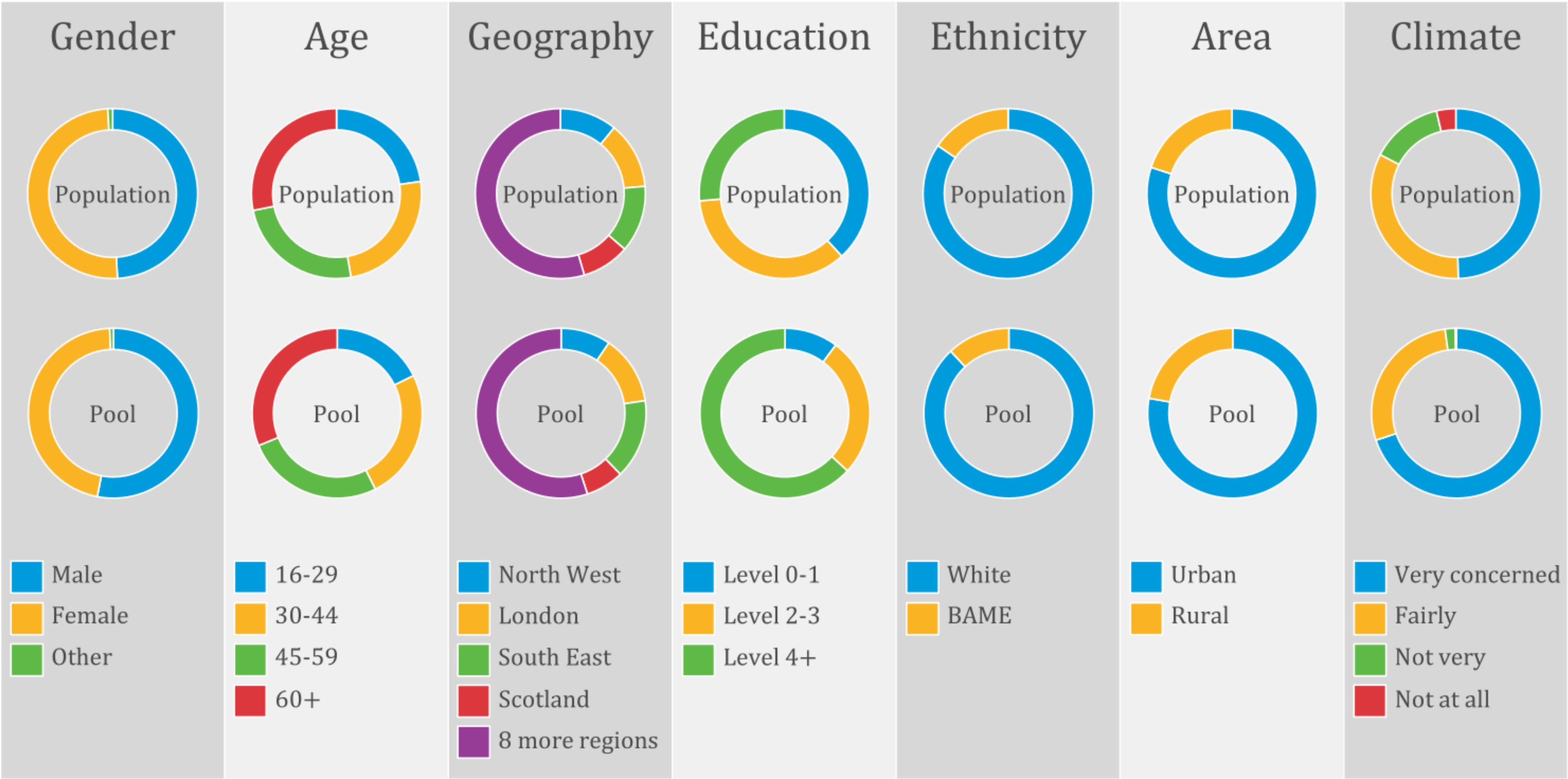

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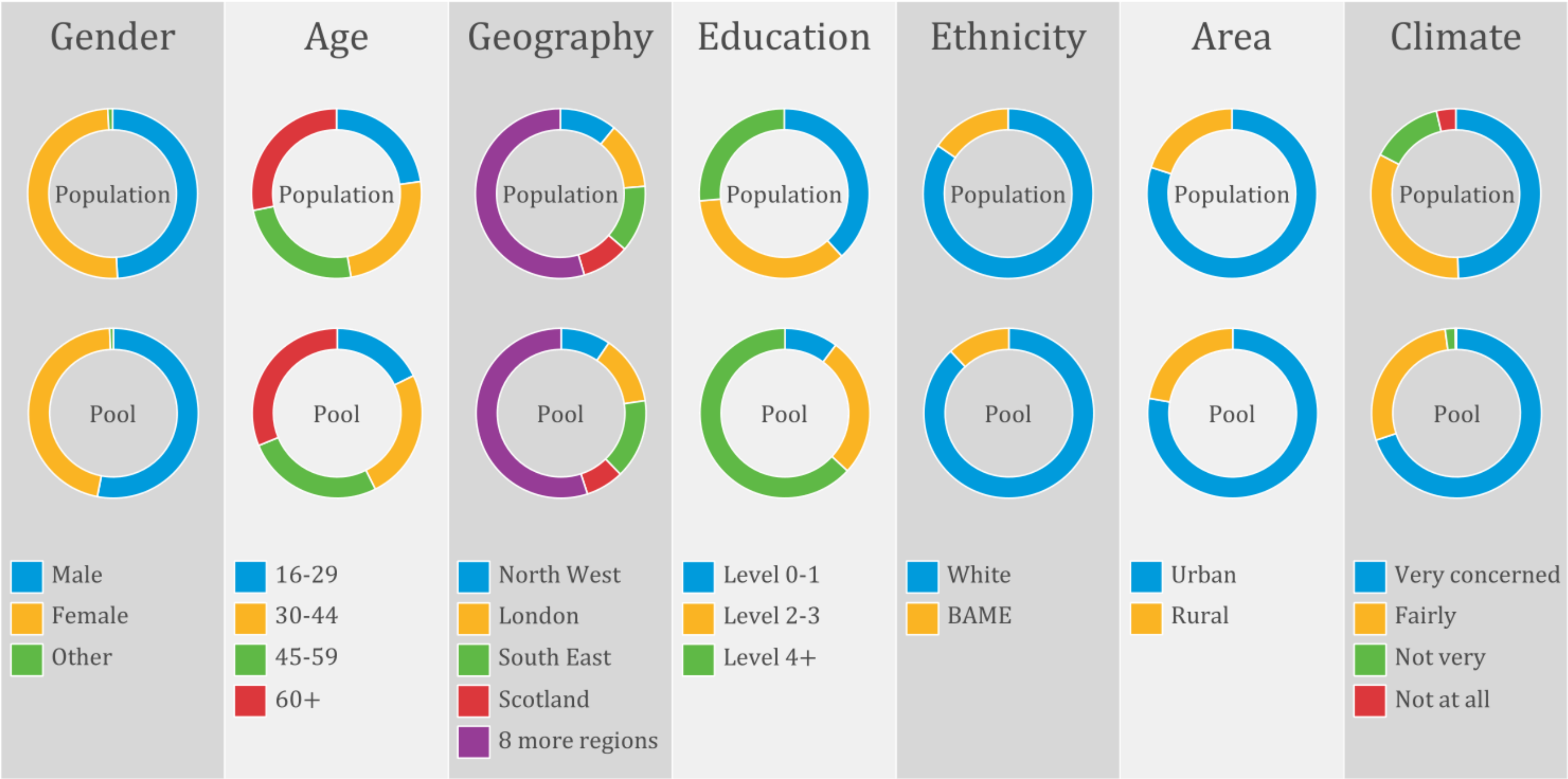

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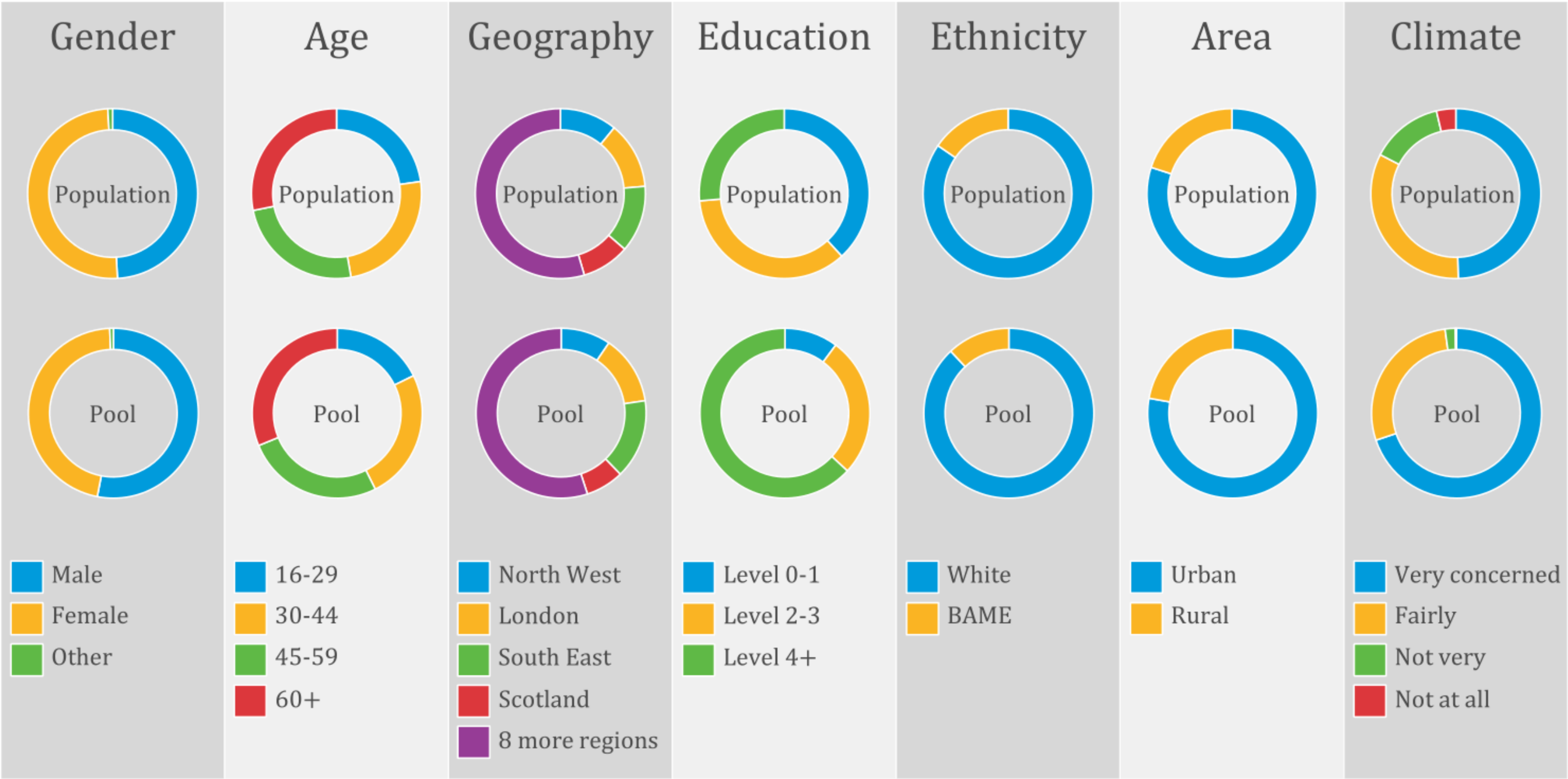

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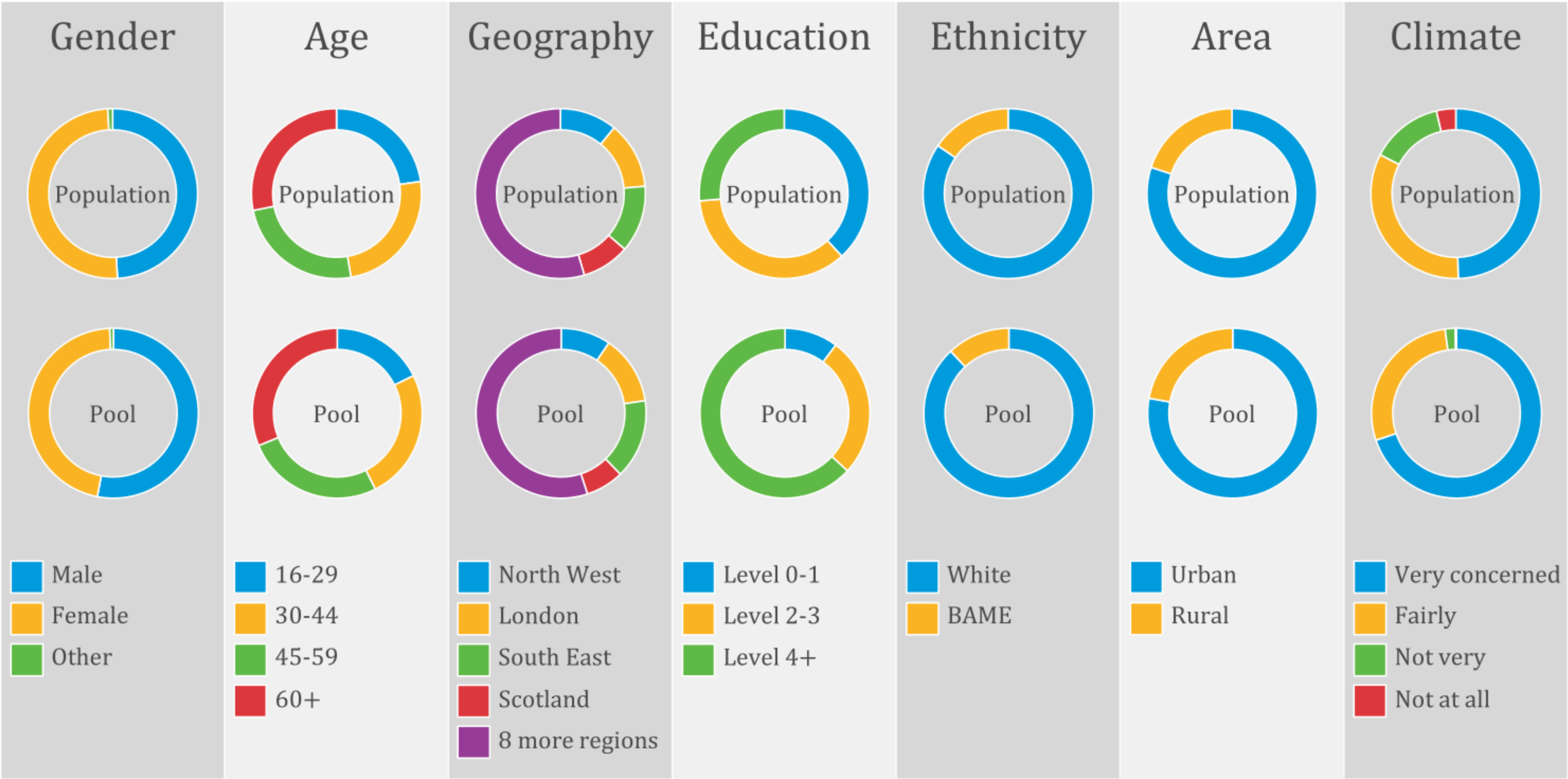

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[Goldilocks (Baharav, Flanigan, 2024)]

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Lecture 18: Deterministic Apportionment

How do you round a list of n rational numbers to integers while preserving their sum?

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How do you round a list of *n* rational numbers to integers while preserving their sum?

Party 1 votes: 110

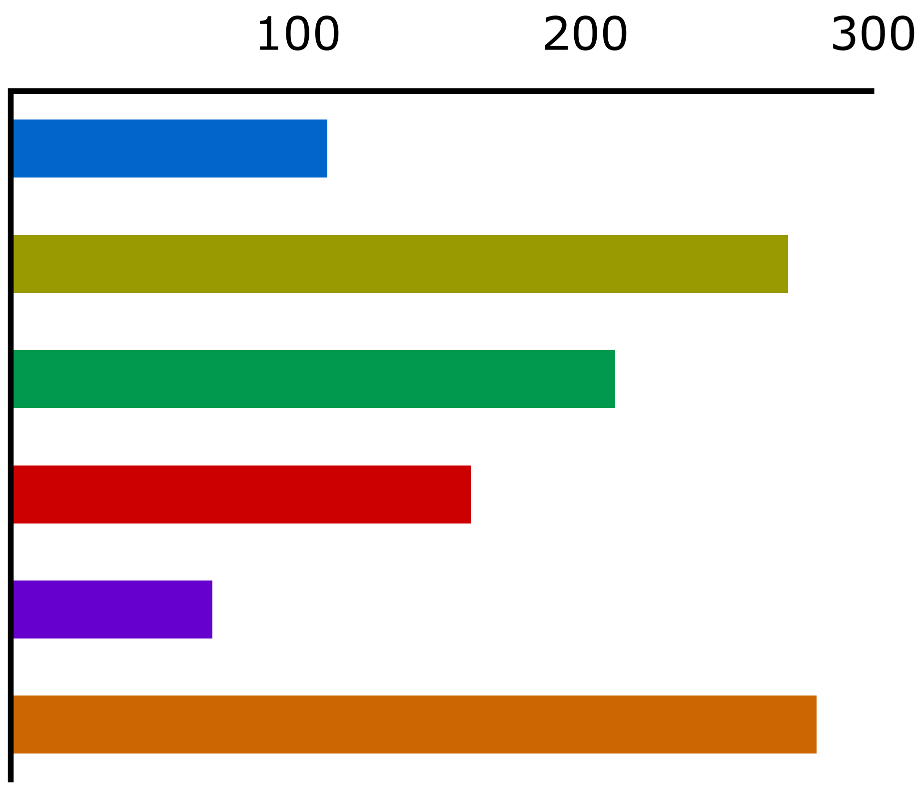
Party 2 votes: 270

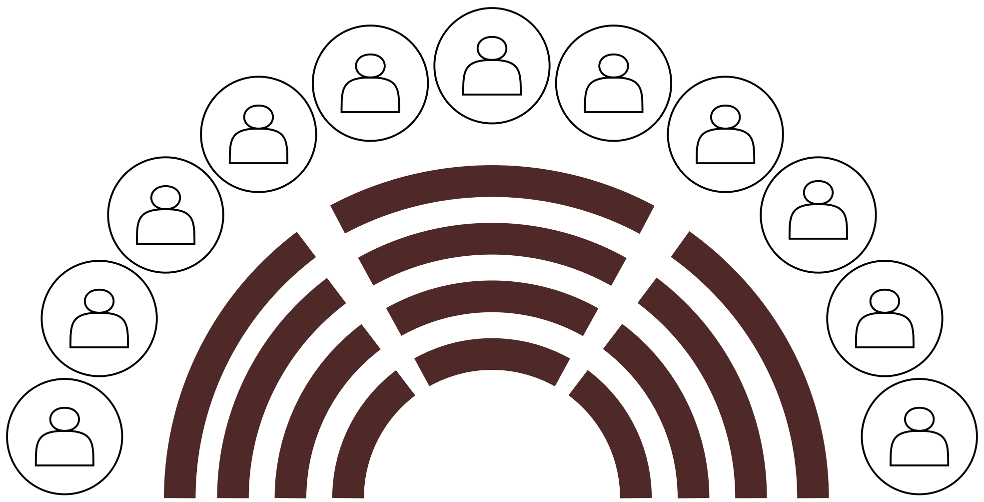
Party 3 votes: 210

Party 4 votes: 160

Party 5 votes: 70

Party 6 votes: 280





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How do you round a list of *n* rational numbers to integers while preserving their sum?

Party 1 share: 1.1

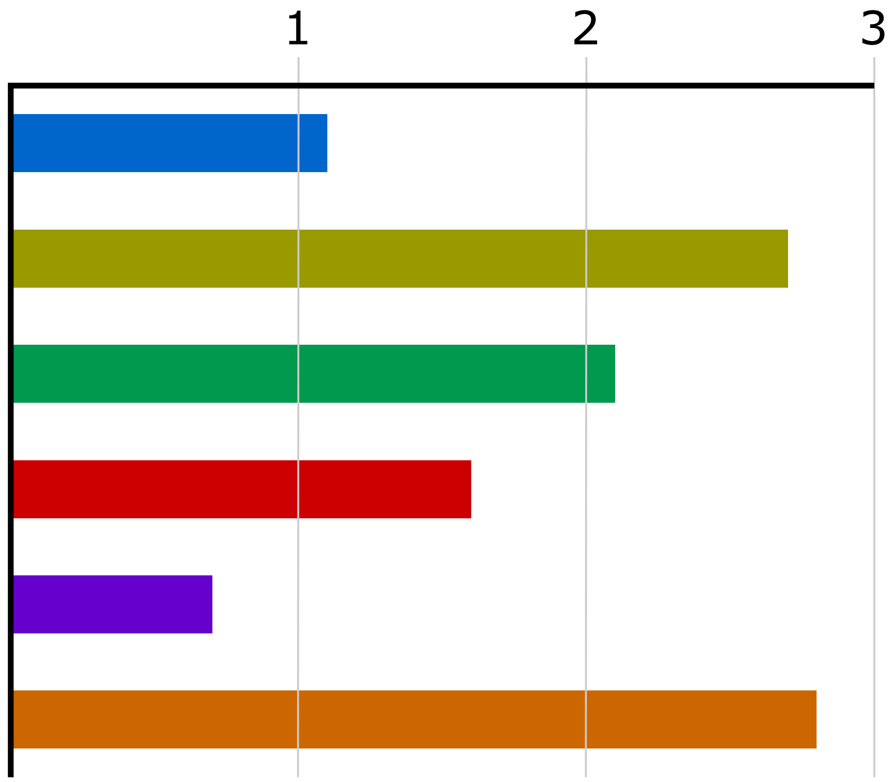
Party 2 share: 2.7

Party 3 share: 2.1

Party 4 share: 1.6

Party 5 share: 0.7

Party 6 share: 2.8



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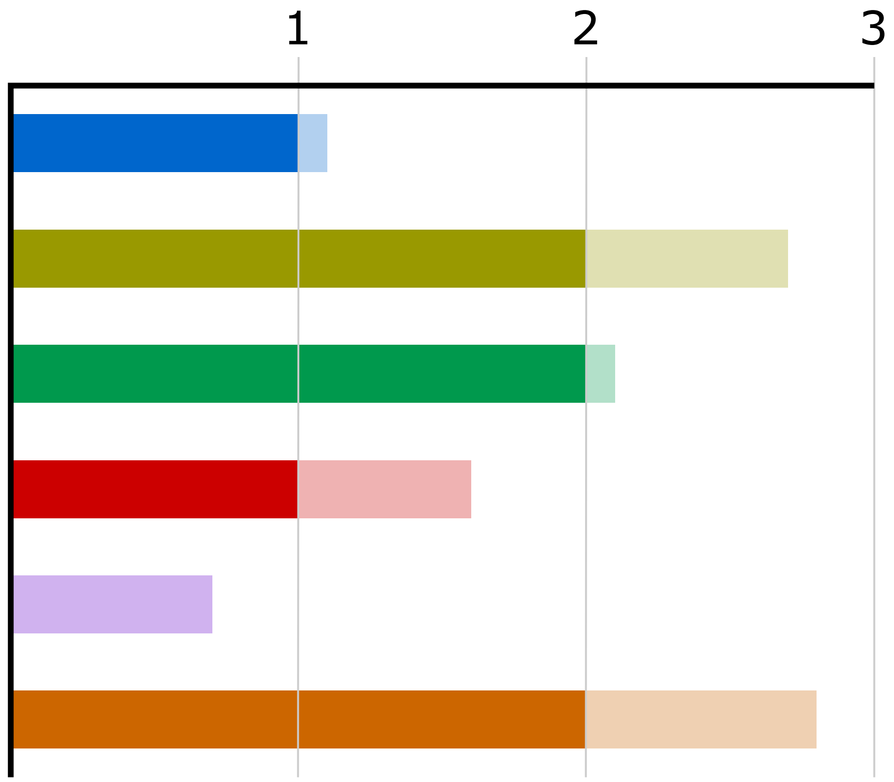

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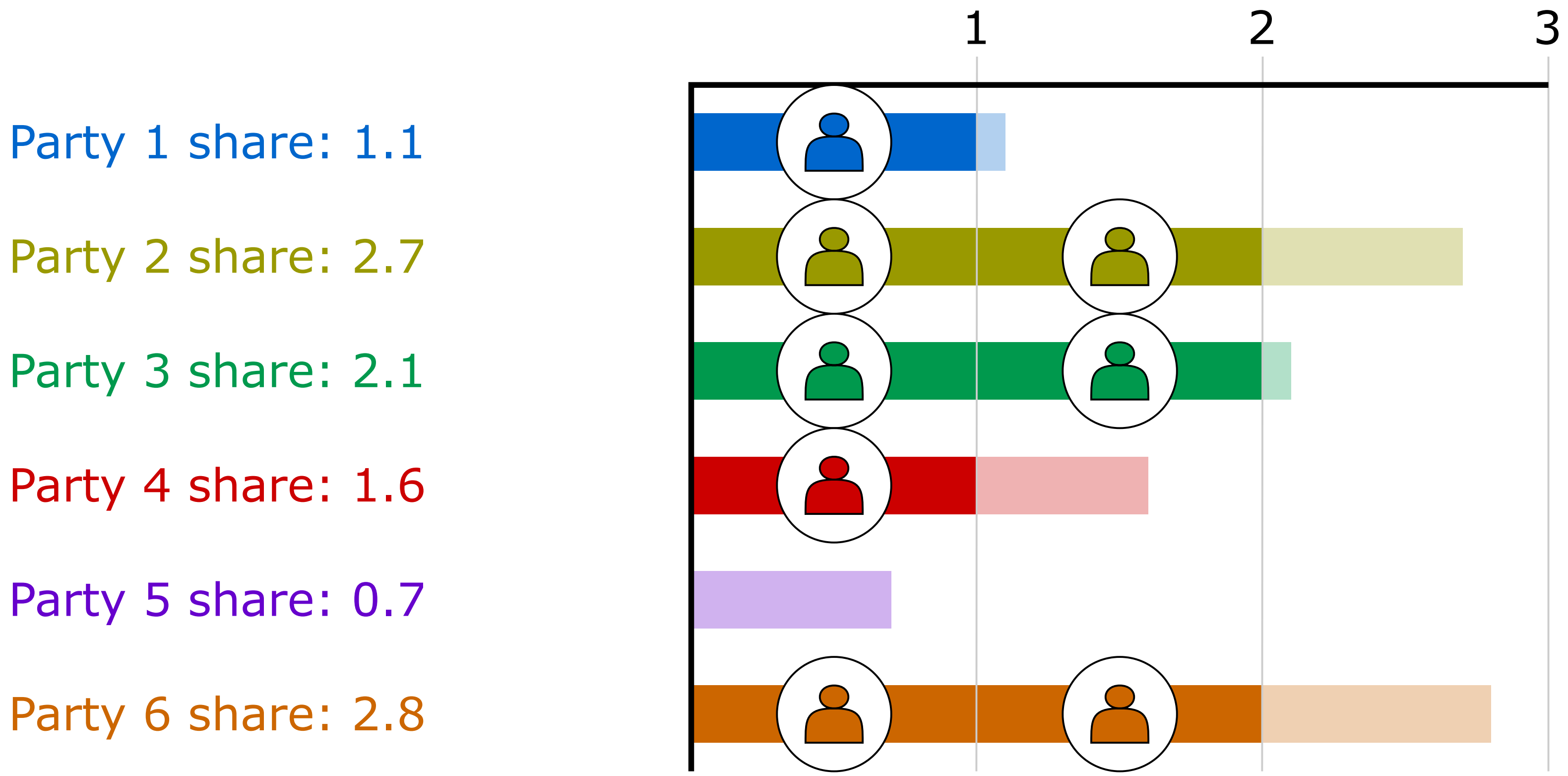

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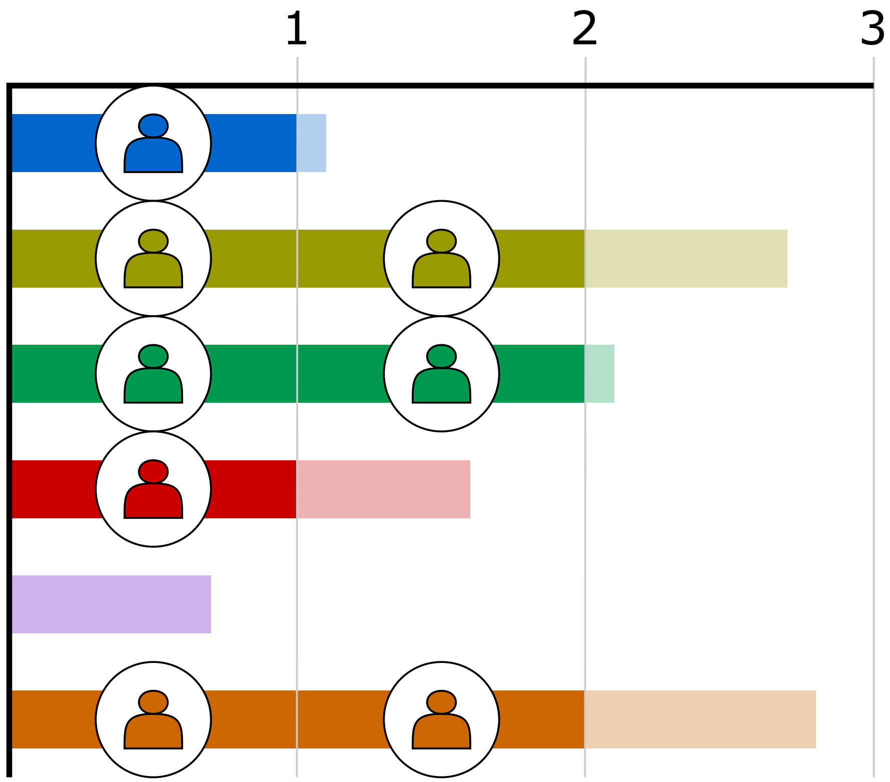

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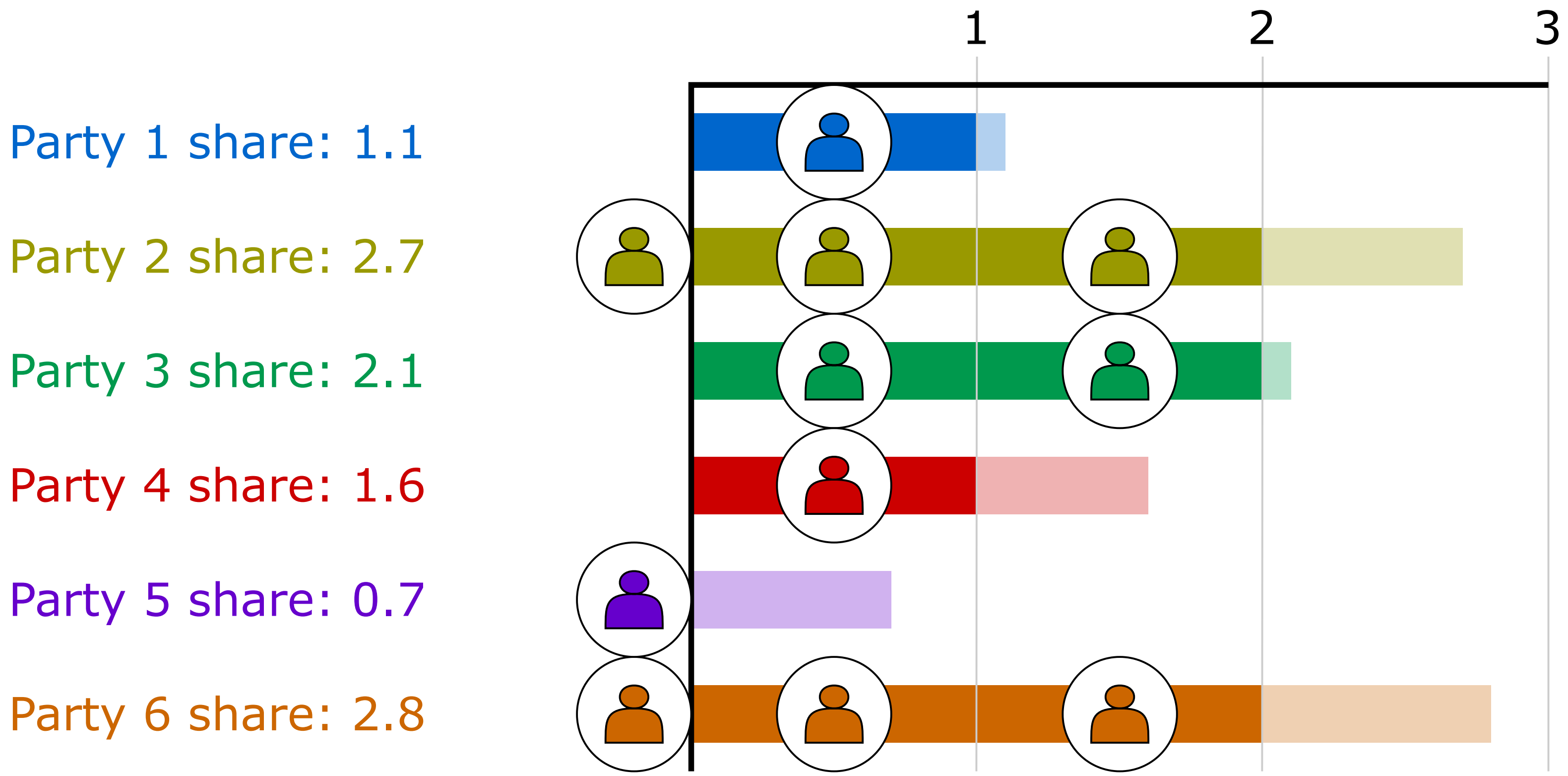

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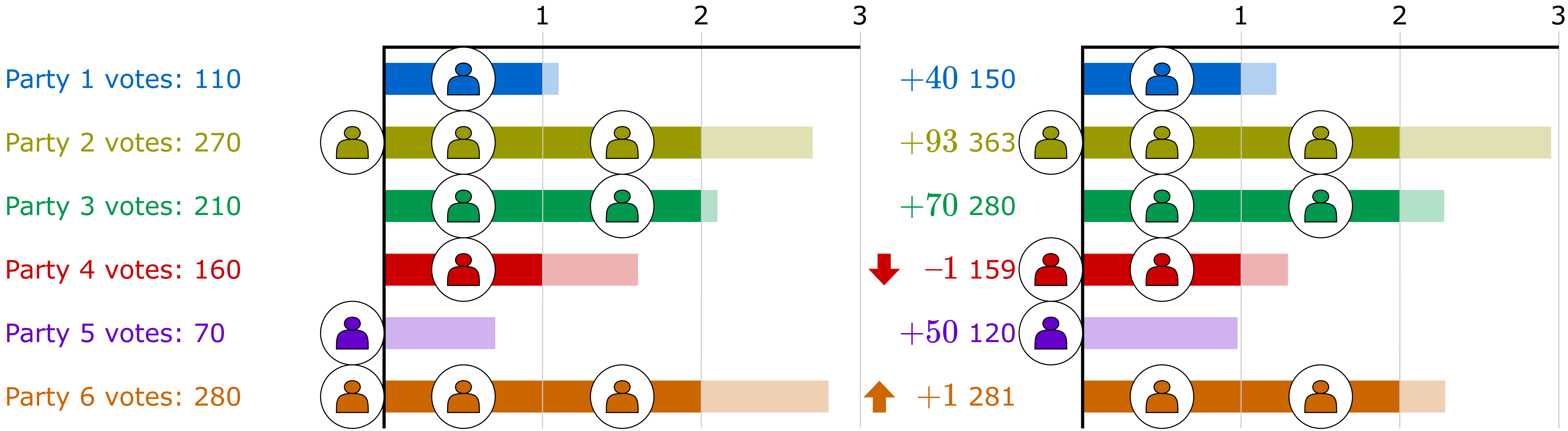

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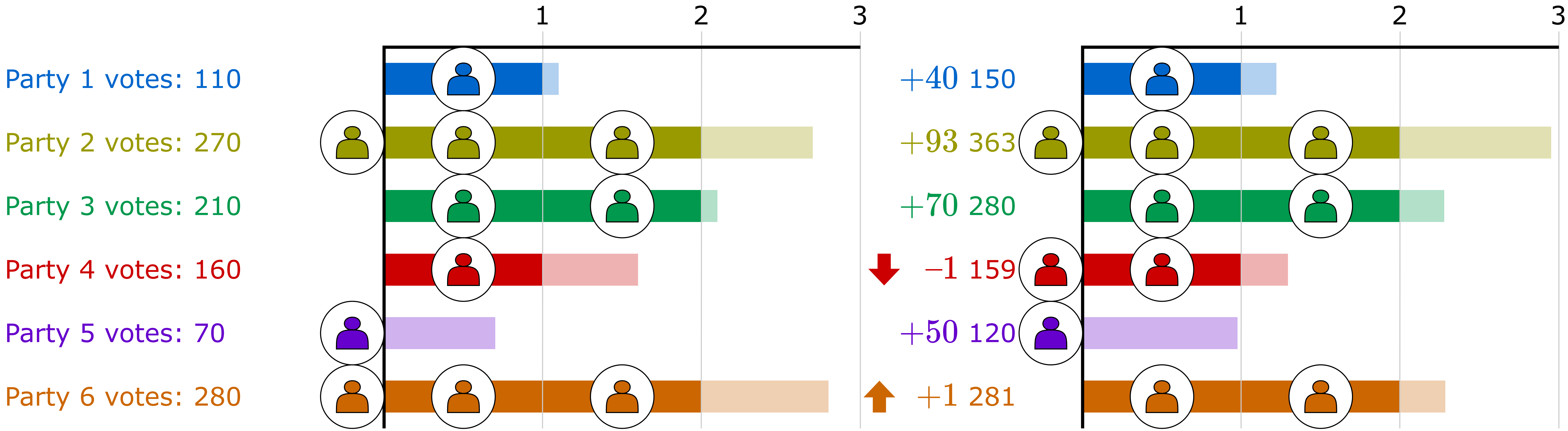

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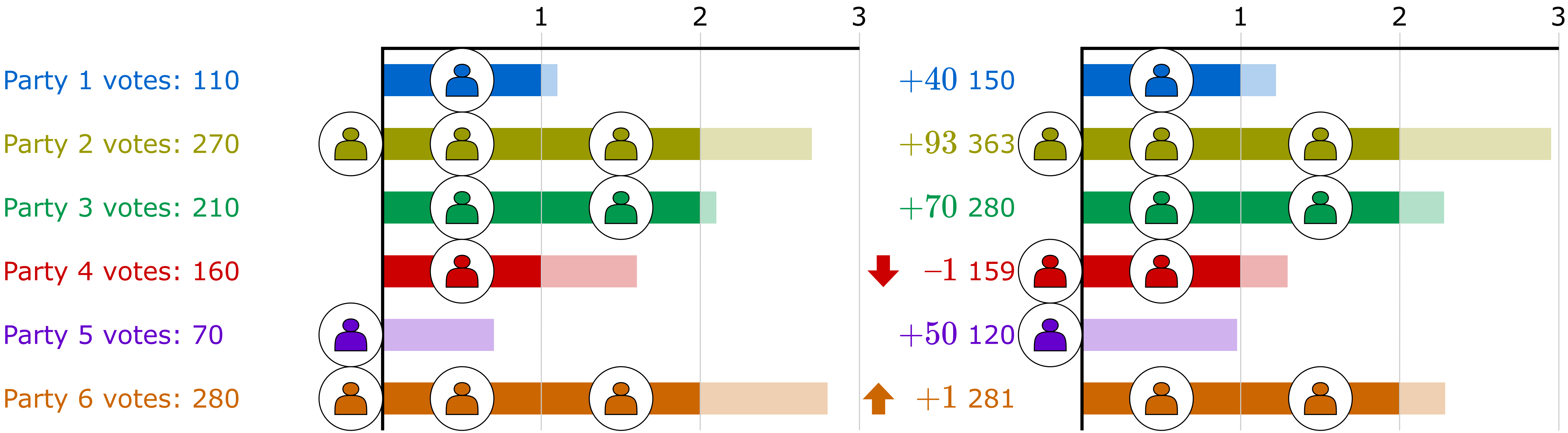

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Multi level apportionment (Schmidt-Kraepelin, Suksompong, Wijaya, 2025)

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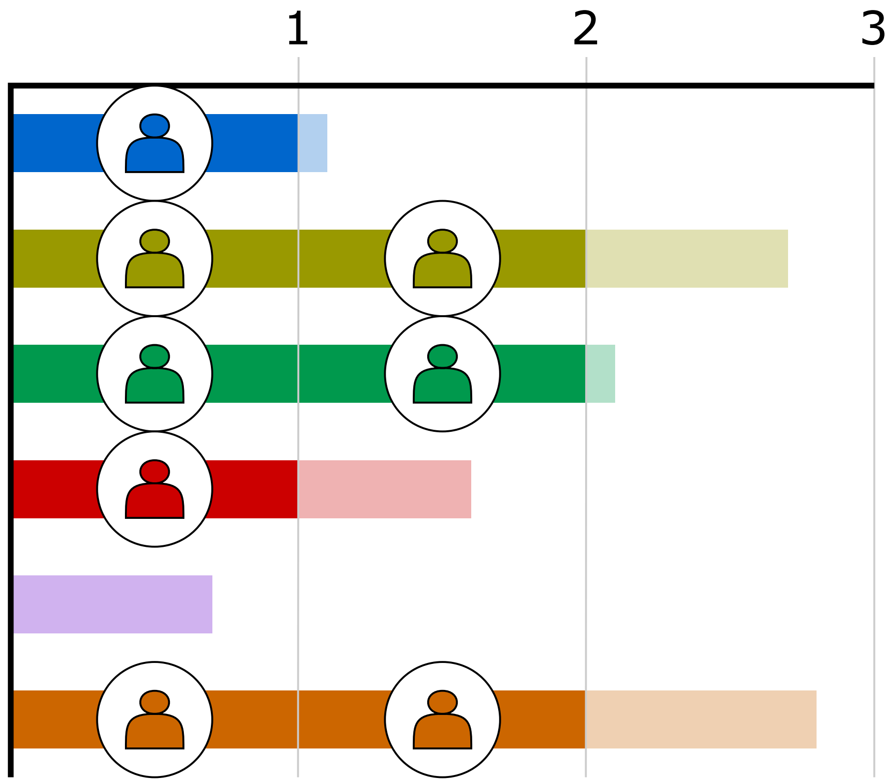

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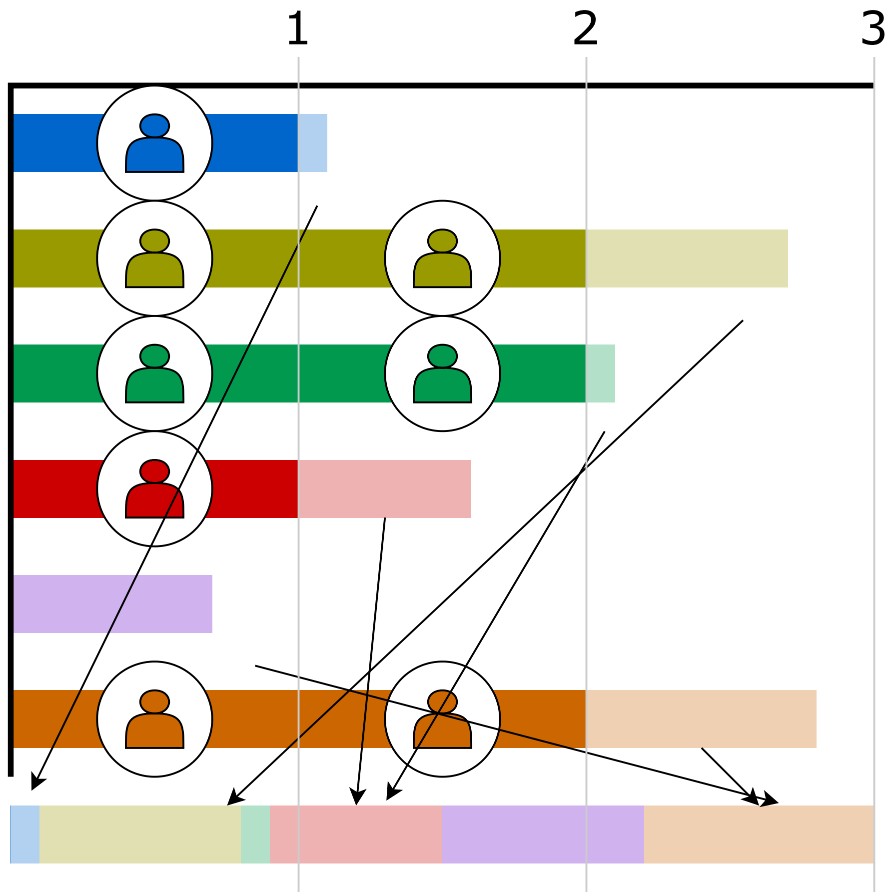

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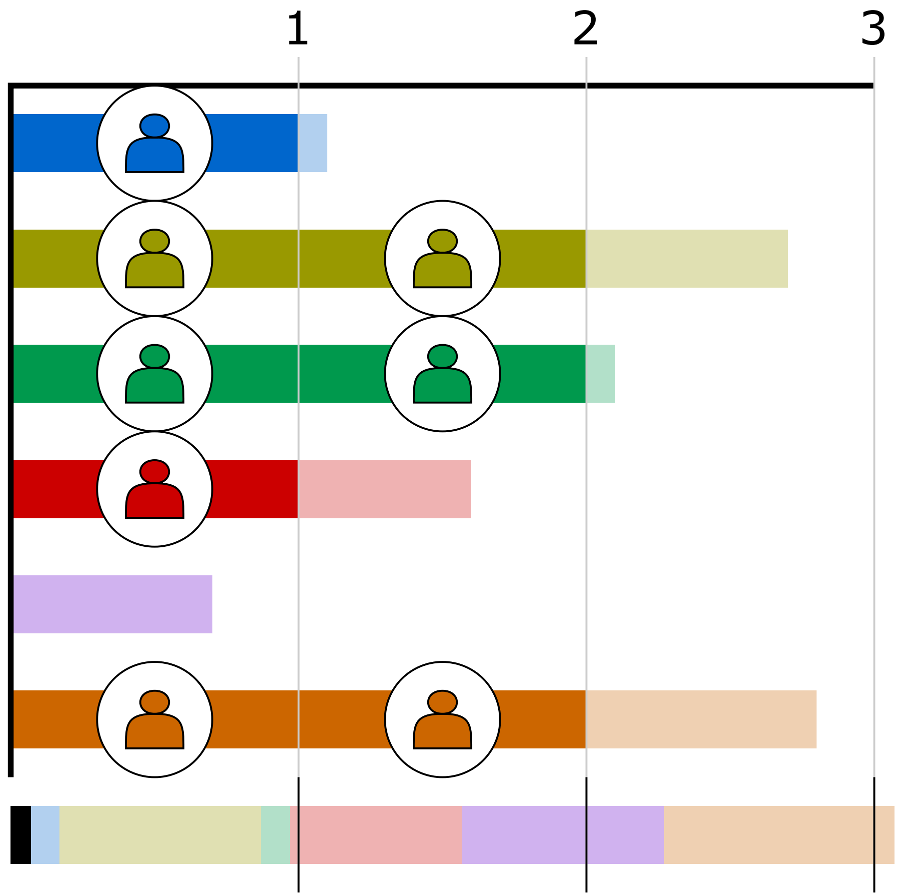

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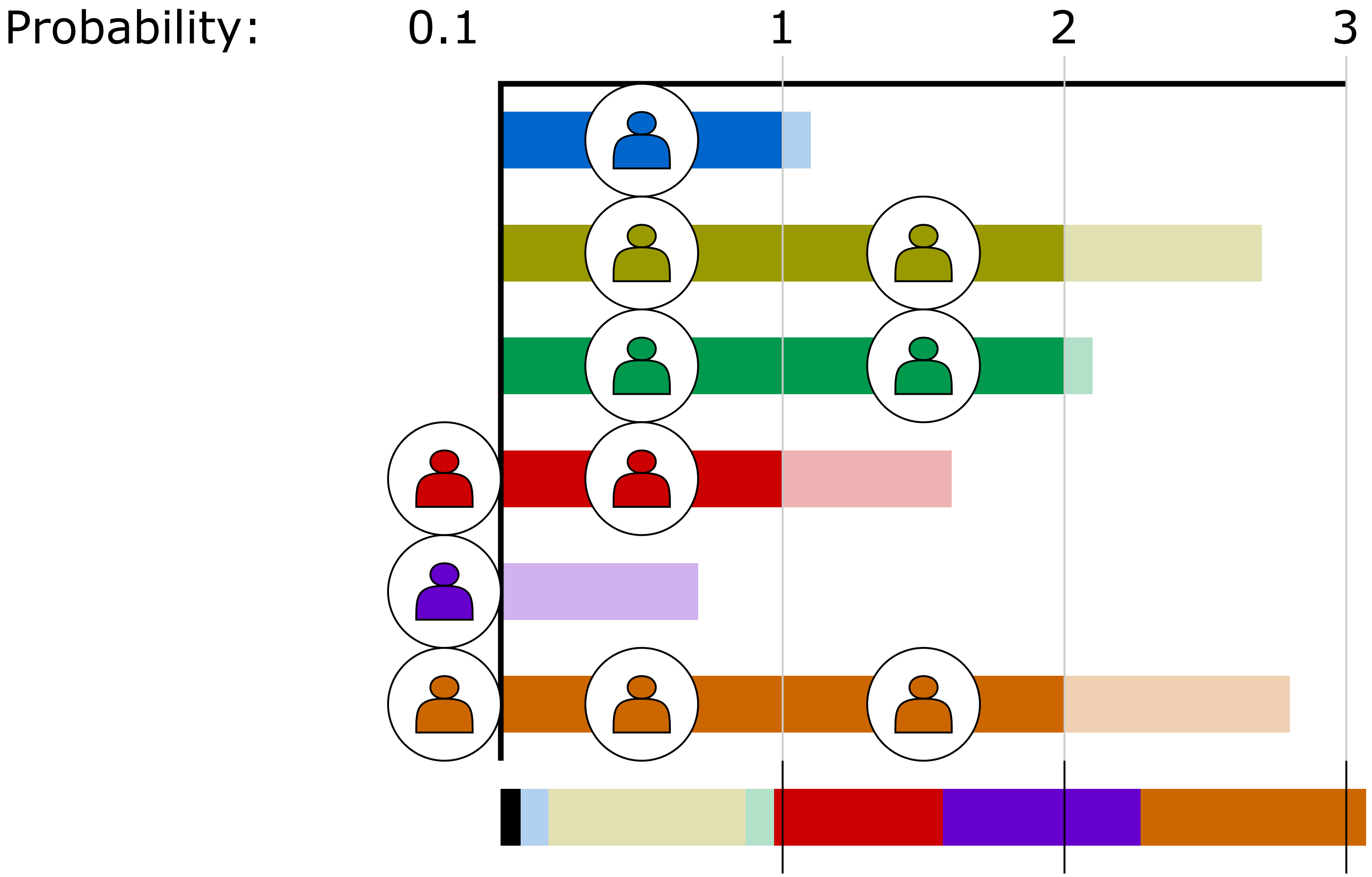

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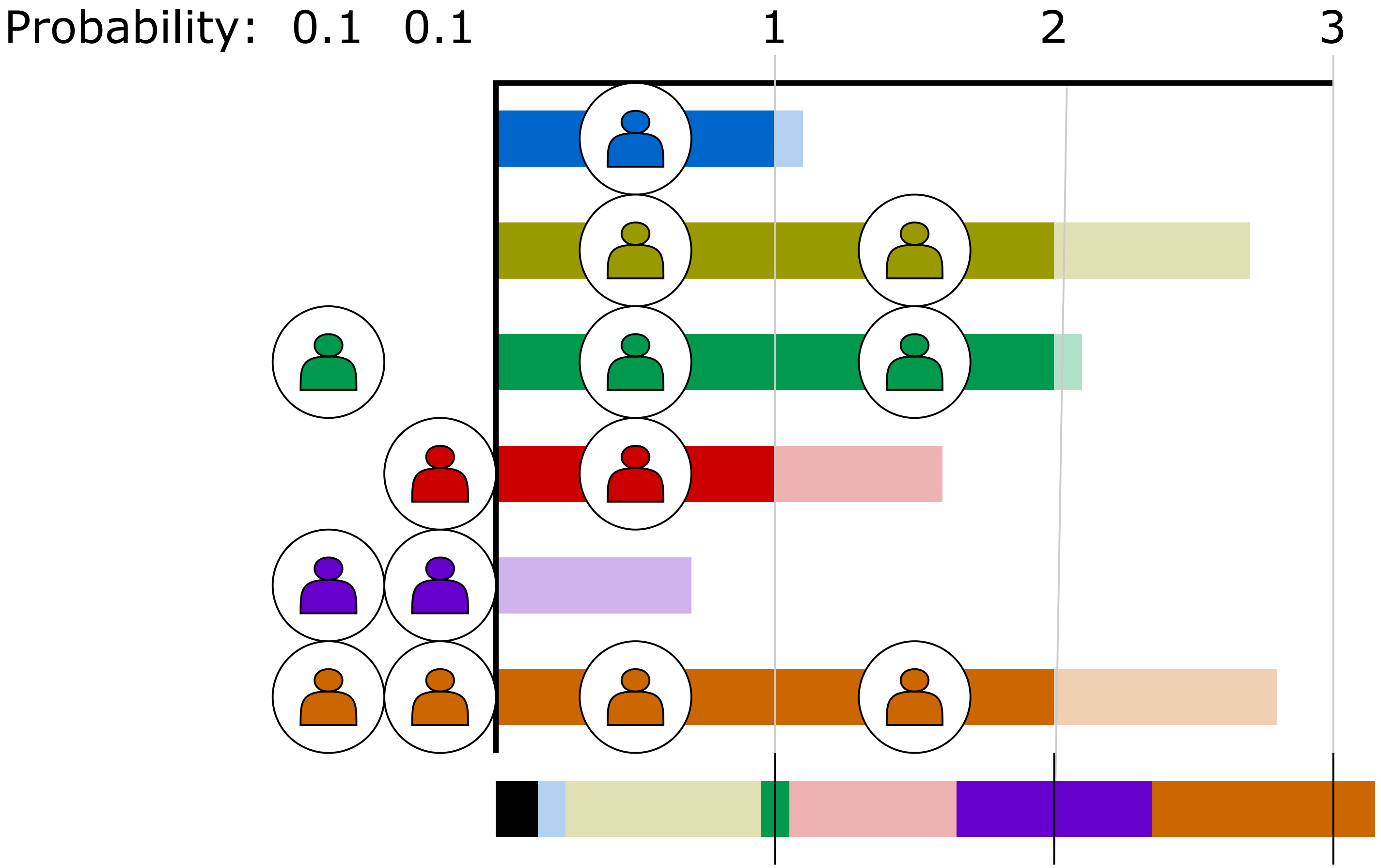

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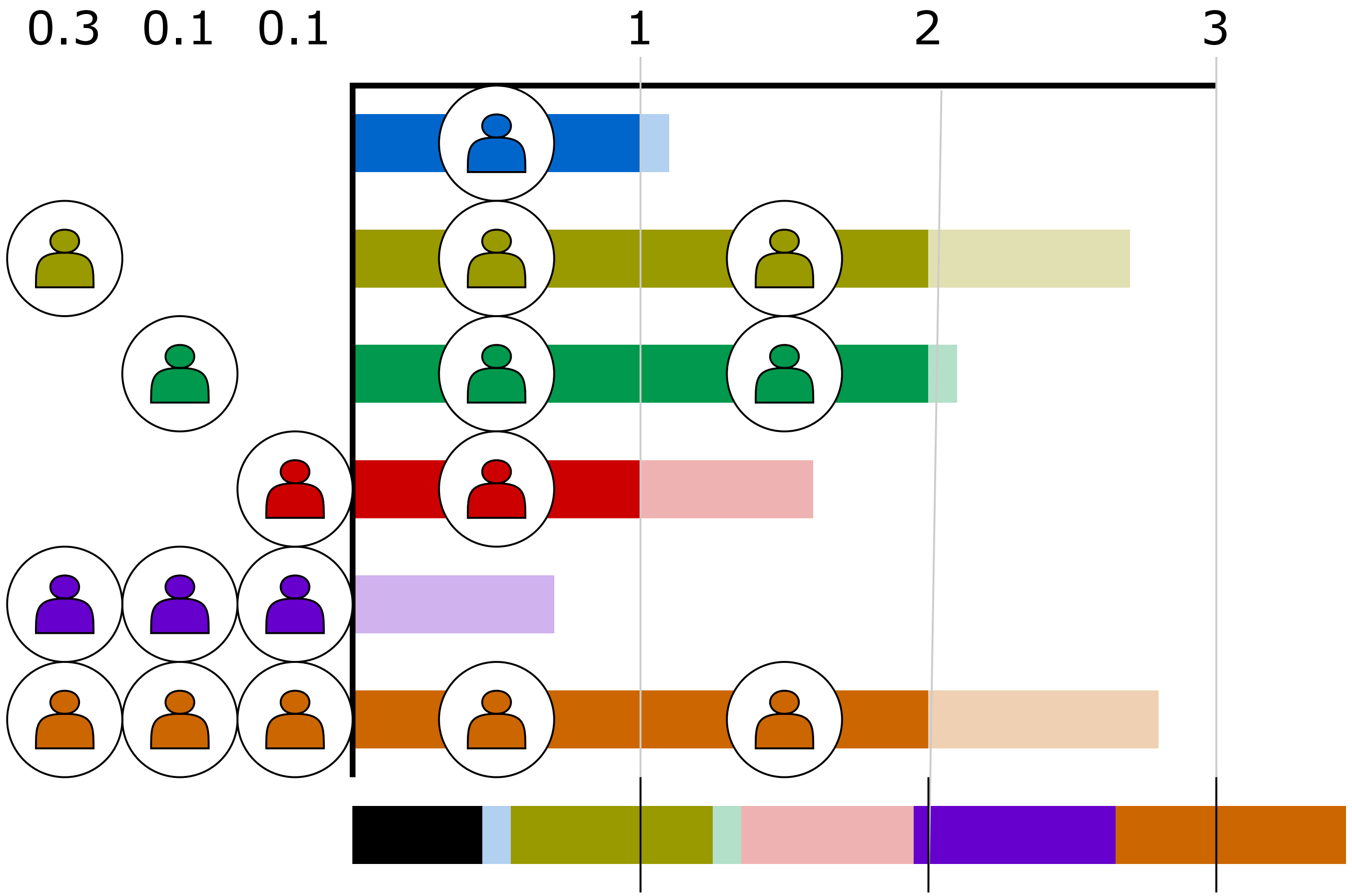

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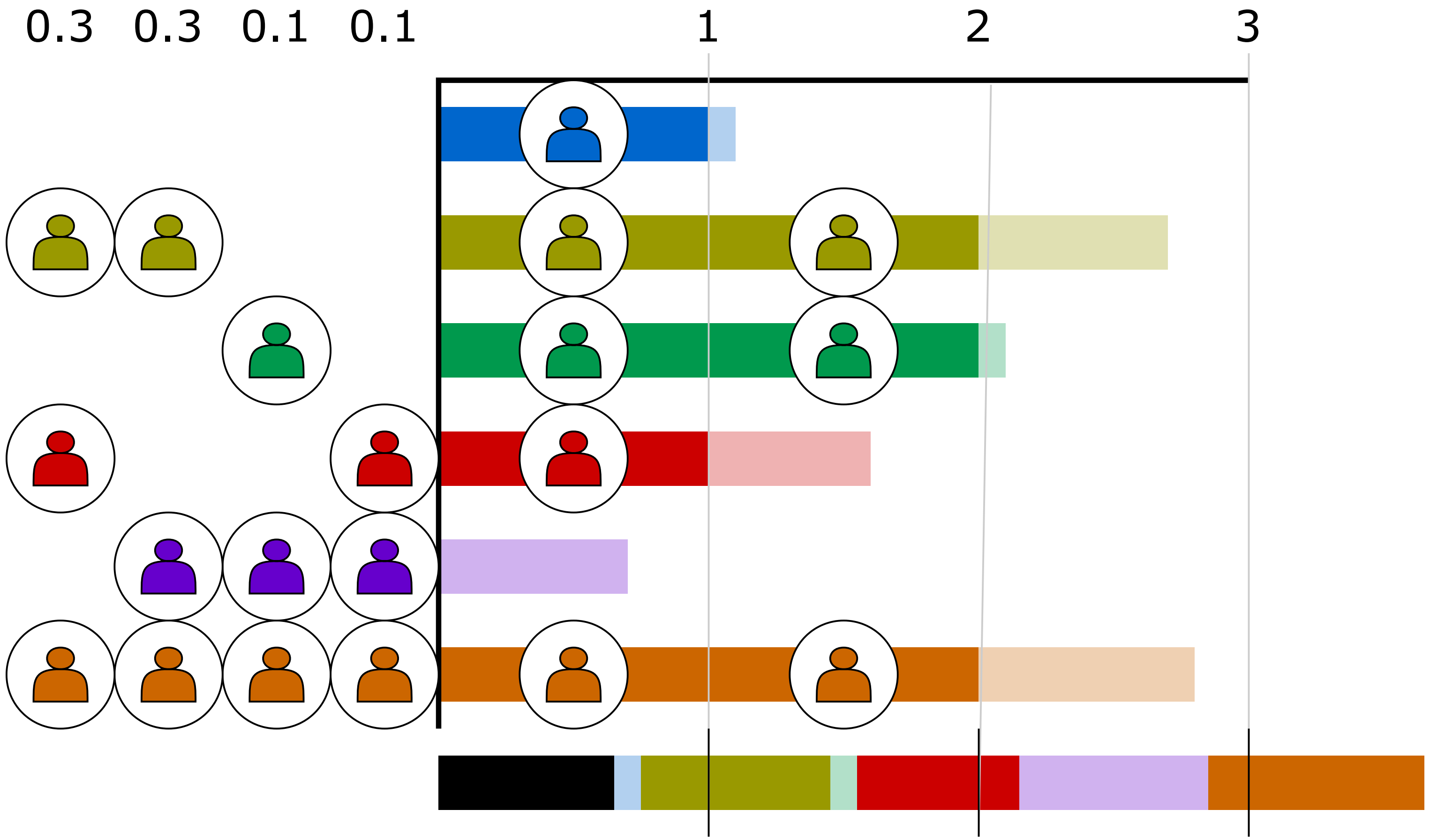

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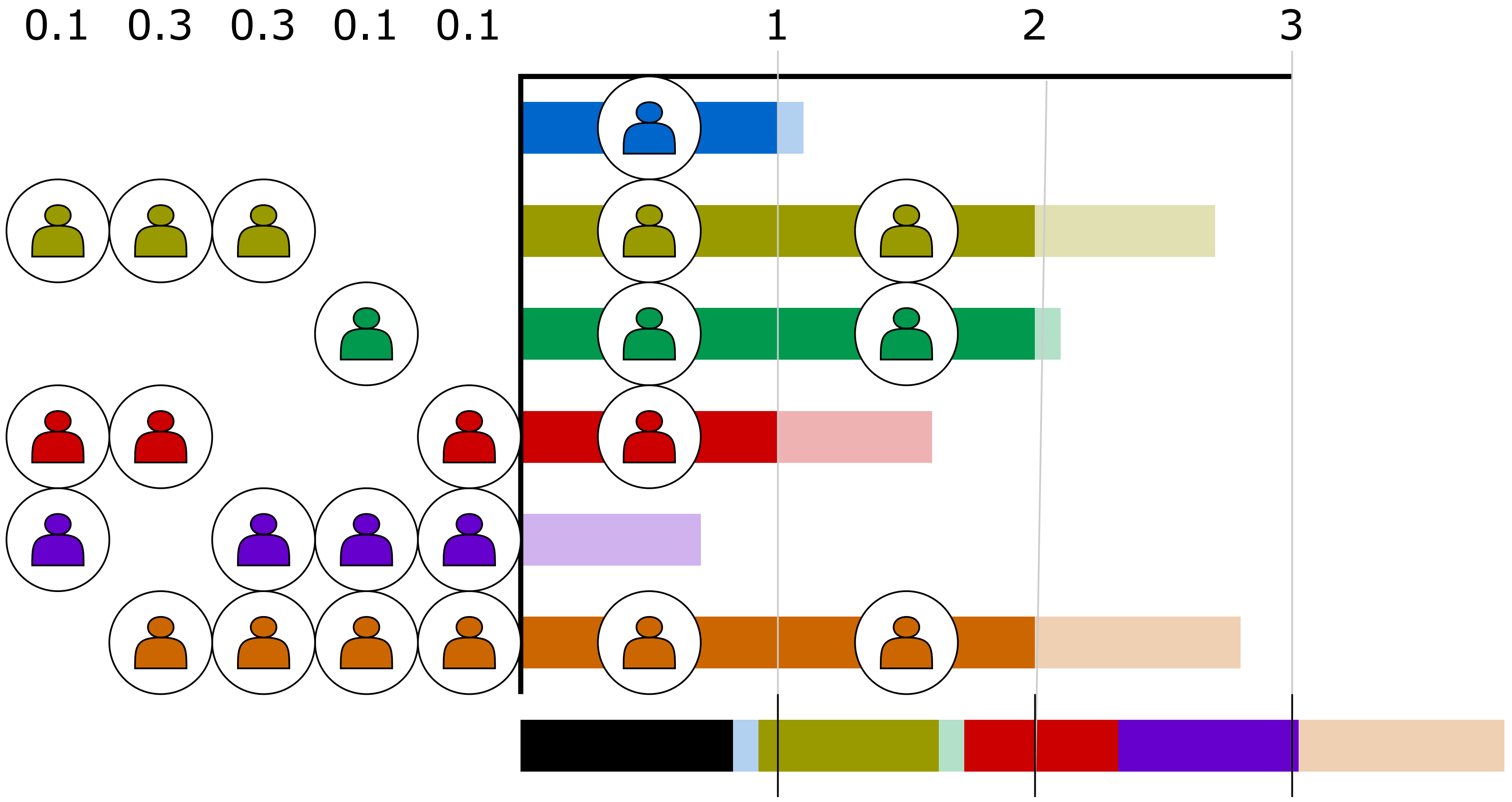

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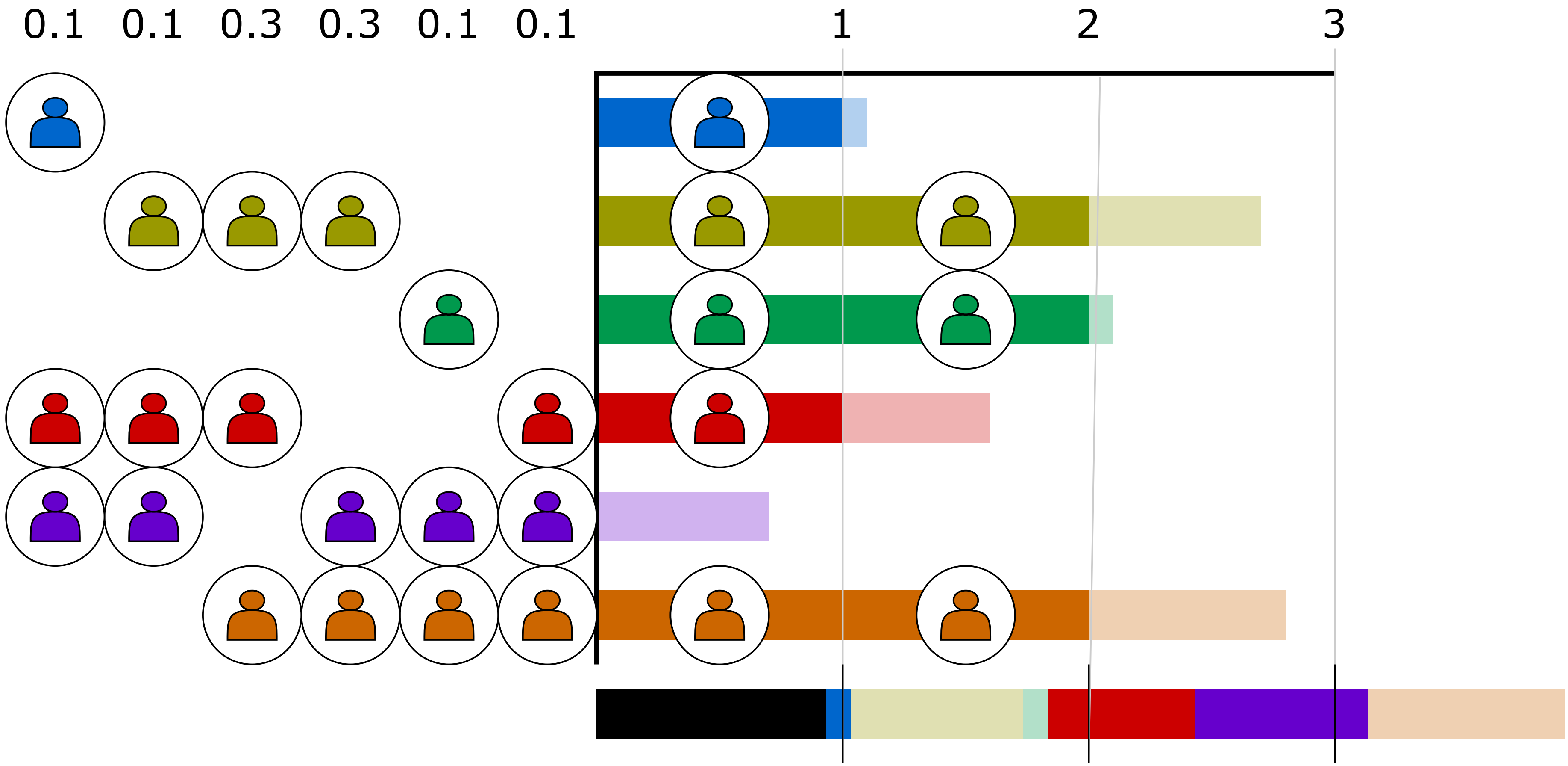

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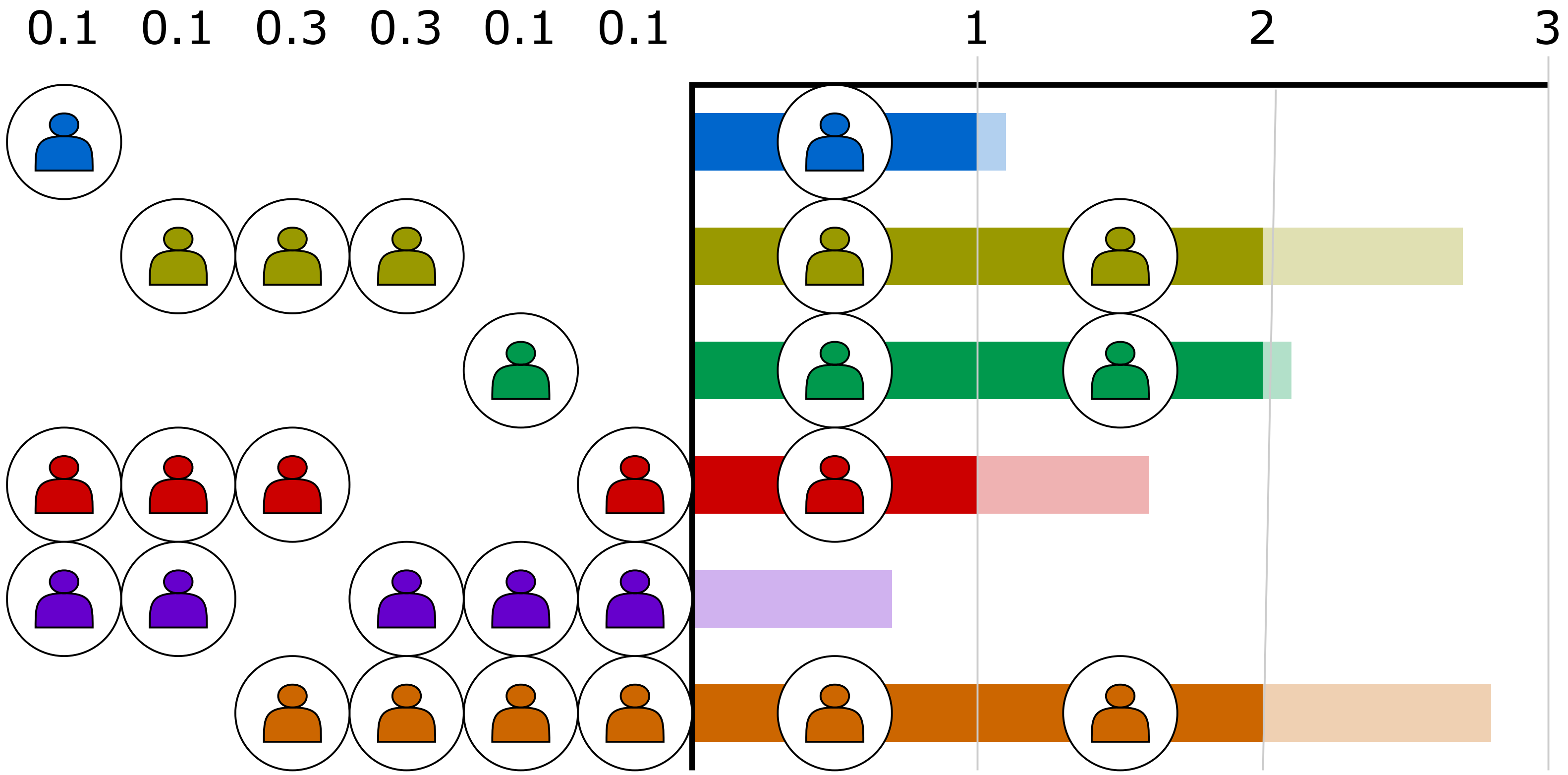

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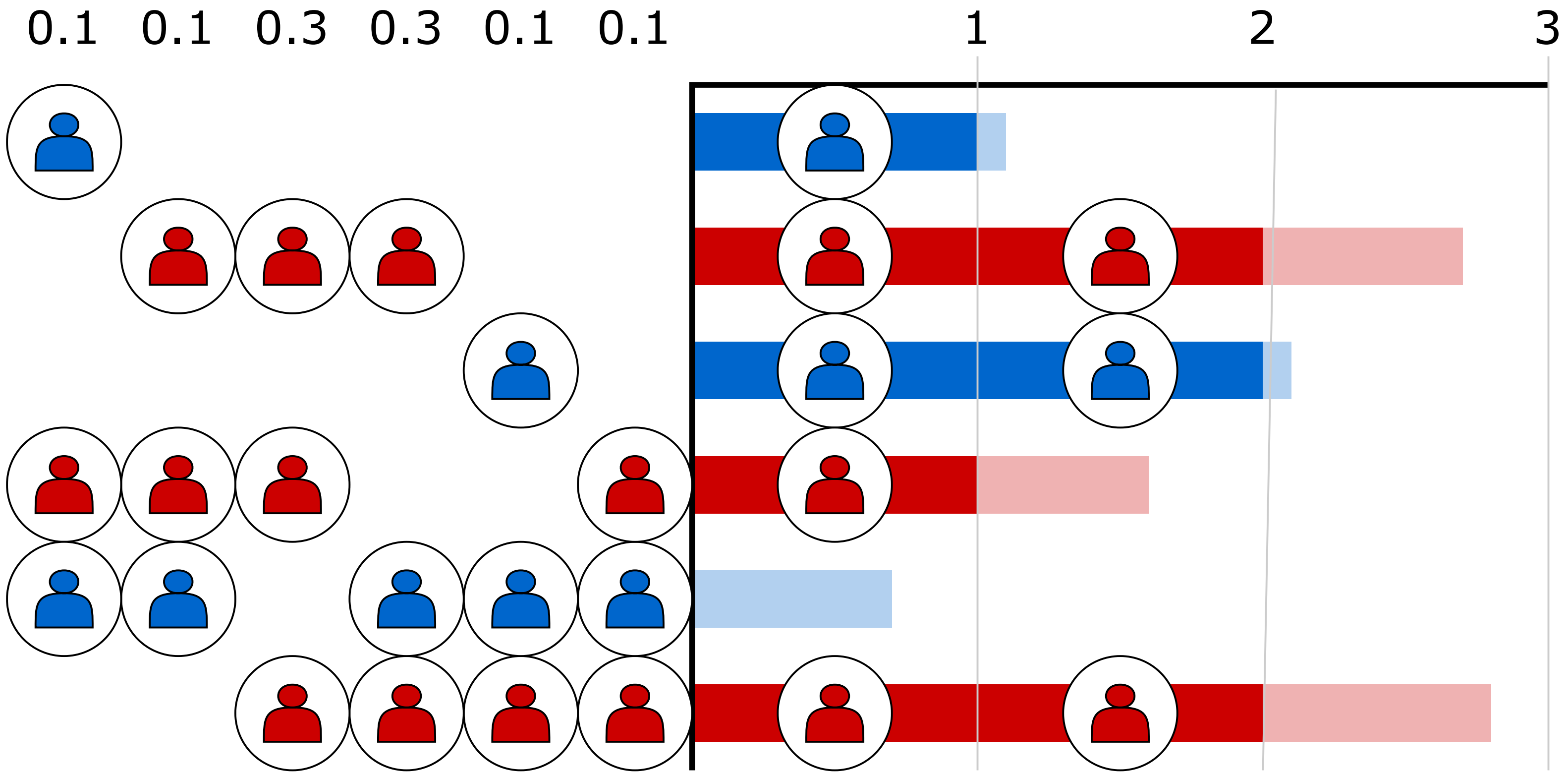

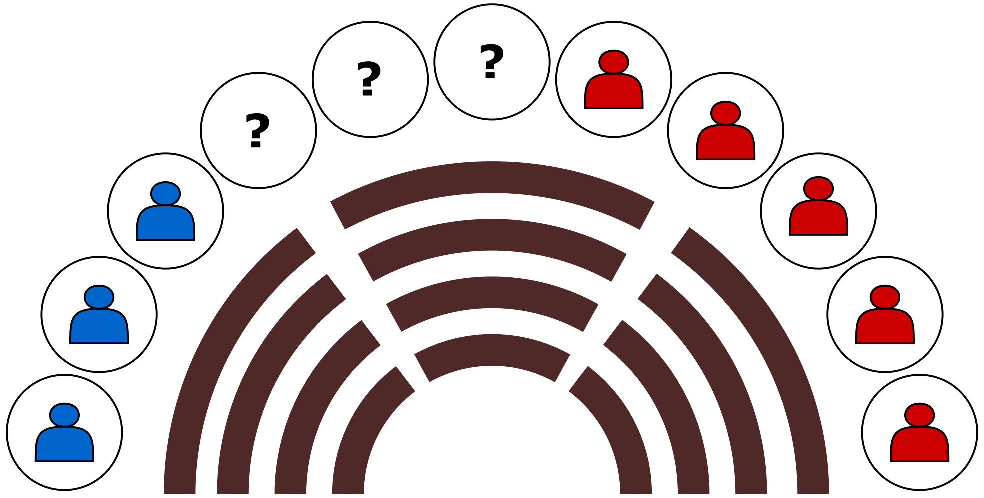

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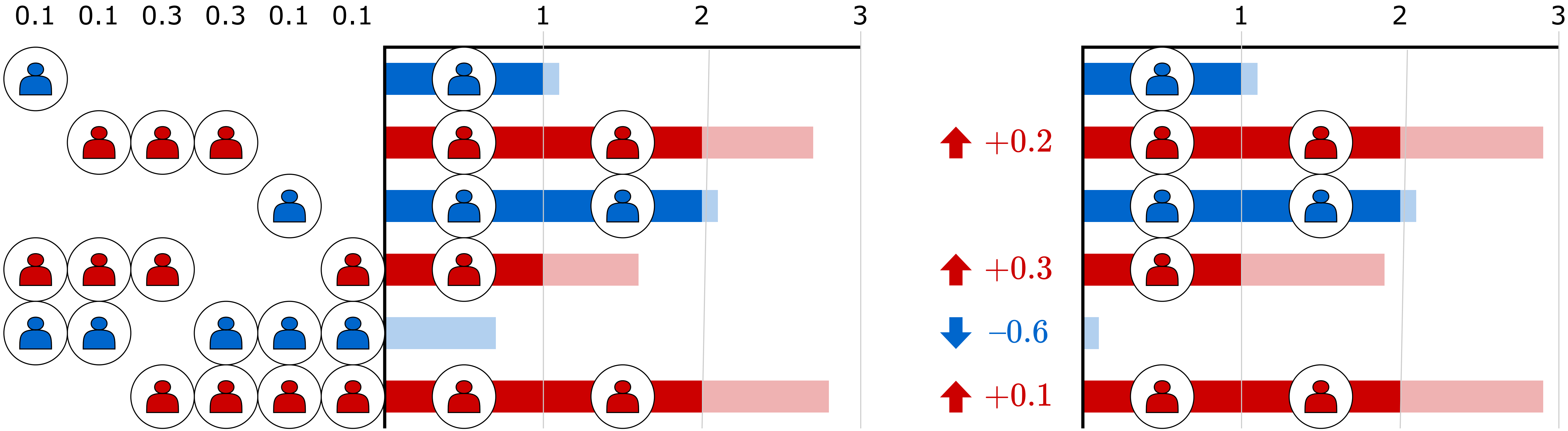

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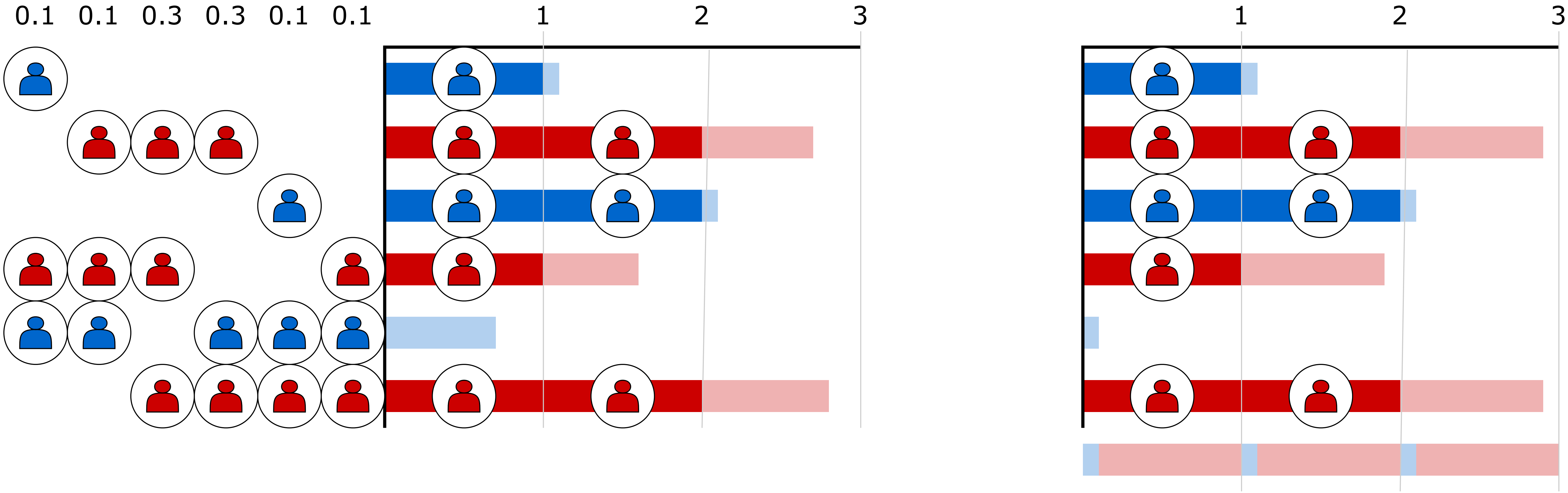

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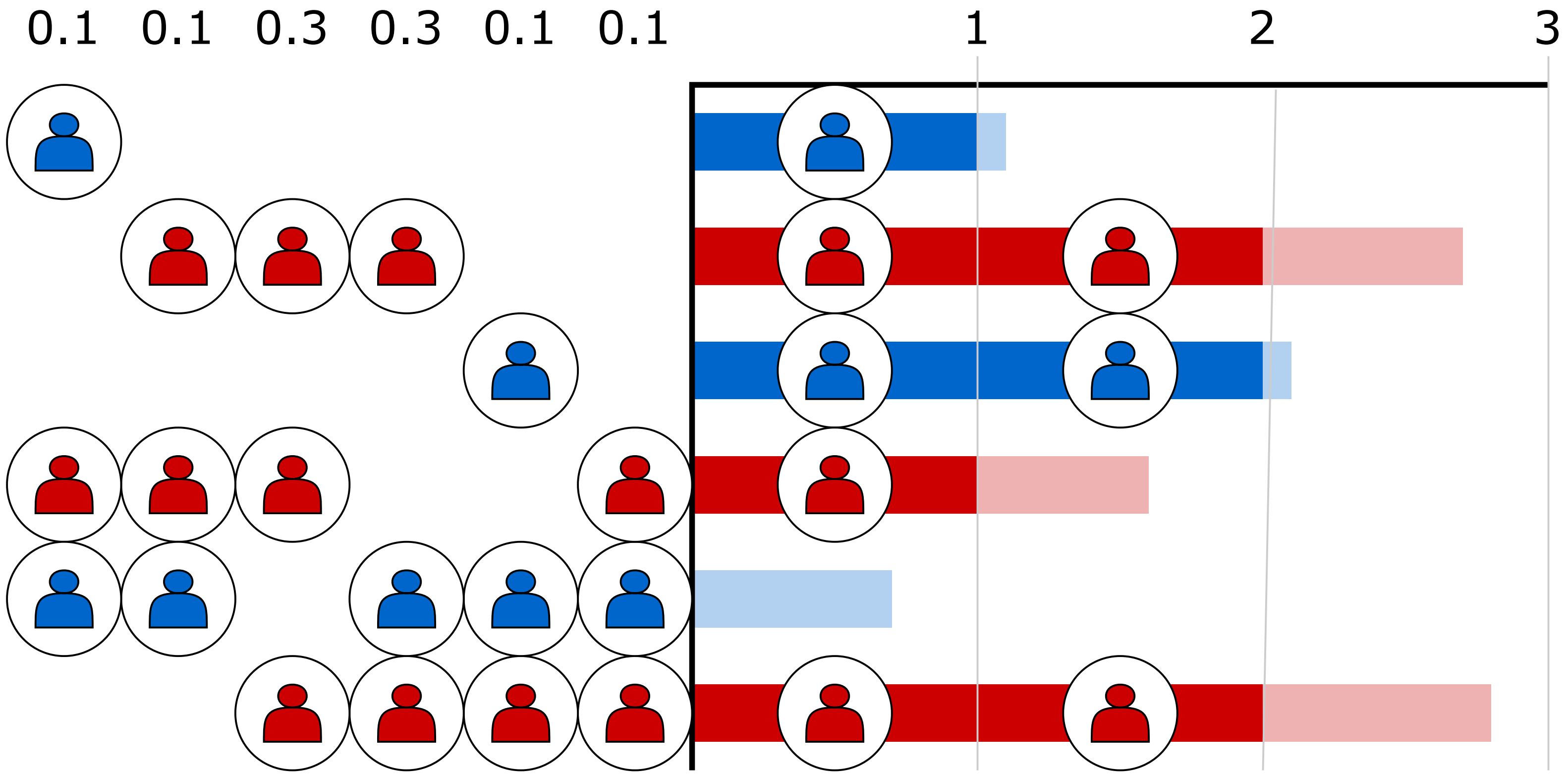

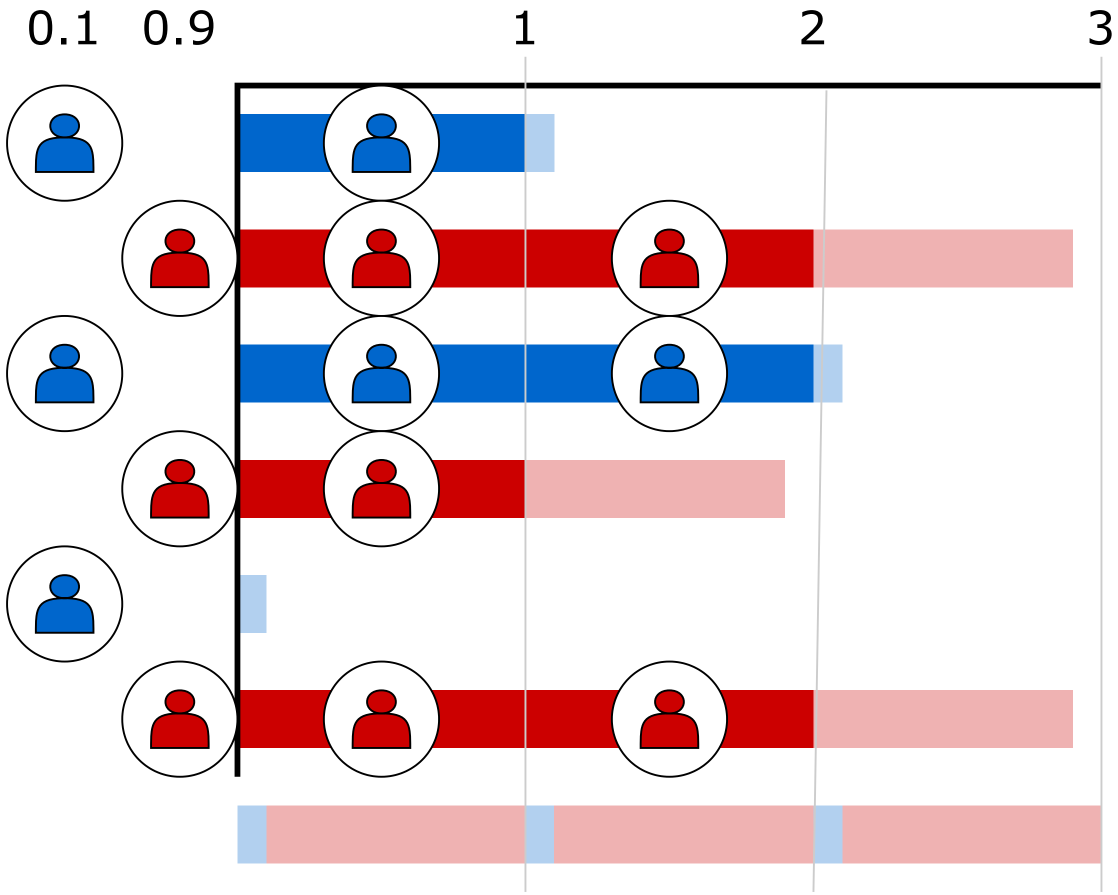

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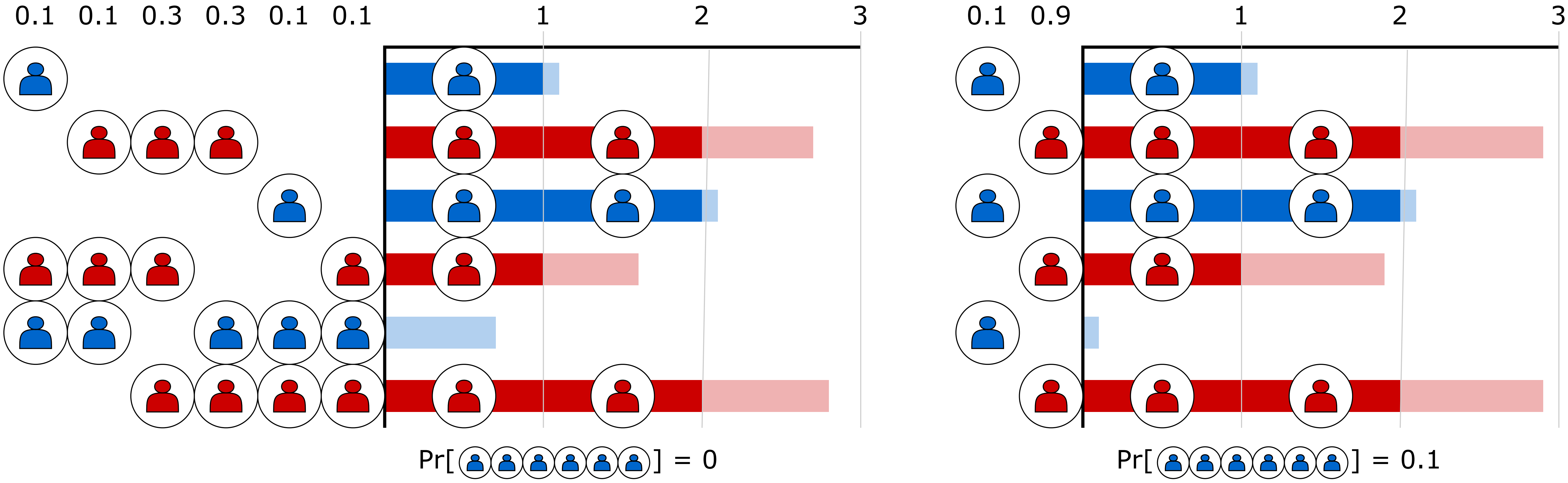

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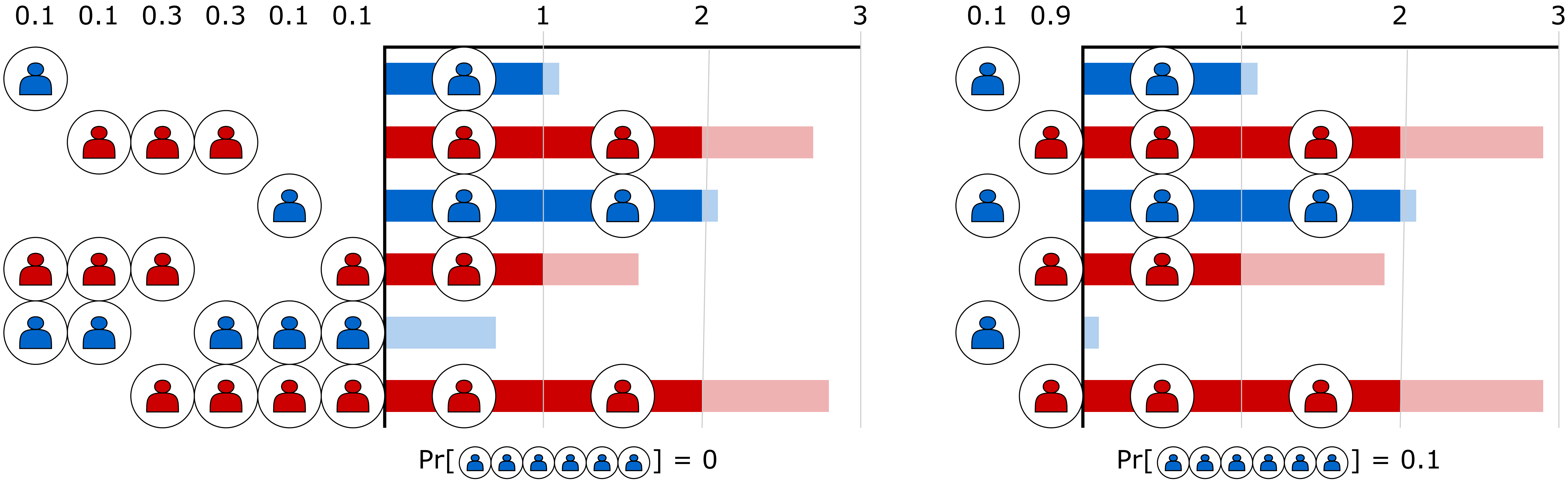

Randomization solves some problems but creates new ones!

House monotonicity (Gölz, Peters, Procaccia, 2024)

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Alternative to apportionment: One member per party/state, but votes are weighted.

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Town of Naples	2403	0.02137	2211
Town of Seneca	2644	0.02351	2421
Town of West Bloomfield	2740	0.02437	2506
Town of Richmond	3360	0.02988	3123
Town of Geneva	3473	0.03088	3204
Town of East Bloomfield	3640	0.03237	3340
City of Geneva (5,6)	3679	0.03272	3373
City of Geneva (3,4)	3921	0.03487	3582
Town of Hopewell	3931	0.03496	3589
Town of Gorham	4106	0.03651	3740
City of Canandaigua (2,3)	5140	0.04571	4682
City of Geneva (1,2)	5210	0.04633	4740
City of Canandaigua (1,4)	5436	0.04834	4932
Town of Phelps	6637	0.05902	6000
Town of Manchester	9404	0.08362	8381
Town of Canandaigua	11109	0.09879	9748
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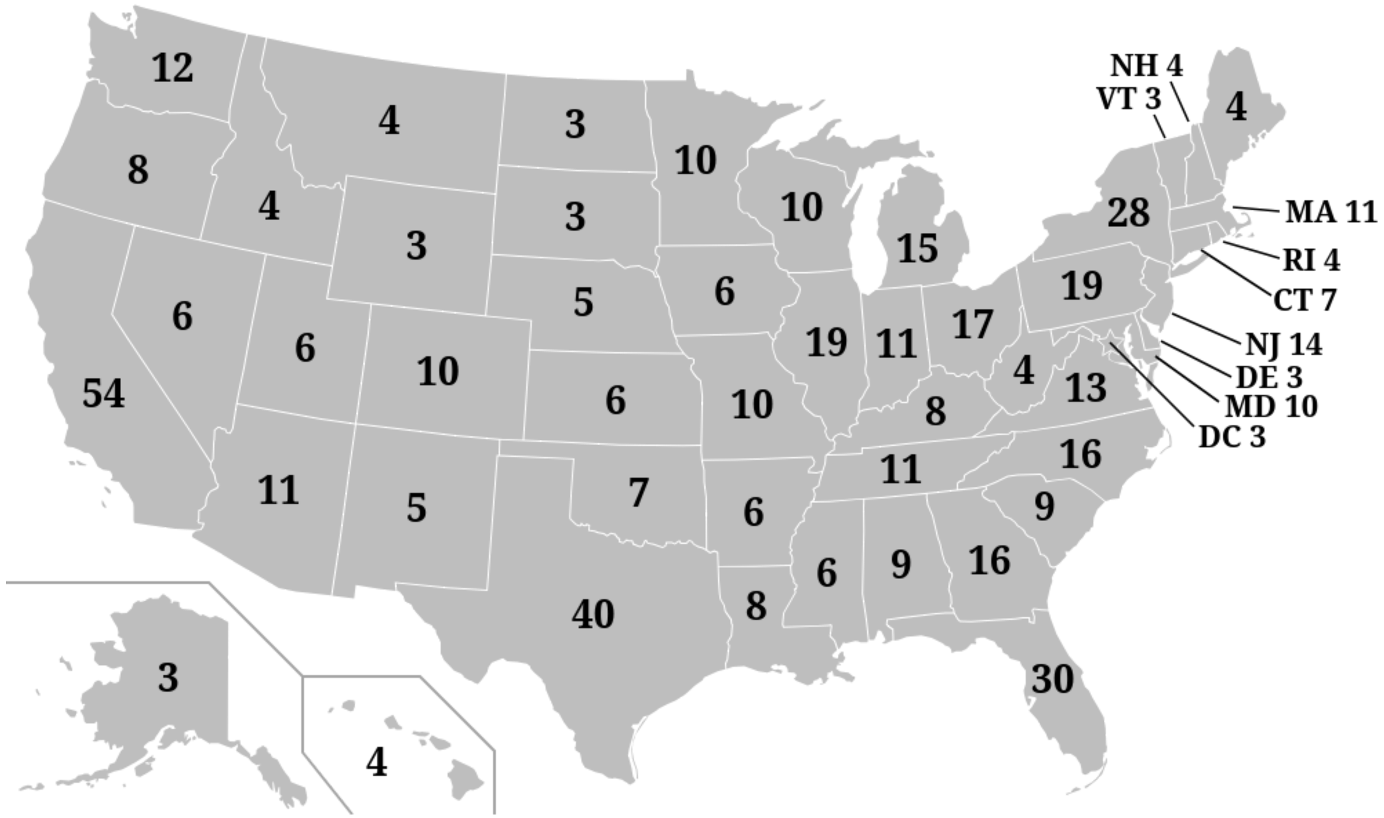

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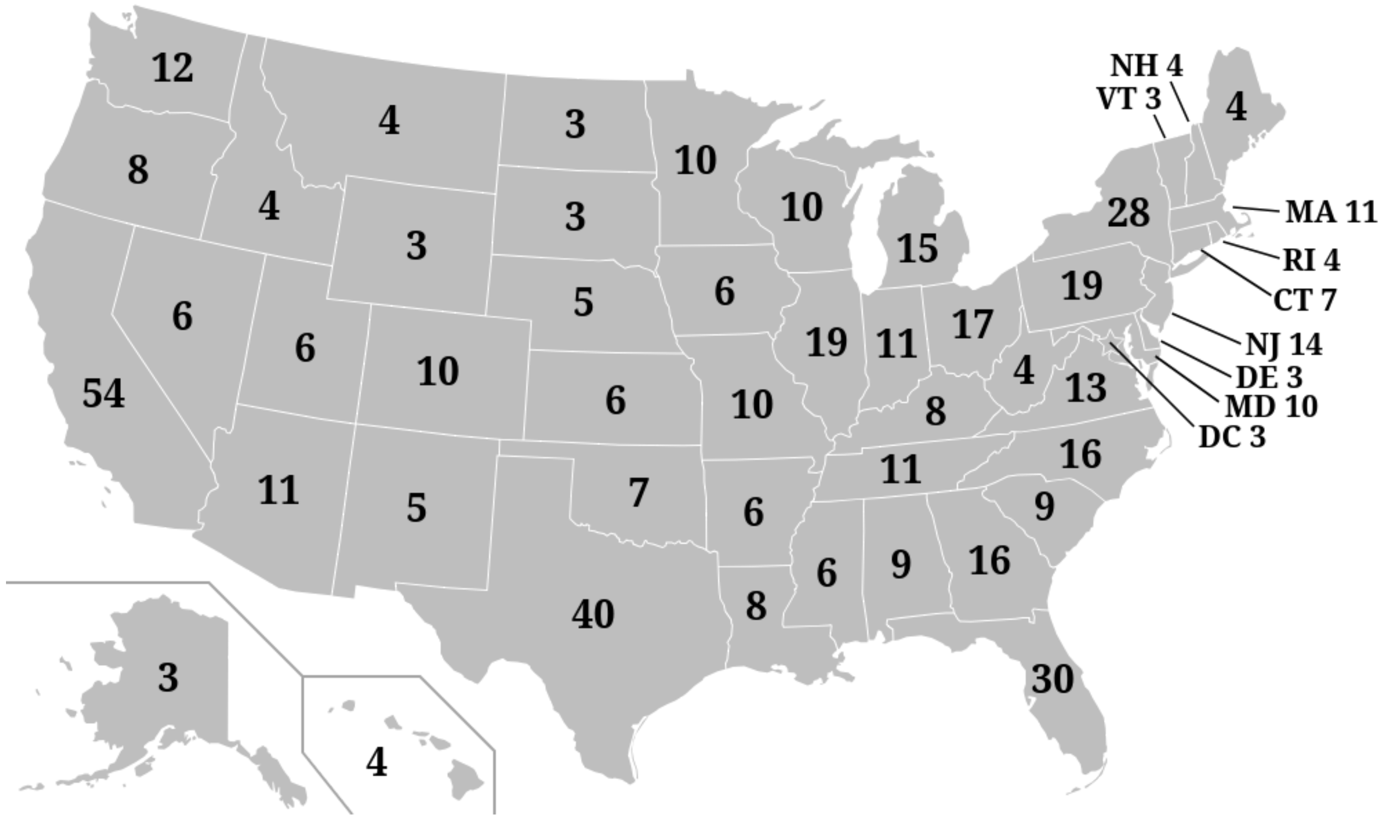

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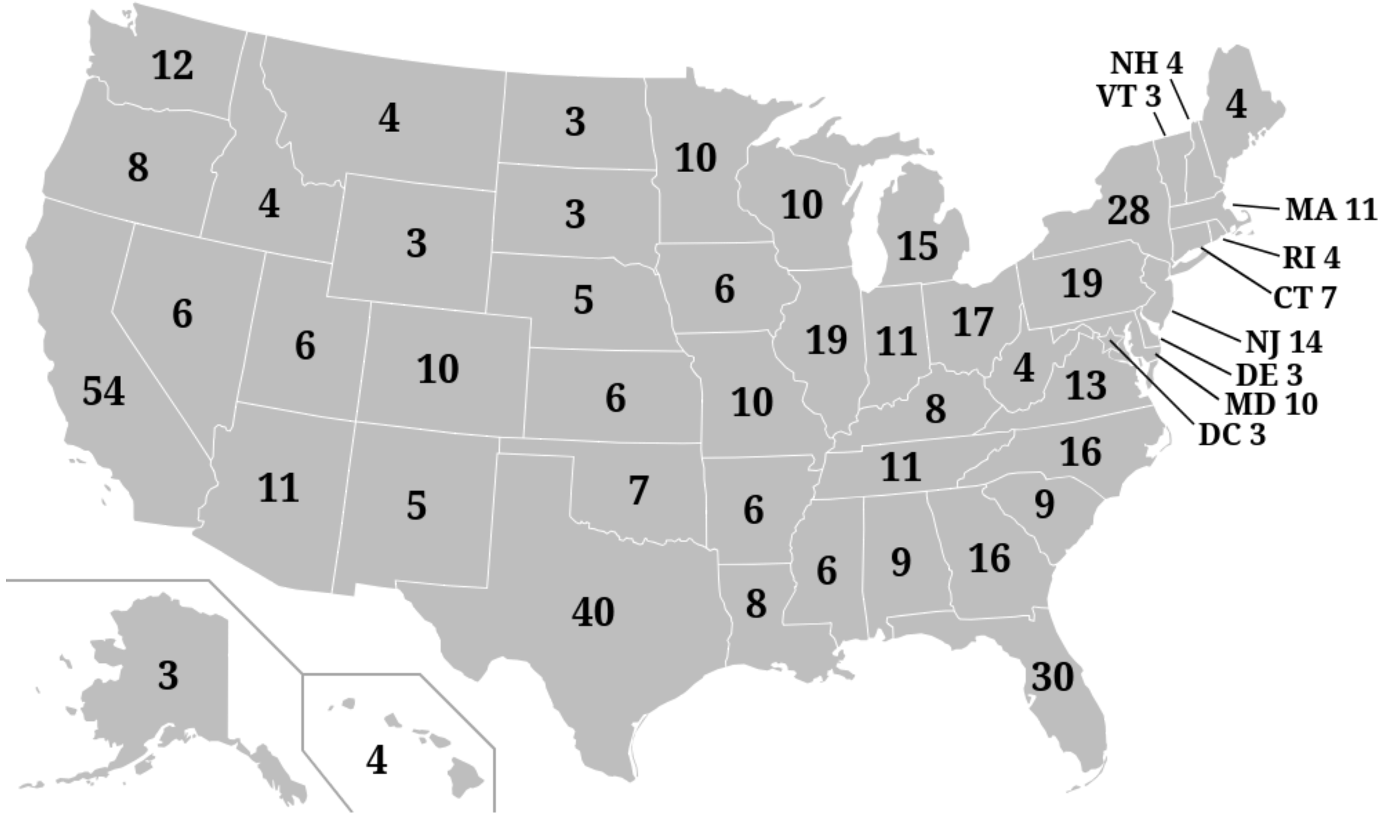

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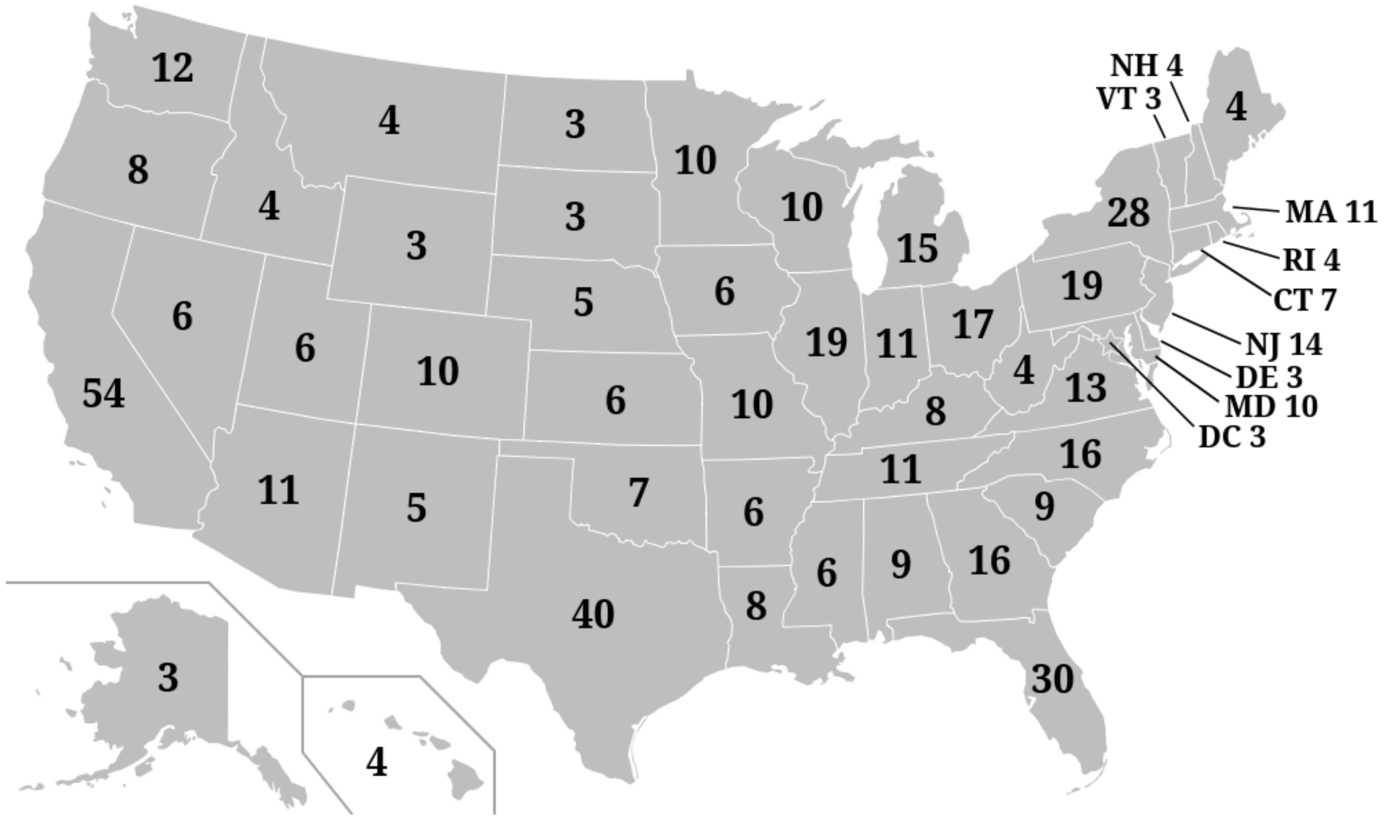

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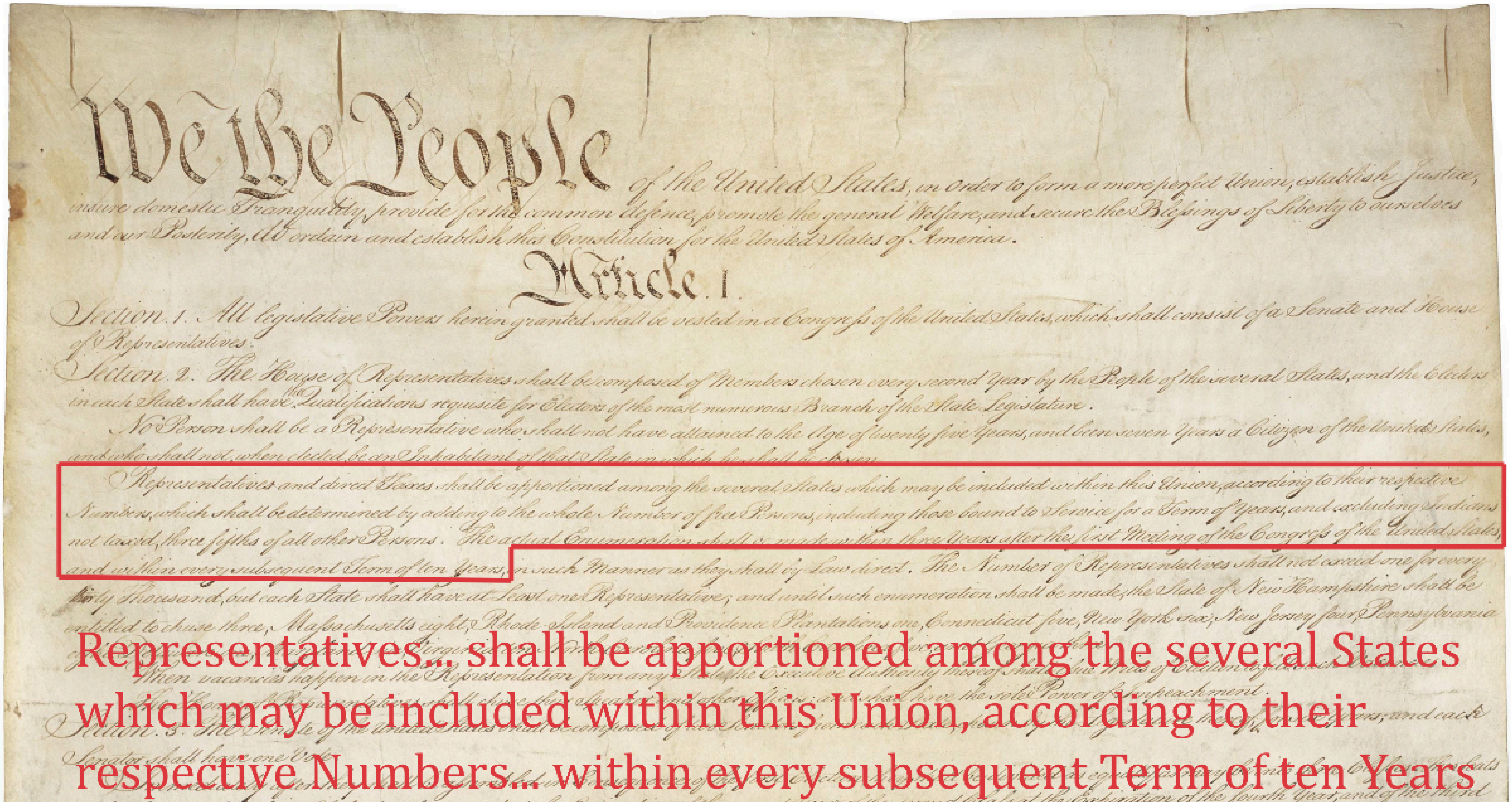
This equalizes the "voting power" of the counties.

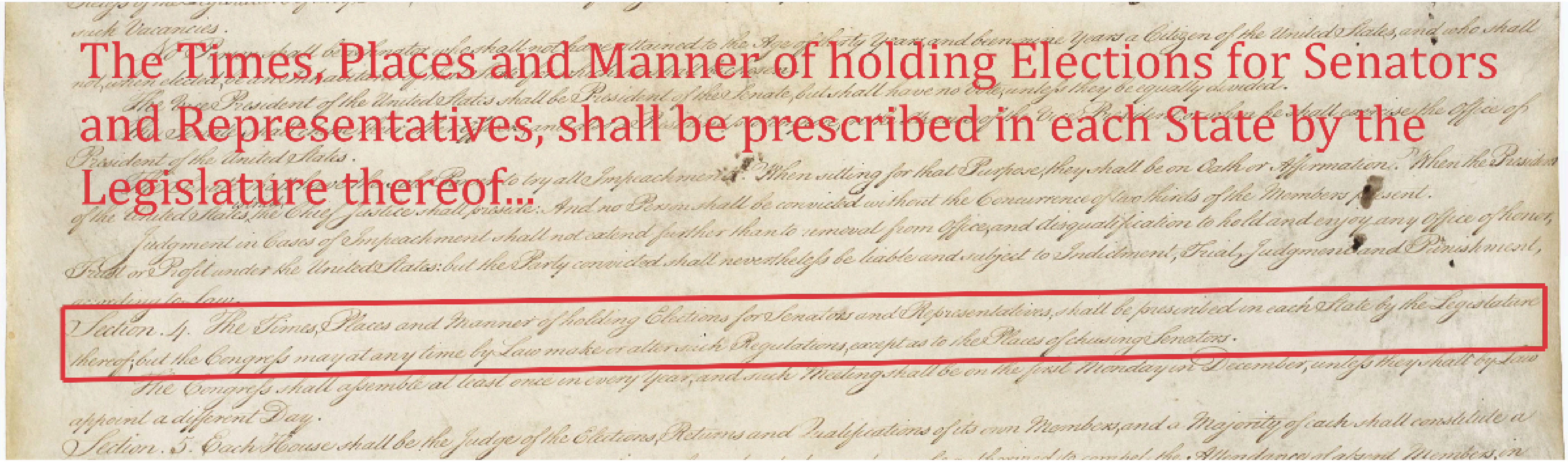
[Inverse semivalue approximation (Diakonikolas, Pavlou, Peebles, Stewart, 2022)]

Lectures 21, 22, 23 - Redistricting

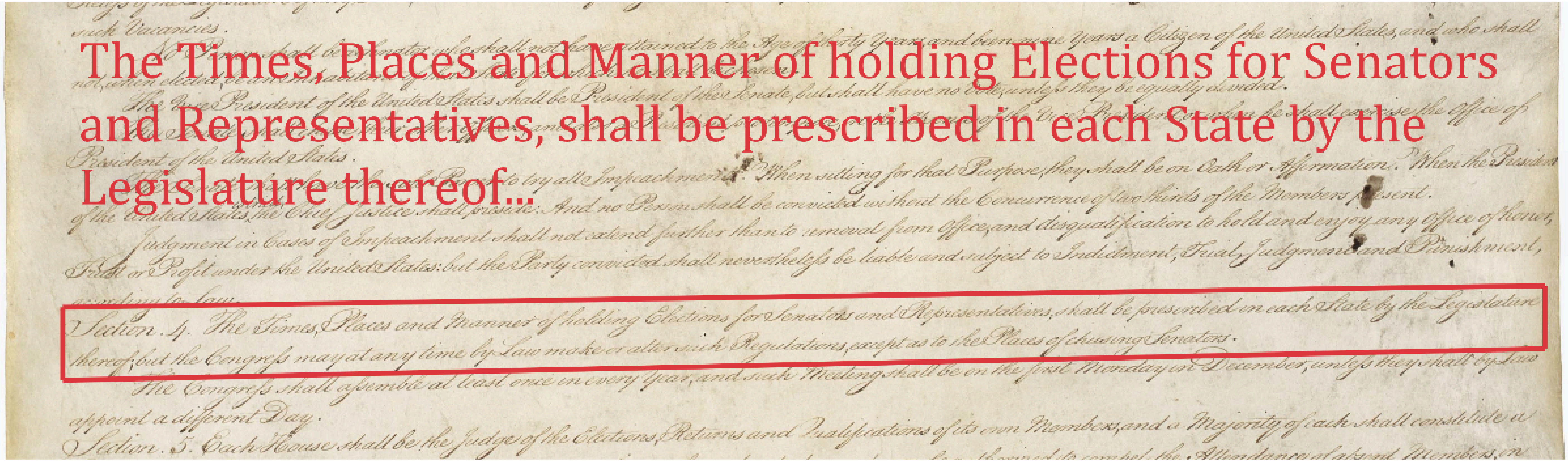
Lectures 21, 22, 23 - Redistricting

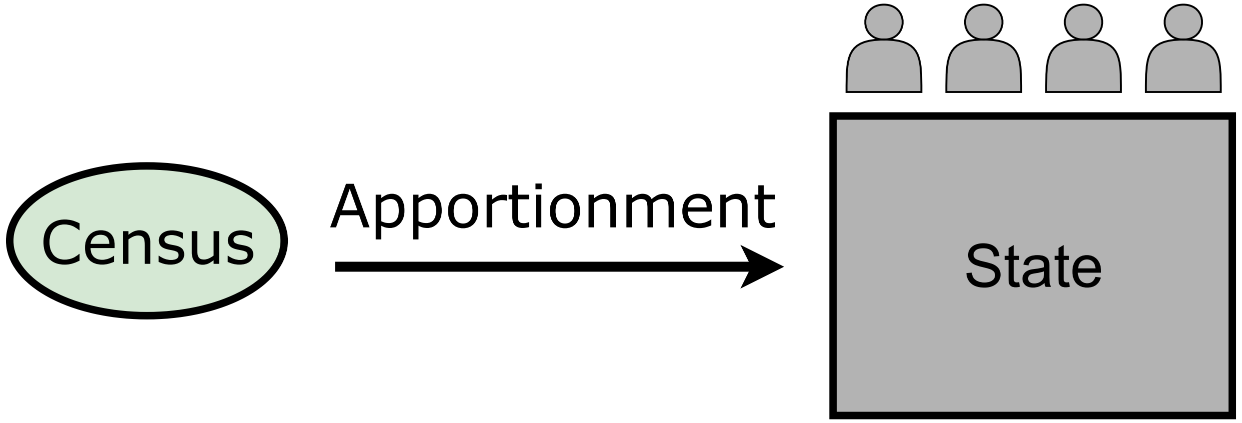


Representatives… shall be apportioned among the several States which may be included within this Union, according to their respective Numbers… within every subsequent Term of ten Years

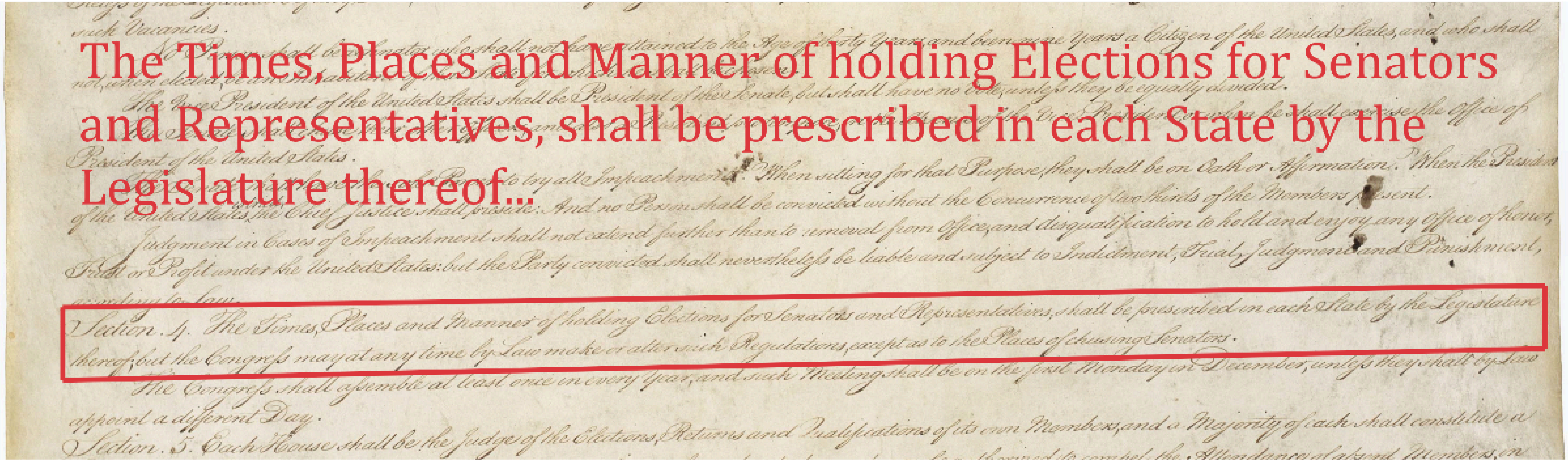

The Times, Places and Manner of holding Elections for Senators and Representatives, shall be prescribed in each State by the Legislature thereof…
Section. 4. The Times, Places and Manner of holding Elections for Senators and Representatives, shall be prescribed in each State by the Legislature thereof; but the Congress may at any time by Law make or alter such Regulations, except as to the Places of chusing Senators.

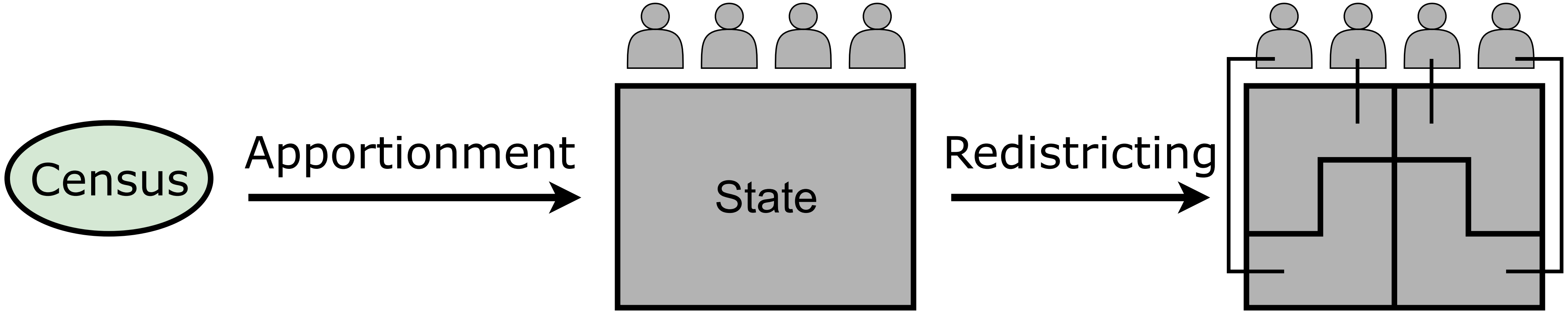
Lectures 21, 22, 23 - Redistricting



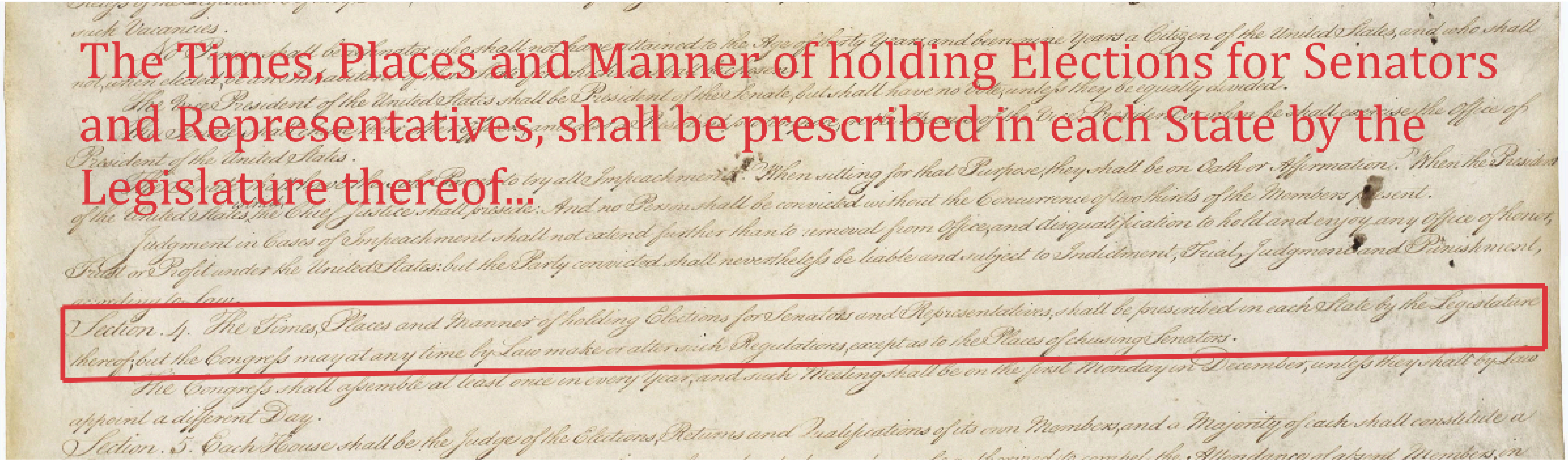


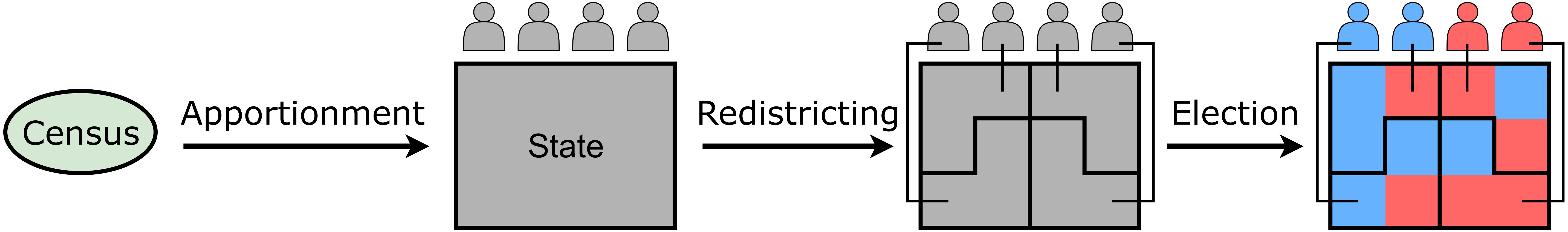
Lectures 21, 22, 23 - Redistricting



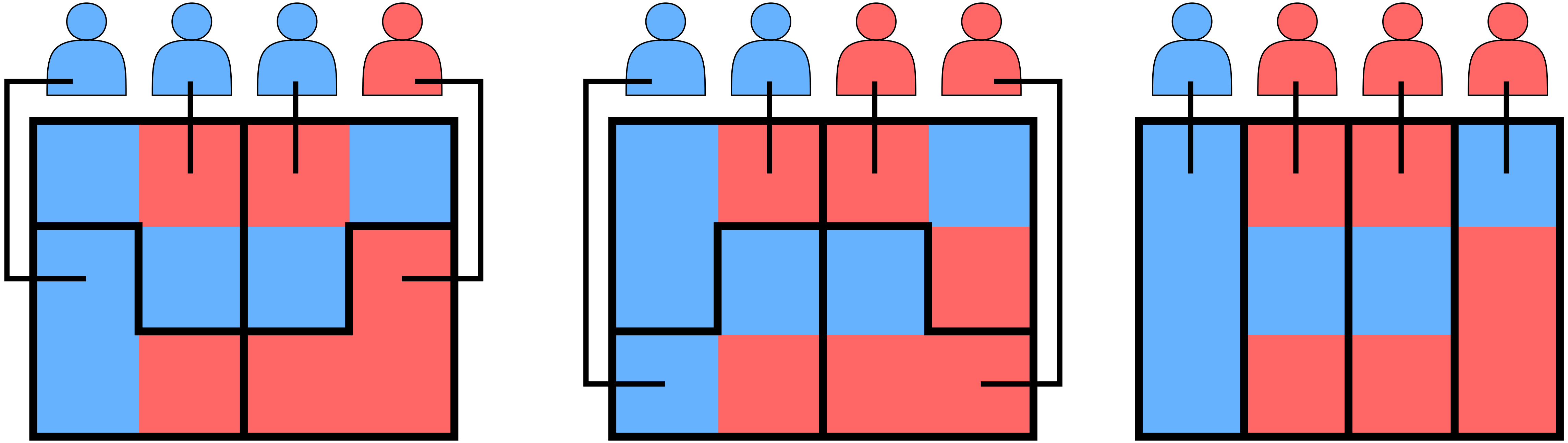


Lectures 21, 22, 23 - Redistricting



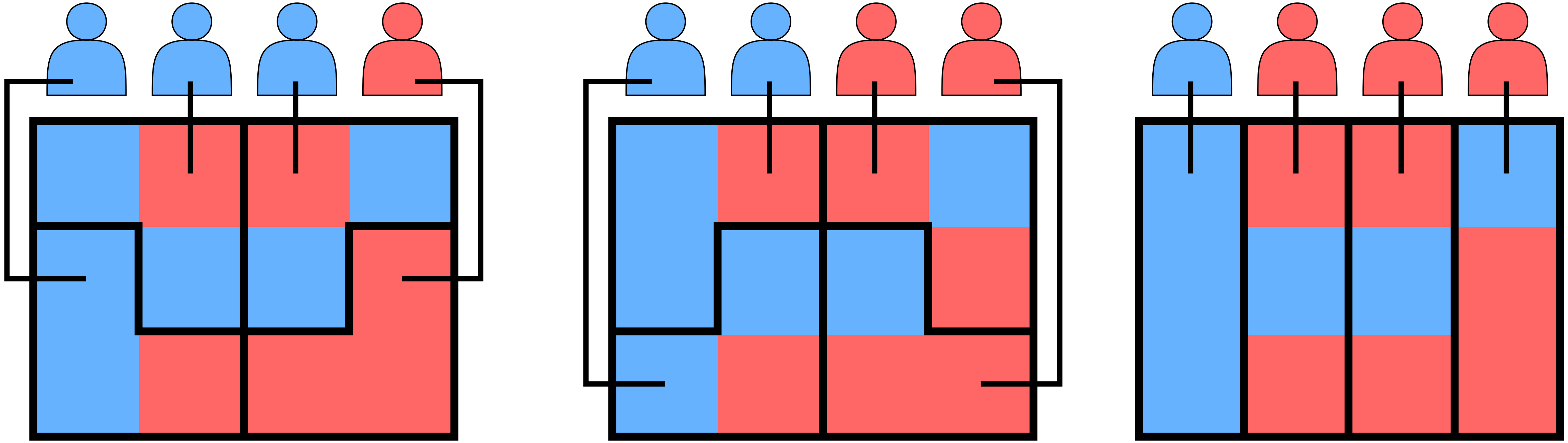


Lectures 21, 22, 23 - Redistricting



Lectures 21, 22, 23 - Redistricting

Gerrymandering: Drawing district lines to advantage one party or group over another.



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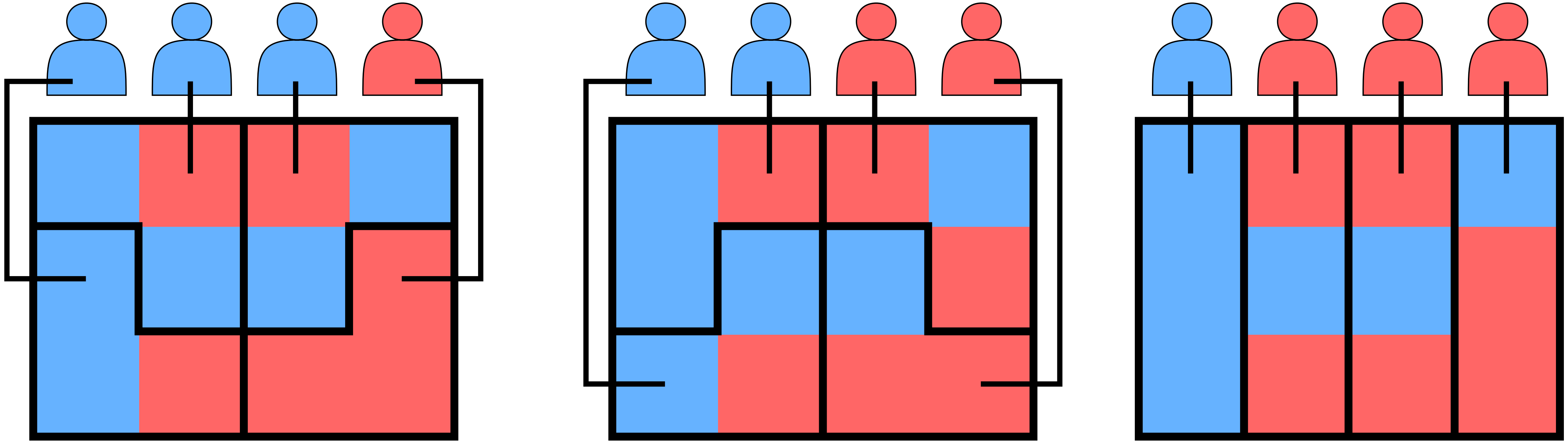
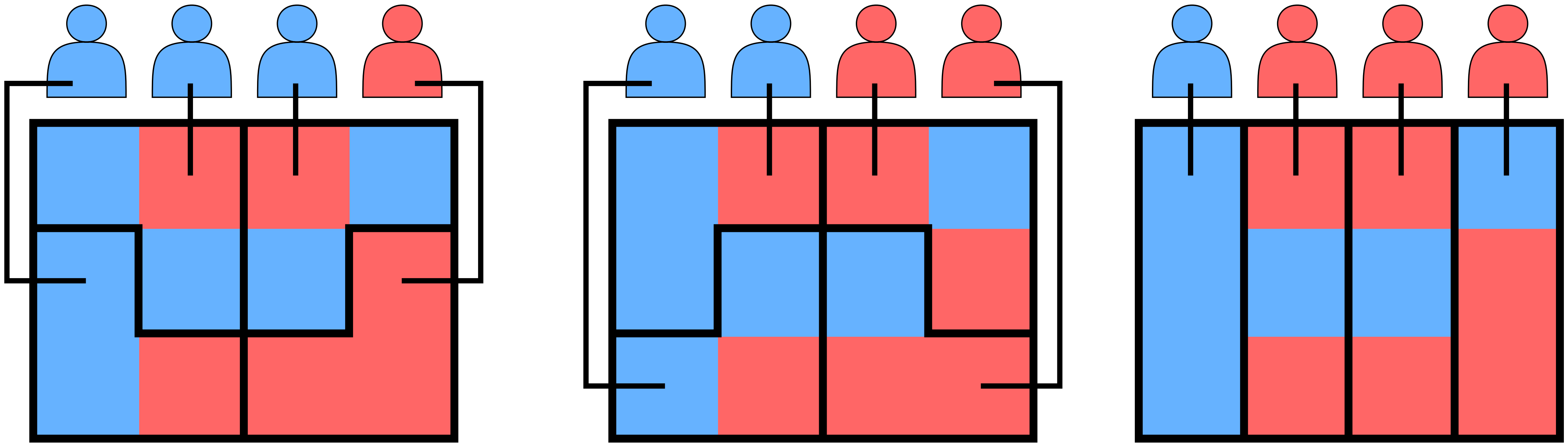
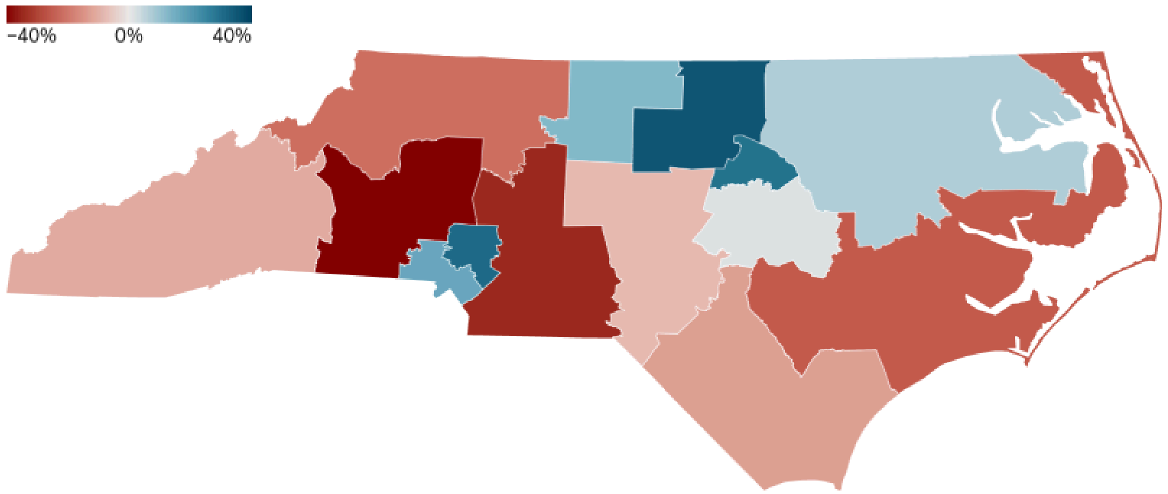


Lectures 21, 22, 23 - Redistricting

Gerrymandering: Drawing district lines to advantage one party or group over another.





Lectures 21, 22, 23 - Redistricting

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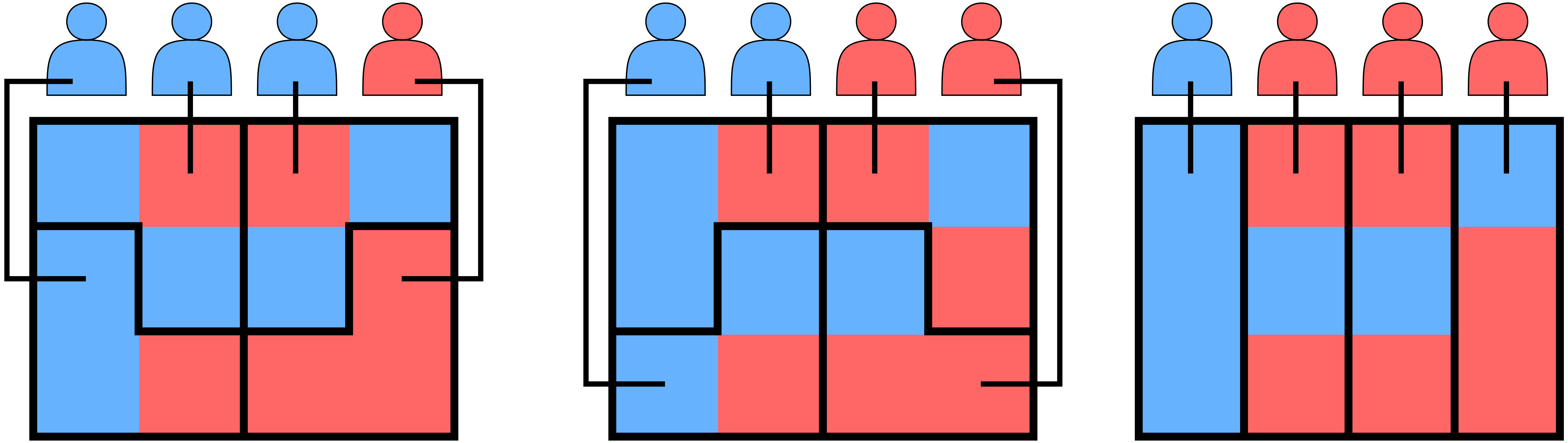

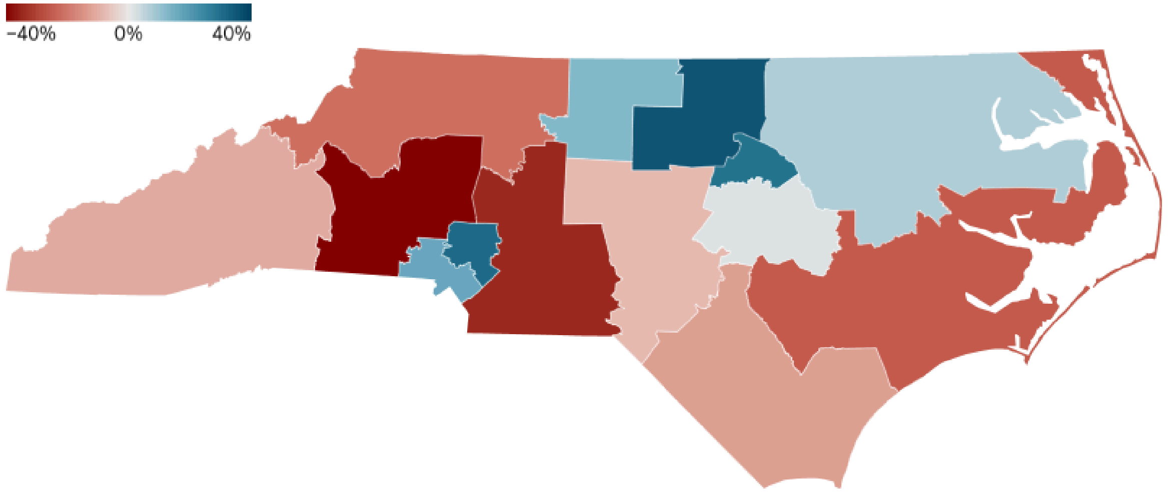

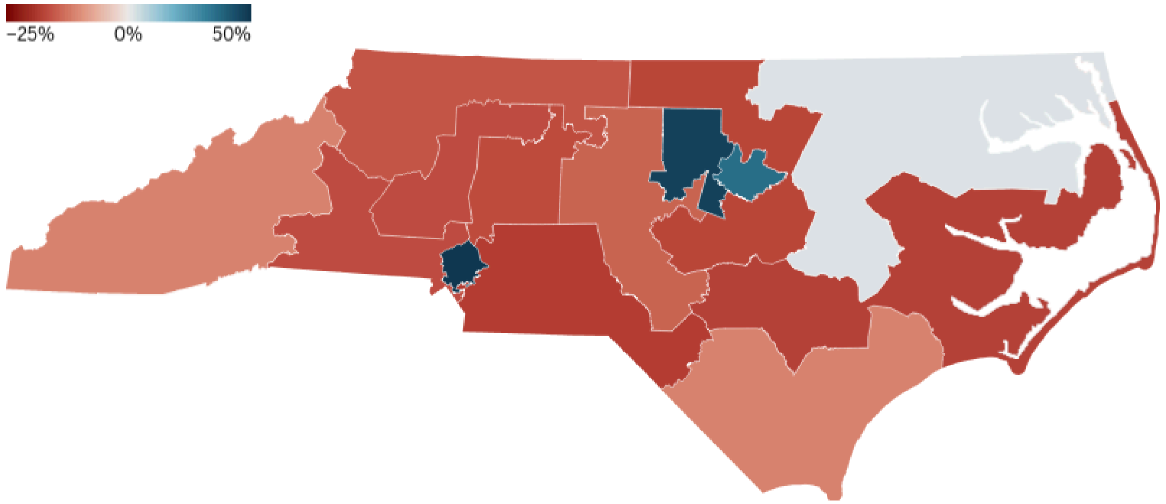

Approaches:

$$\min \sum_{e \in E} y_e$$

$$\text{s.t. } x_{i1} + x_{i2} = 1$$

$$Lk' \leq \sum_{i \in V} p_i x_{i1} \leq Uk'$$

$$L(k-k') \leq \sum_{i \in V} p_i x_{i2} \leq U(k-k')$$

$$x_{u1} - x_{v1} \leq y_e \text{ and } x_{v1} - x_{u1} \leq y_e$$

$$x_{r1} = 1$$

$$x,\ y \text{ binary.}$$

Treat as an optimization problem

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Treat as an optimization problem

Achieving no county splits
(Buchanan, Ezazipour, Shahmizad, 2023)

Design a game

Approaches:

$$
\begin{aligned}
\min \sum_{e \in E} y_e \\
\text{s.t. } & x_{i1} + x_{i2} = 1 \\
& Lk' \le \sum_{i \in V} p_i x_{i1} \le Uk' \\
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Treat as an optimization problem

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Treat as an optimization problem

Fair majority voting (Balinski, 2018)

Achieving no county splits (Buchanan, Ezazipour, Shahmizad, 2023)

Lectures 21, 22, 23 - Redistricting

Approaches:

Design a game

$$\min \sum_{e\in E} y_e$$
$$\text{s.t. } x_{i1}+x_{i2}=1$$
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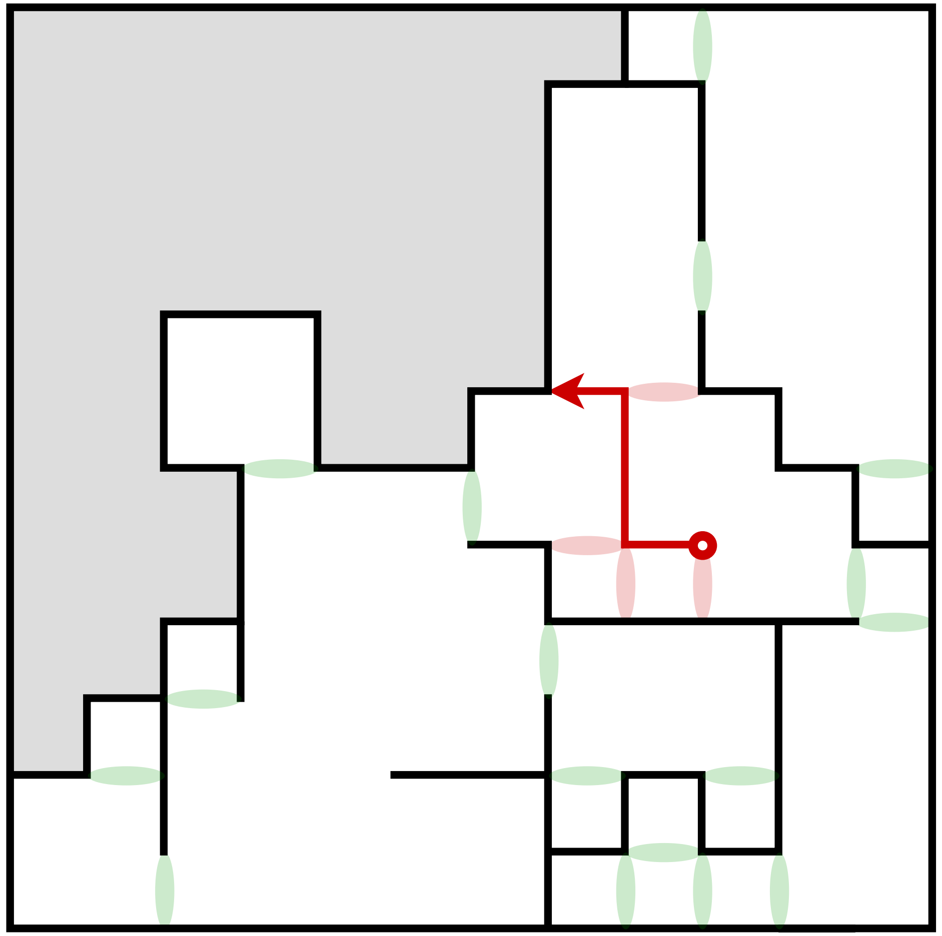
Treat as an optimization problem



Sample many "random" maps

Fair majority voting (Balinski, 2018)

Achieving no county splits (Buchanan, Ezazipour, Shahmizad, 2023)

Lecture 21 - Design a Game

Does Divide-and-Choose work?

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GAME:

1. Party 1 splits the state into two pieces
2. Party 2 chooses a piece
3. Each party redistricts the piece they control

Lecture 21 - Design a Game

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Turns out that this gives the divider (party 1) too much power.

Lecture 21 - Design a Game

Does Divide-and-Choose work?

GAME:

1. Party 1 splits the state into two pieces
2. Party 2 chooses a piece
3. Each party redistricts the piece they control

Turns out that this gives the divider (party 1) too much power.

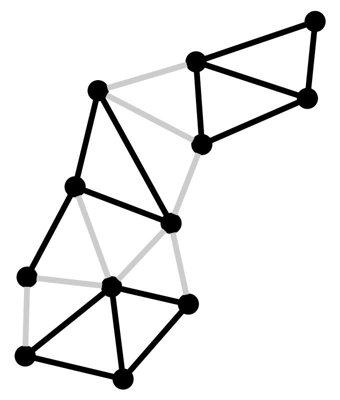
The *State-Cutting* game

GAME:

1. Each party i draws a region X_i of arbitrary size
2. Whichever player i drew the region containing a smaller population is the divider, and the other party, j, is the chooser
3. Party i divides X_i into two pieces
4. Party j chooses a piece
5. Party i redistricts their part of X_i and party j redistricts the rest of the state.

Algorithm (ReCom)

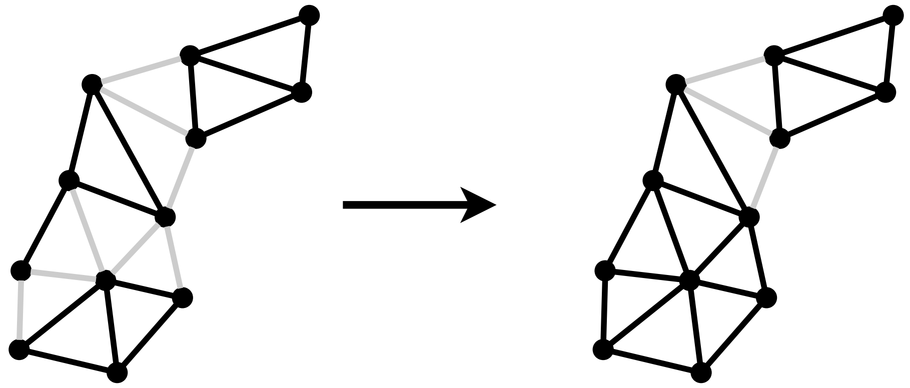
Starting from an arbitrary balanced *k*-partition, repeatedly:



Algorithm (ReCom)

Starting from an arbitrary balanced *k*-partition, repeatedly:

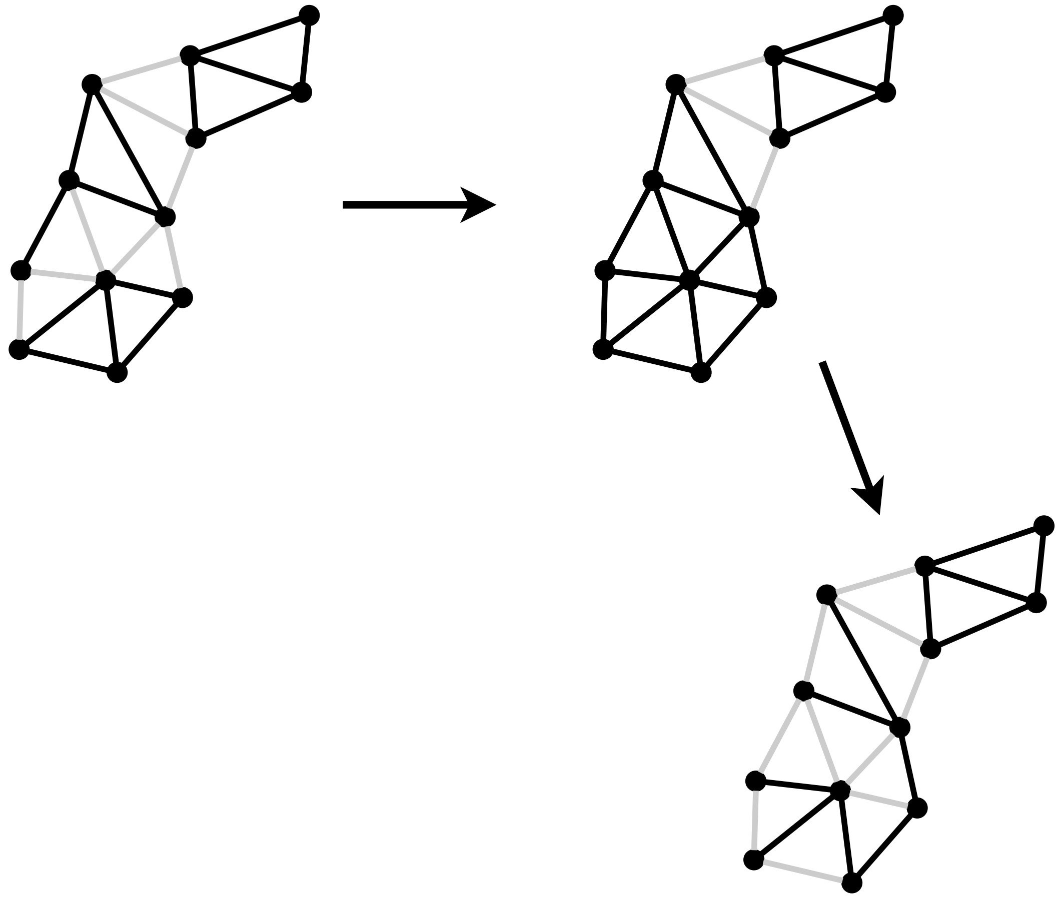
1. Merge two random districts.



Algorithm (ReCom)

Starting from an arbitrary balanced *k*-partition, repeatedly:

1. Merge two random districts.
2. Sample a random spanning tree T on the merged district.



Algorithm (ReCom)

Starting from an arbitrary balanced k-partition, repeatedly:

1. Merge two random districts.
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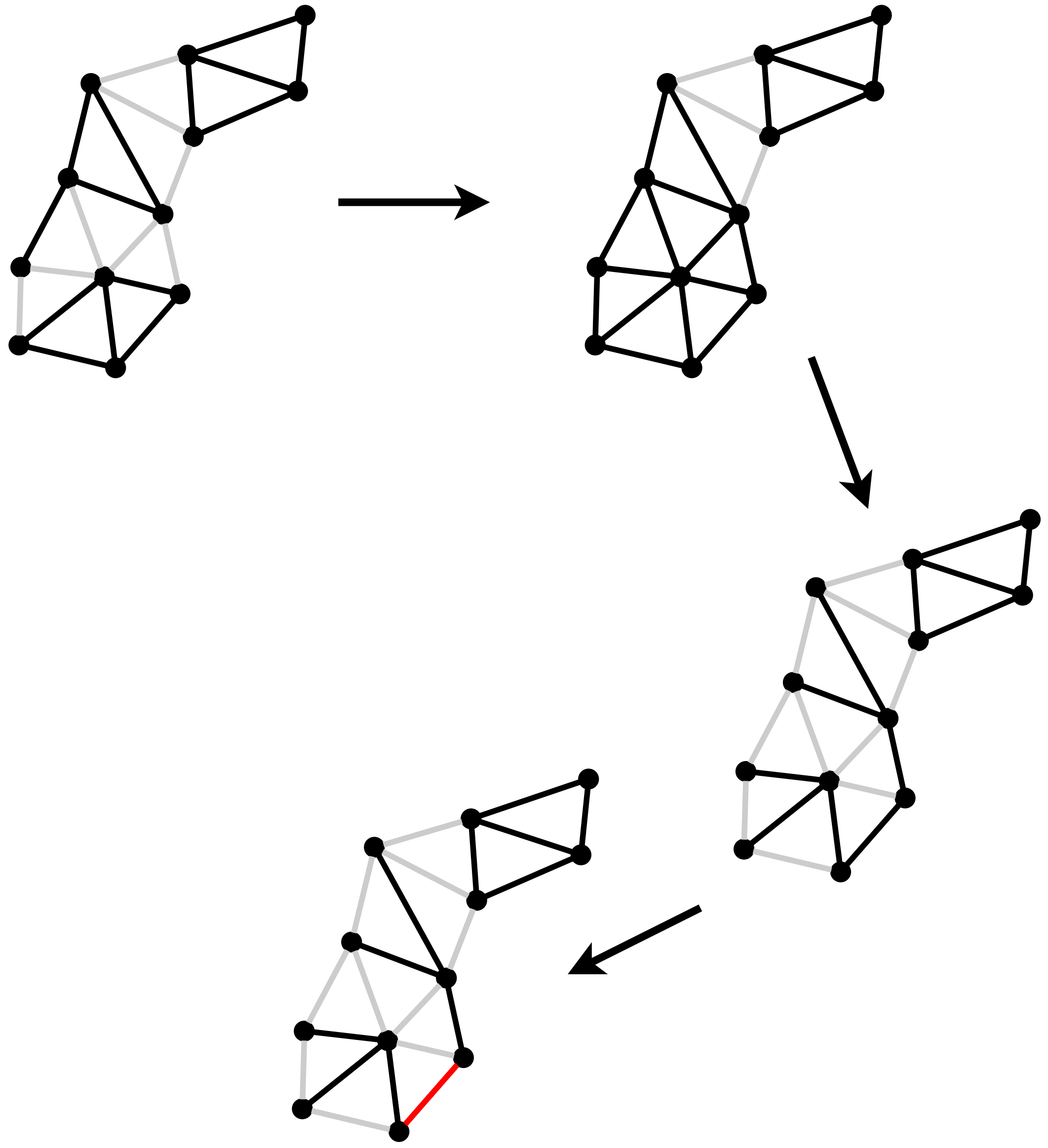
3. Try to split T on a single edge into balanced subtrees. If not possible, start over.



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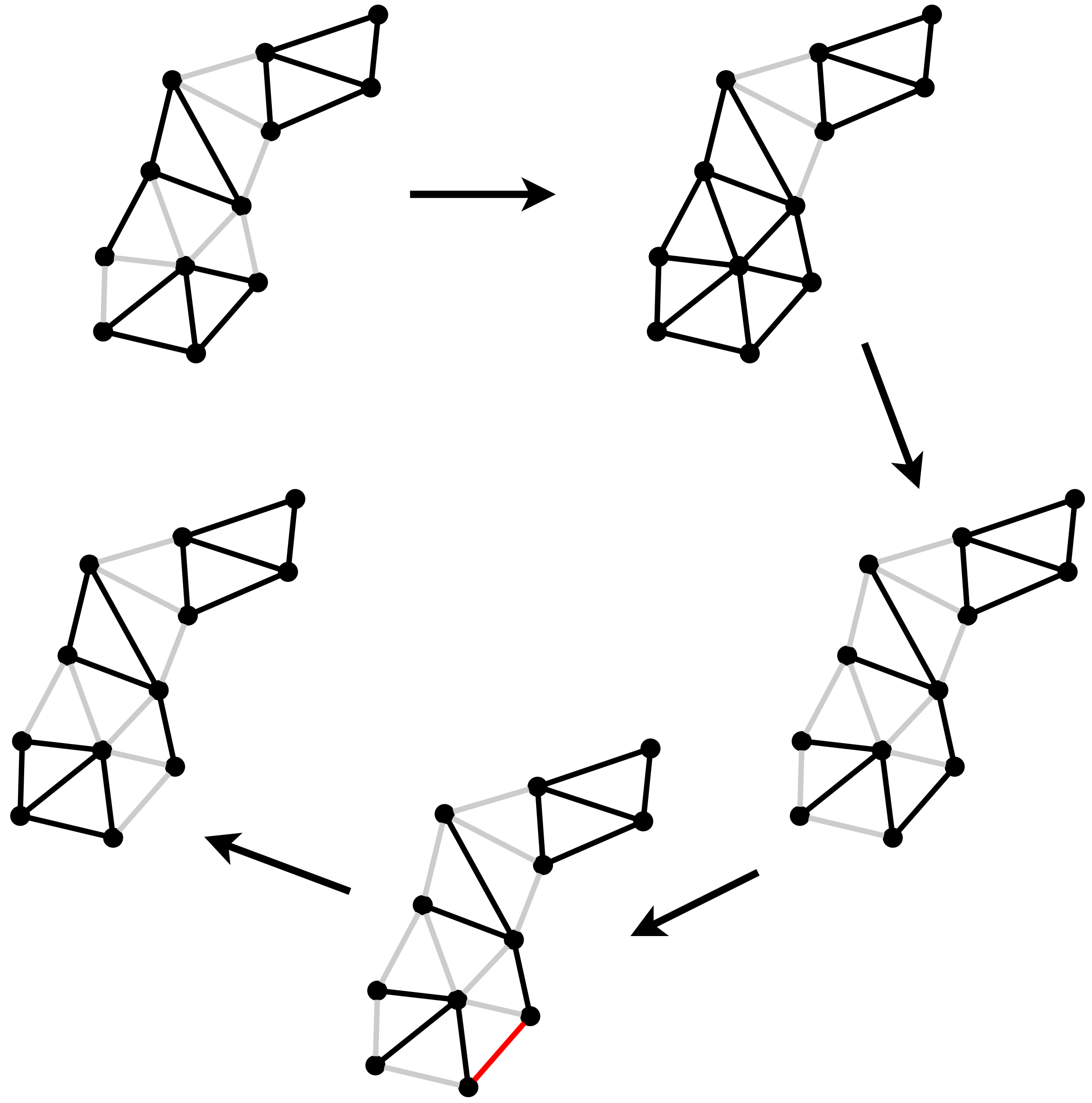
4. Update the k-partition, new parts induced by subtrees.



Lectures 22 and 23 - Sample Many Random Maps

Algorithm (ReCom)

Starting from an arbitrary balanced k-partition, repeatedly:

1. Merge two random districts.
2. Sample a random spanning tree T on the merged district.
3. Try to split T on a single edge into balanced subtrees. If not possible, start over.
4. Update the k-partition, new parts induced by subtrees.

Run for many steps, then output the final partition.