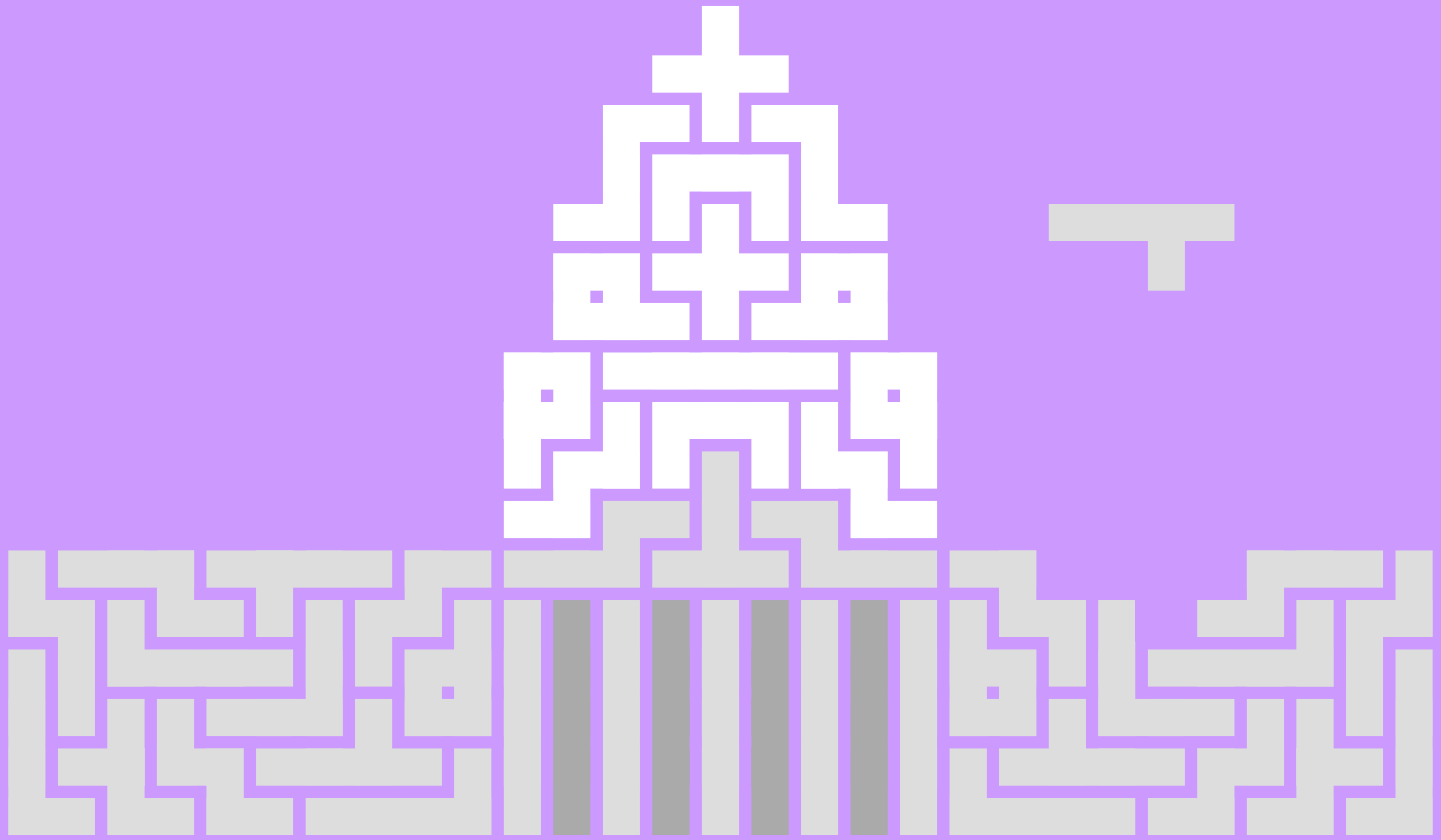

Algorithms For Democratic Decision-Making

Jamie Tucker-Foltz • Yale University • Spring 2026

Lecture 3: **Arrow's Theorem**

Announcements

Office hours: Mondays and Thursdays, 1-2pm
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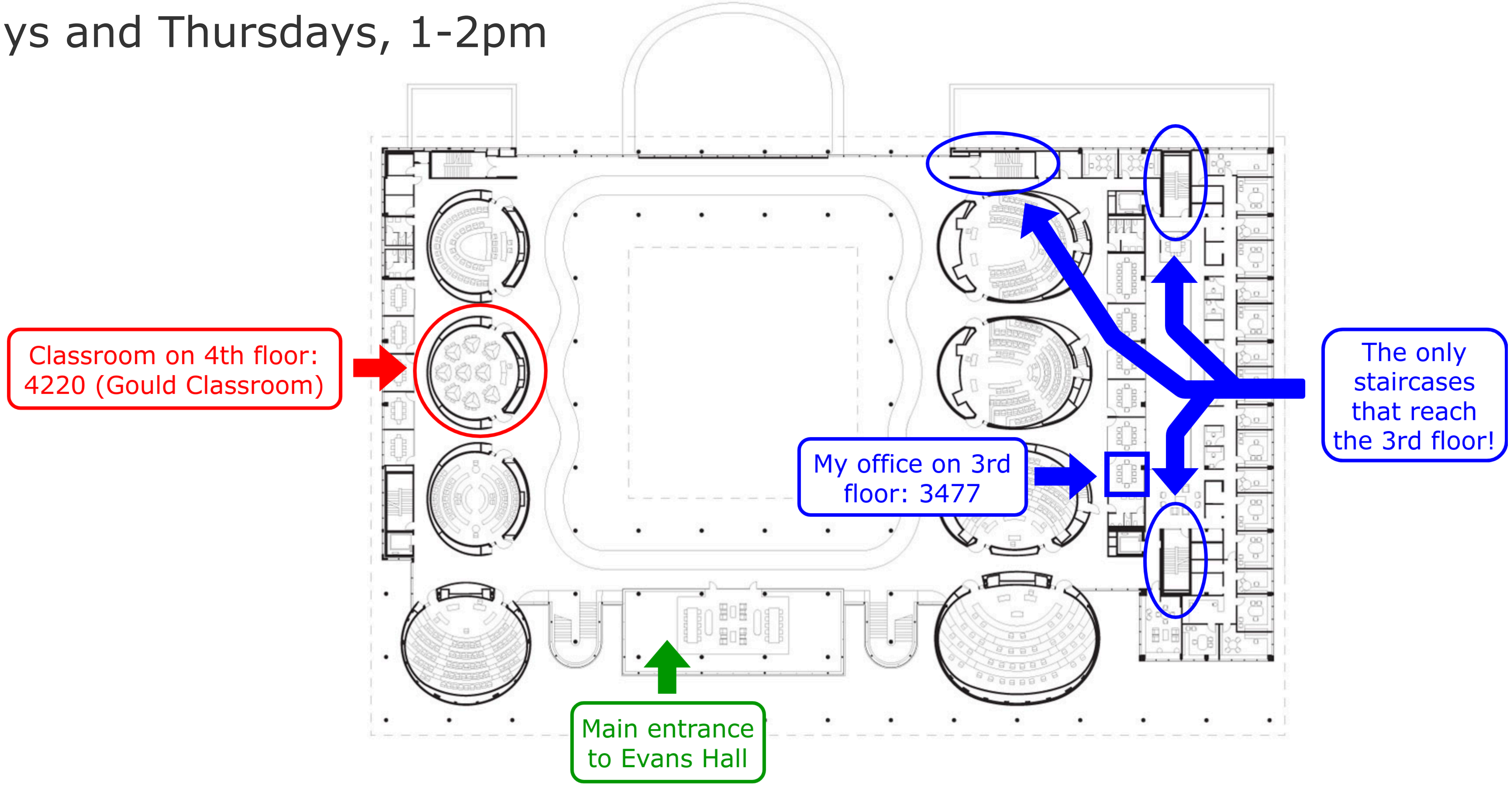

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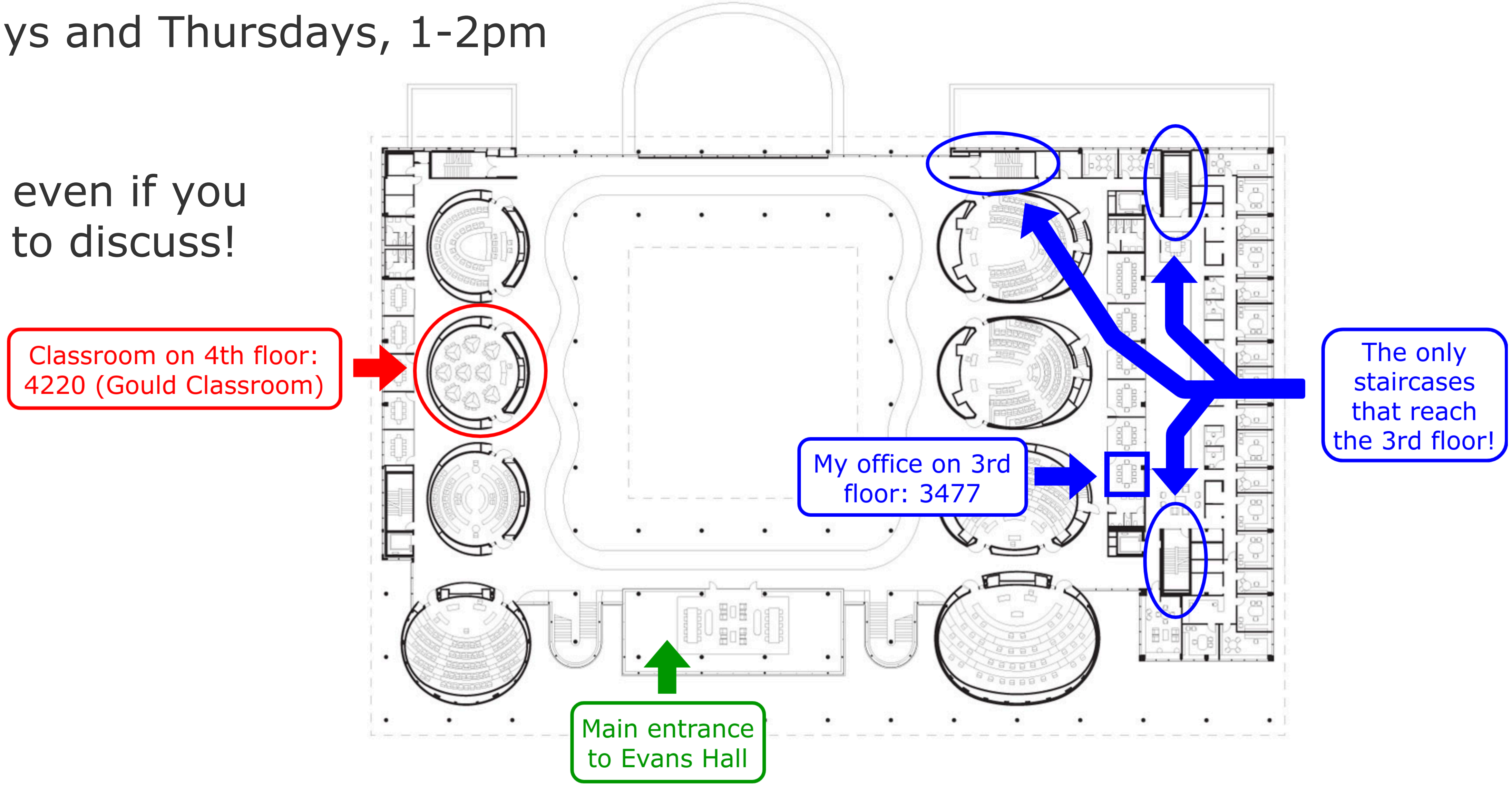

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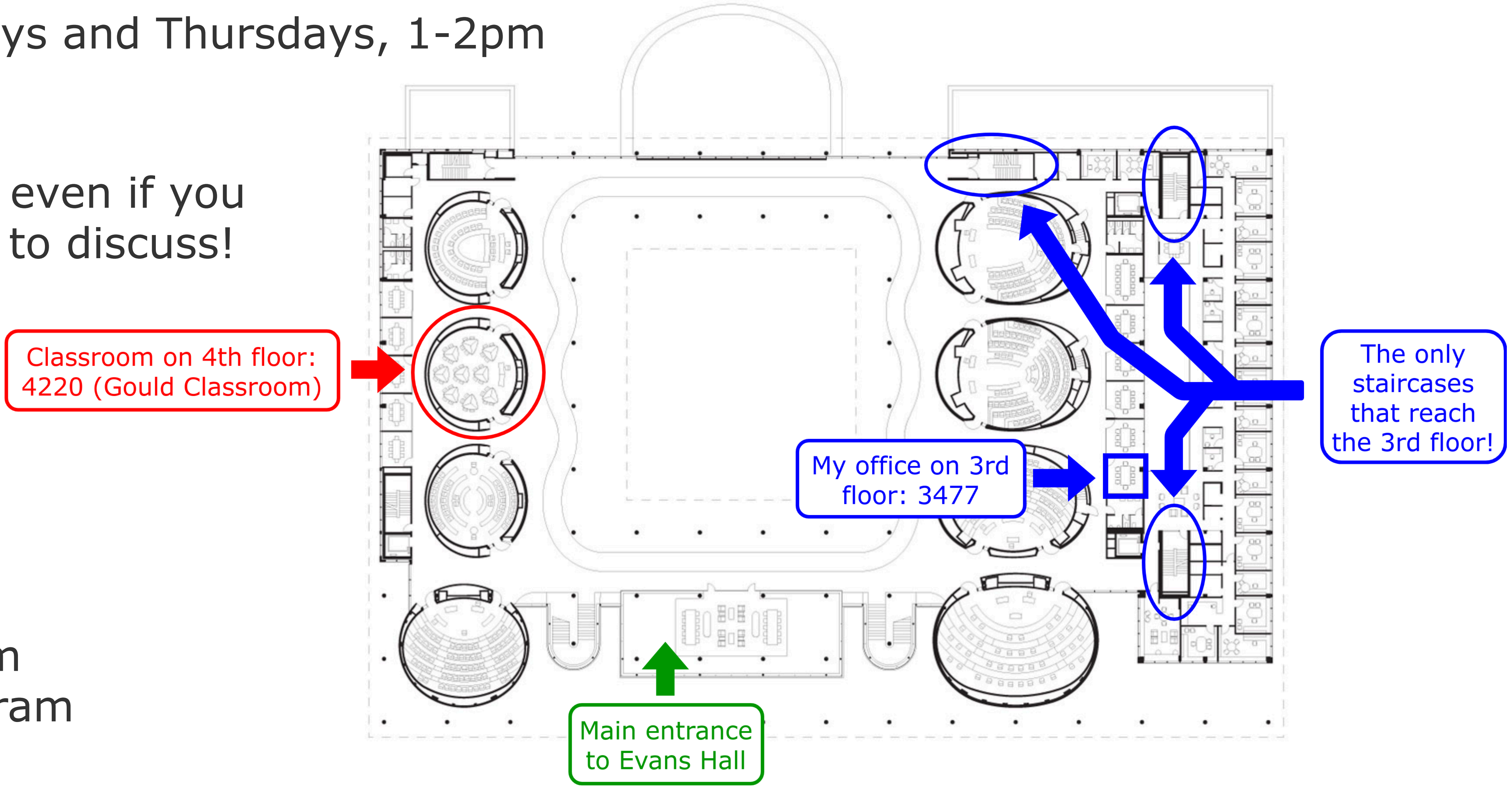

Introductions:

- Name
- Where you're from
- Year in your program
- Department
- Research interest(s)
- Something interesting about you

The birth of modern social choice theory

Kenneth Arrow
(1921-2017)

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Theorem 1: Capitalism works.

Theorem 2: Democracy doesn't.

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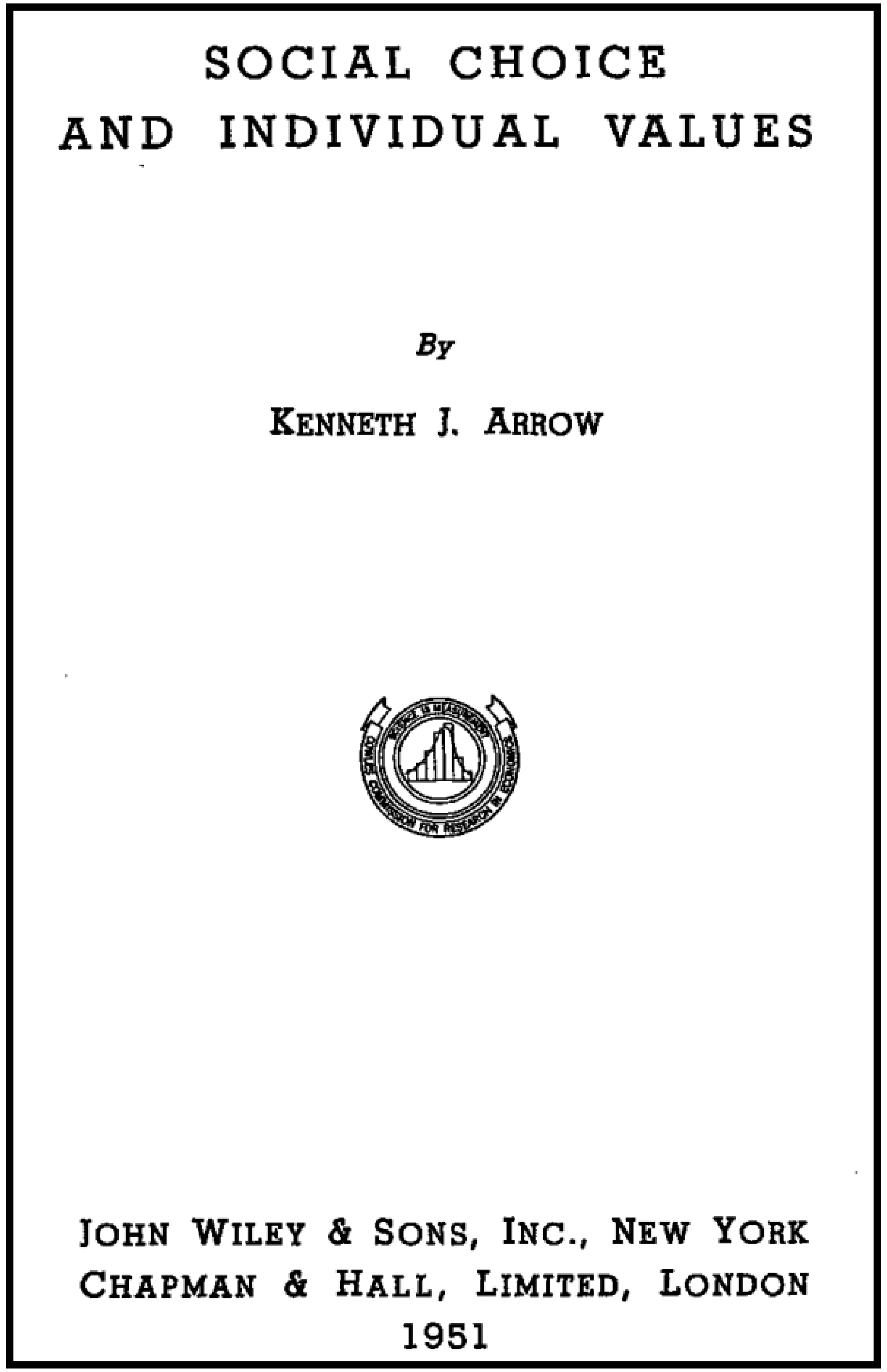

SOCIAL CHOICE
AND INDIVIDUAL VALUES

By

KENNETH J. ARROW

JOHN WILEY & SONS, INC., NEW YORK
CHAPMAN & HALL, LIMITED, LONDON
1951

The "Axiomatic Approach"

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(A) Thunder Bay

(B) Lakeview

(C) The Lakeview

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15,870 votes	(A) Thunder Bay
15,302 votes	(B) Lakeview
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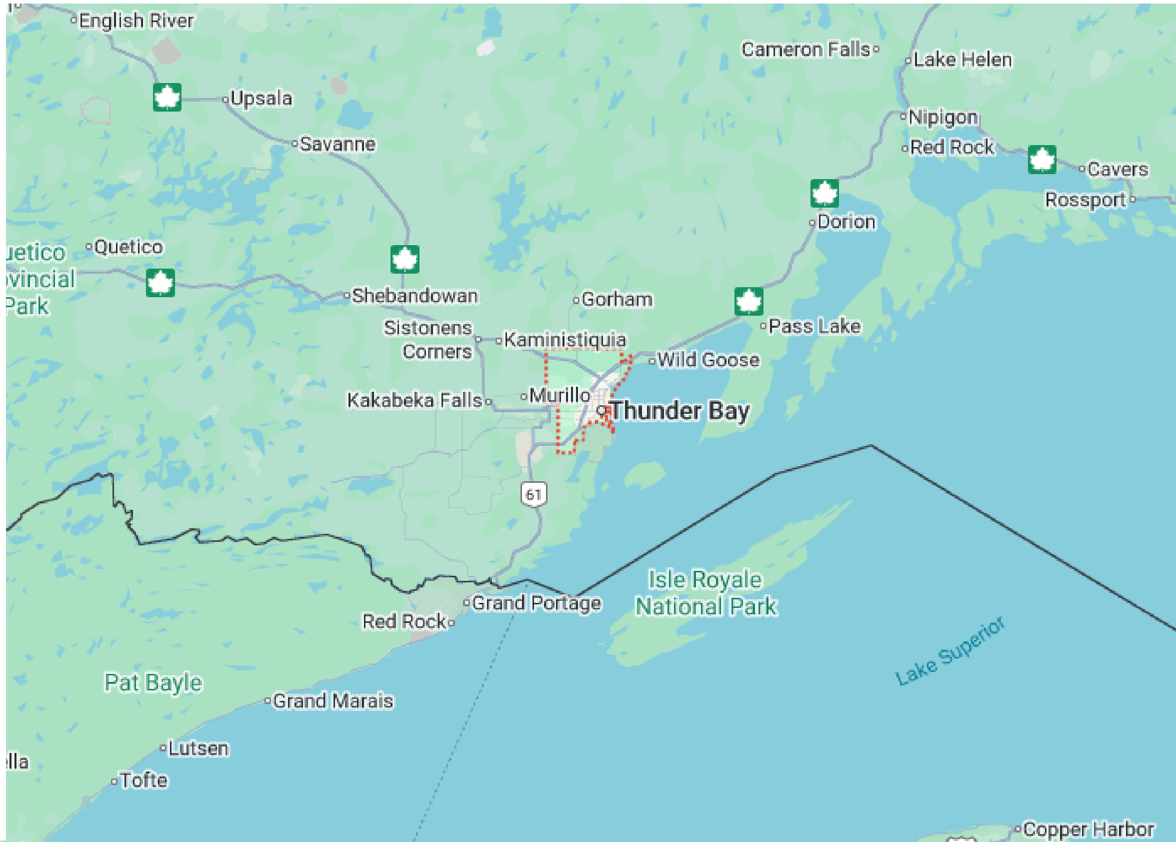

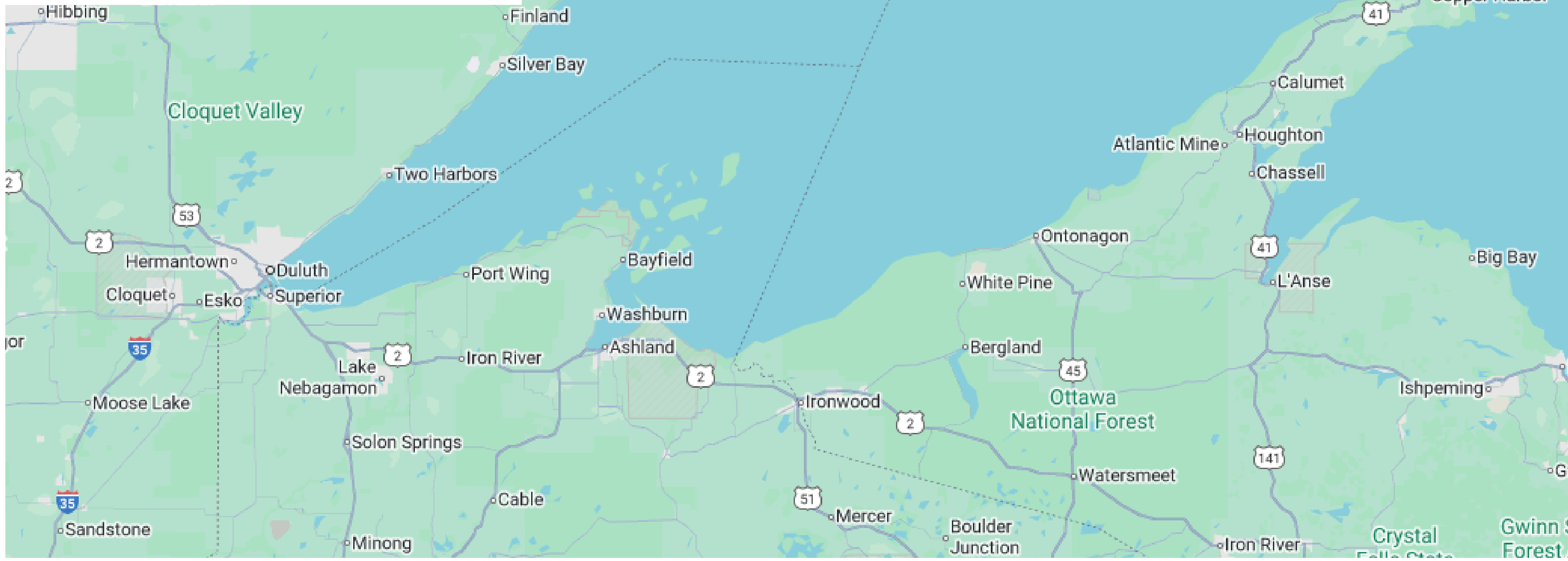

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▸ Which of these axioms does IRV satisfy?
(A) Monotonicity only
(B) Independence of clones only
(C) Both
(D) Neither

Respond at:

pollev.com/jtuckerfoltz255 or

bit.ly/jtfpoll or

text jtuckerfoltz255 to 37607

IRV properties

Proposition

IRV satisfies independence of clones.

Proof. Suppose a and b are clones in P_1 and get merged in P_2. Consider running IRV on P_1 and P_2. The first time they differ, it must be that some clone is eliminated in P_1 but not P_2.

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a	b	c
b	c	a
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IRV → c

Which voting rule is the best?

https://en.wikipedia.org/wiki/Comparison_of_electoral_systems#Compliance_of_selected_single-winner_methods

Two other impossibility theorems

Theorem (Muller-Satterthwaite)

When there are at least 3 alternatives, there is no social choice function satisfying the following three axioms:

- *Pareto Efficiency - If every voter ranks a first, a wins.*

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Note: This is stronger than definition from previous slide!

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Muller-Satterthwaite $\implies$ *Gibbard-Satterthwaite*

Proof. It suffices to show:

(1) SP $\implies$ Monotone
(2) Onto + Monotone $\implies$ PE

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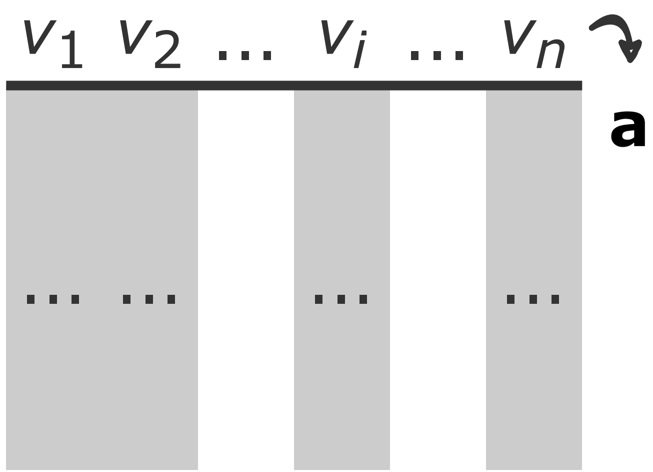

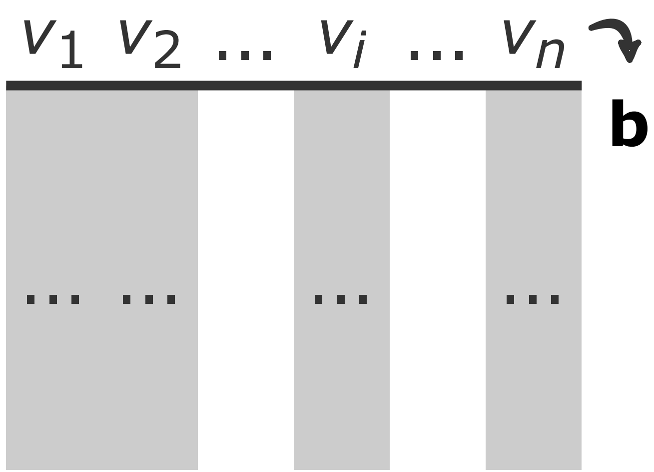

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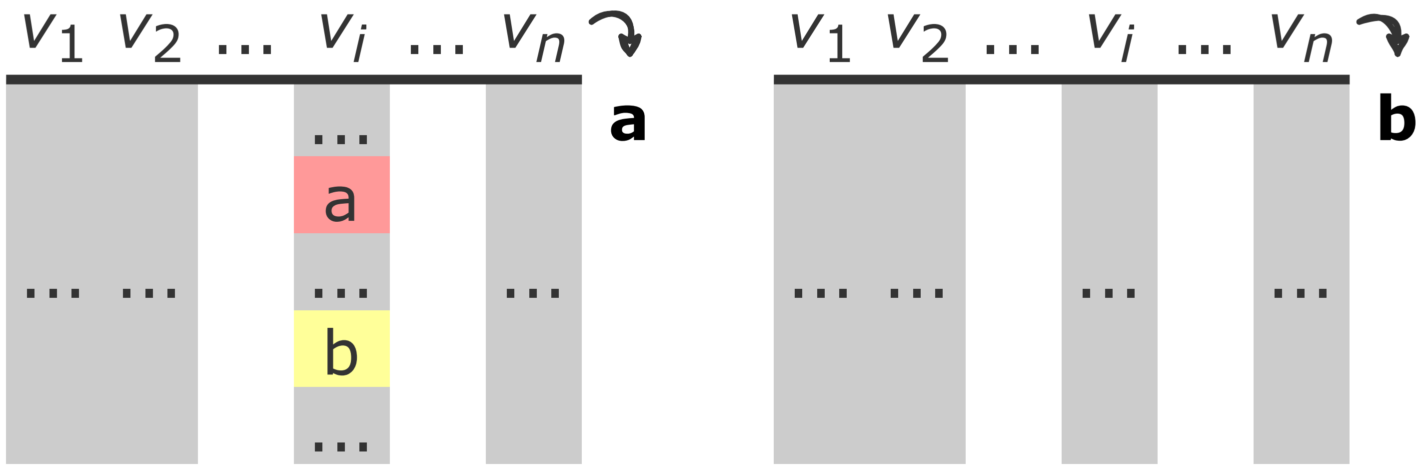

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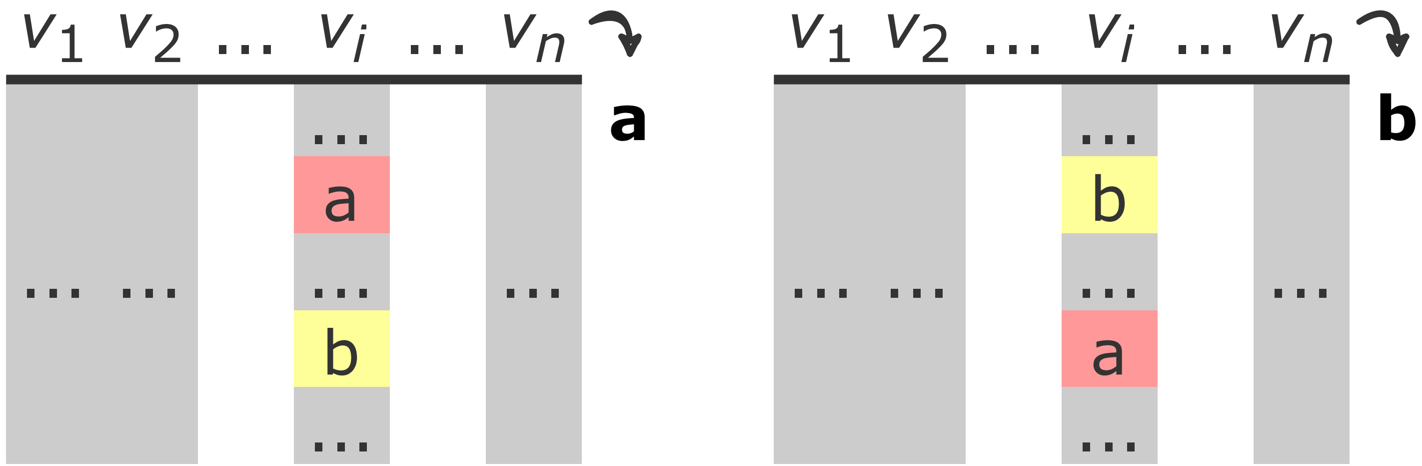

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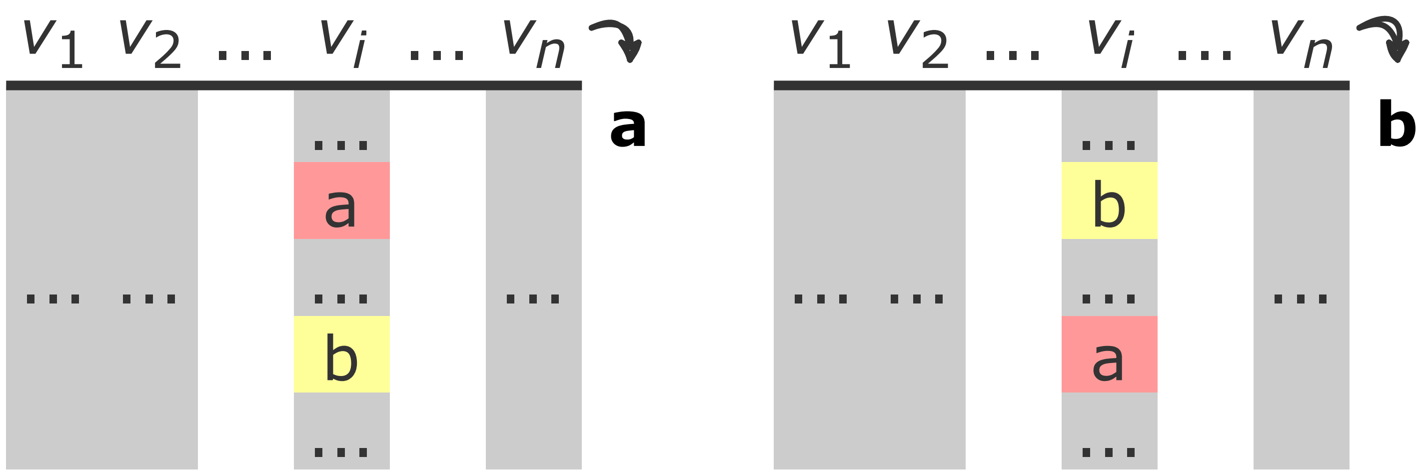

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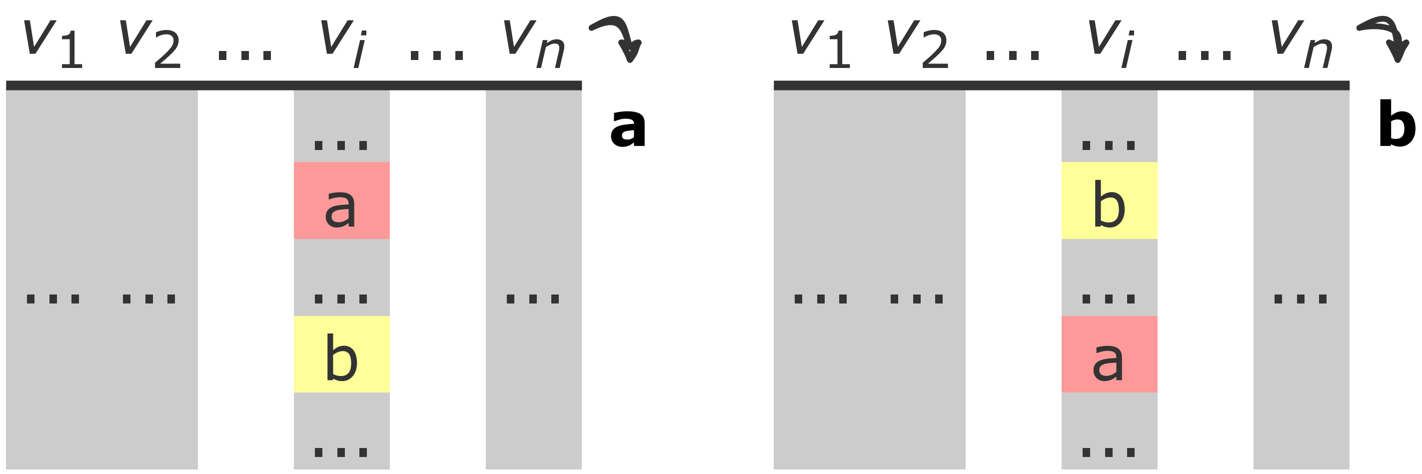

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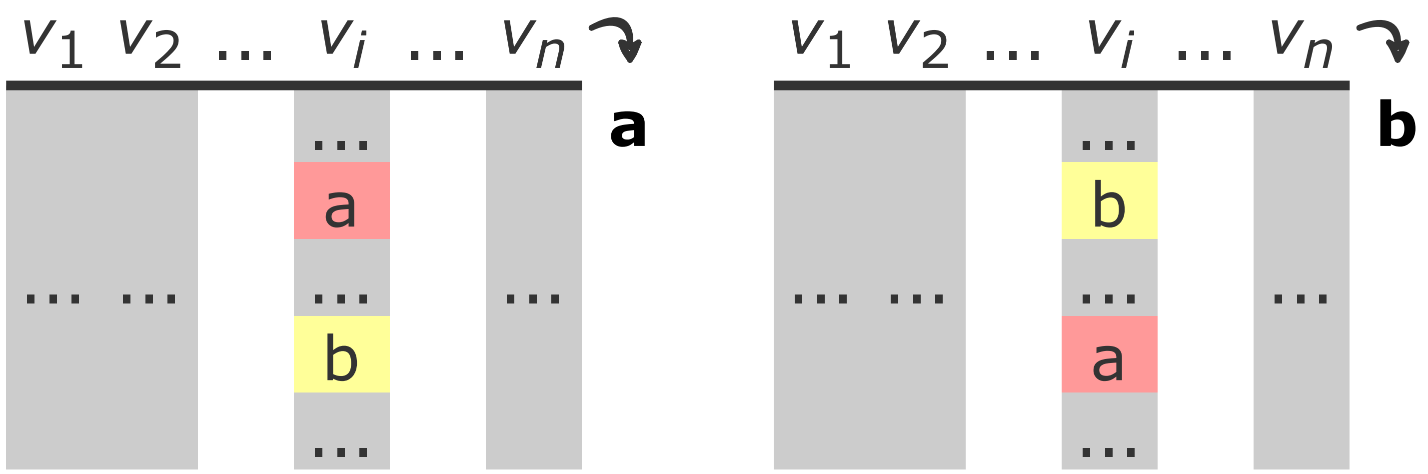

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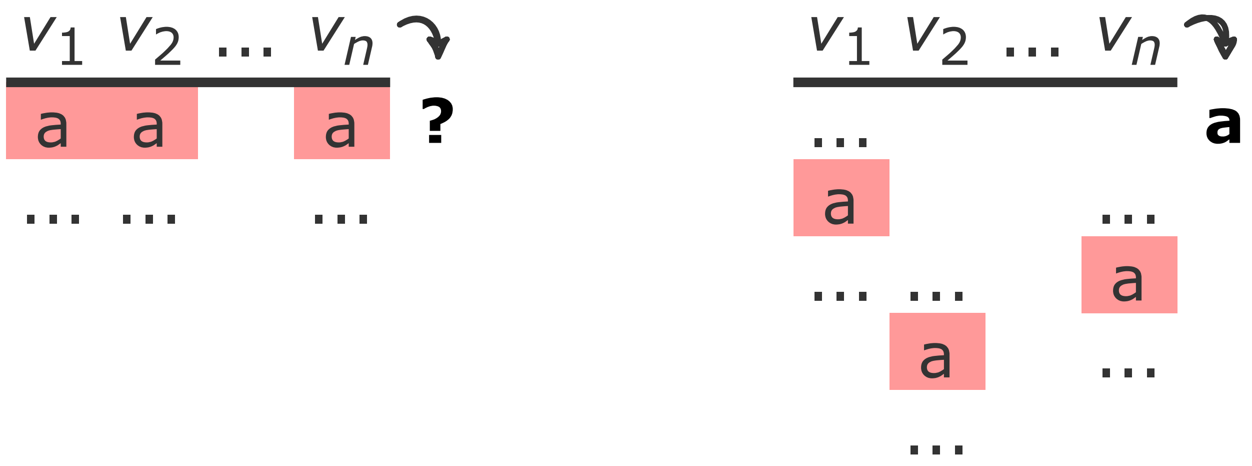

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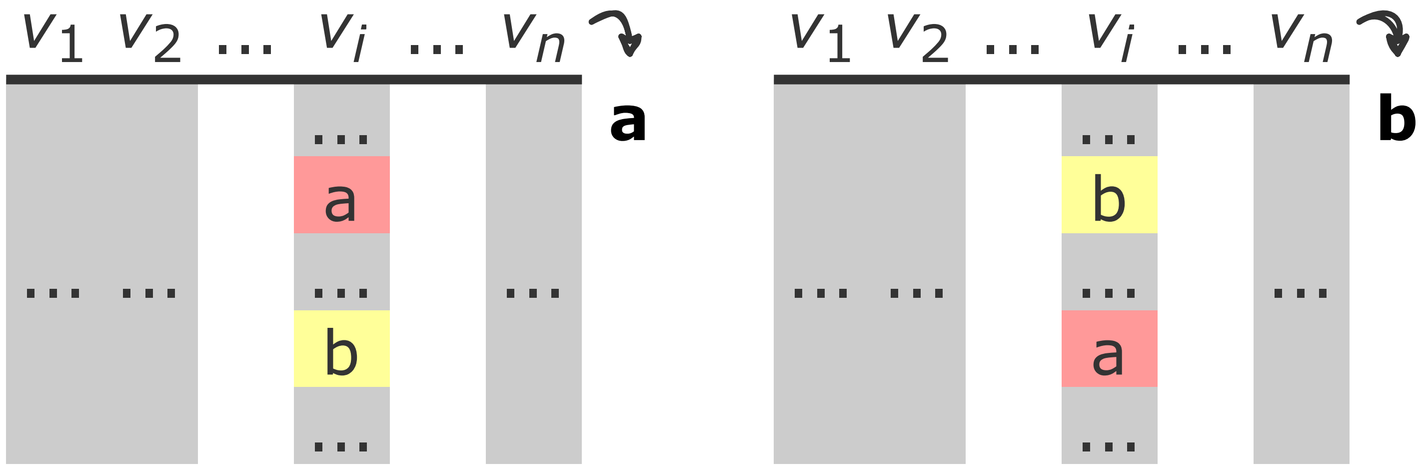

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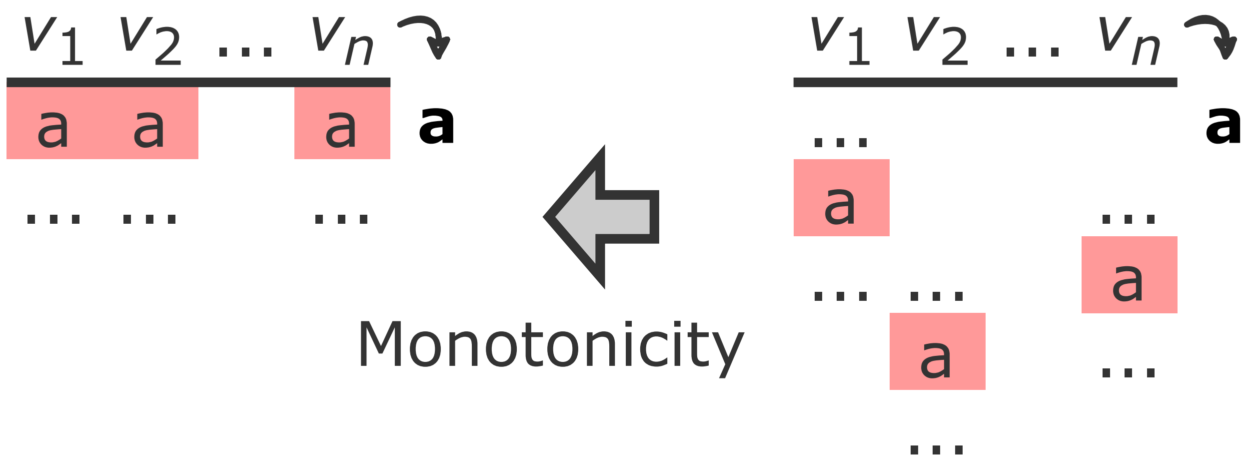

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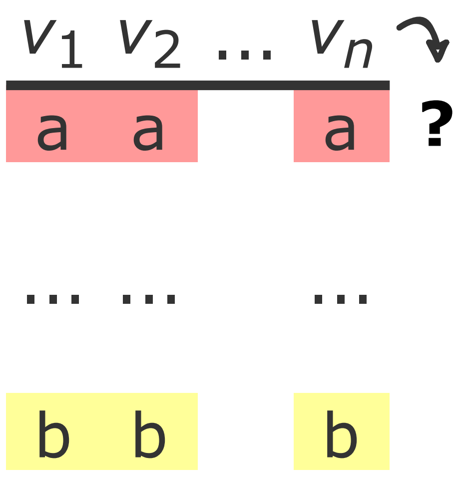

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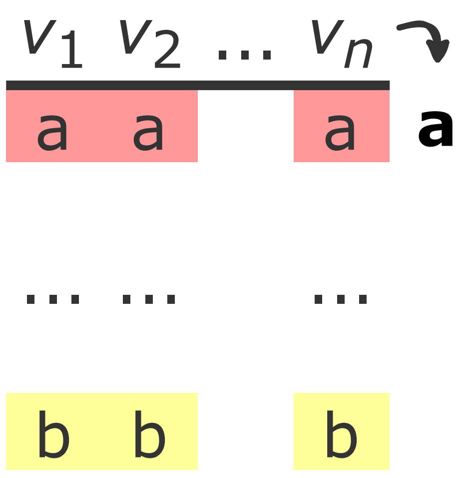

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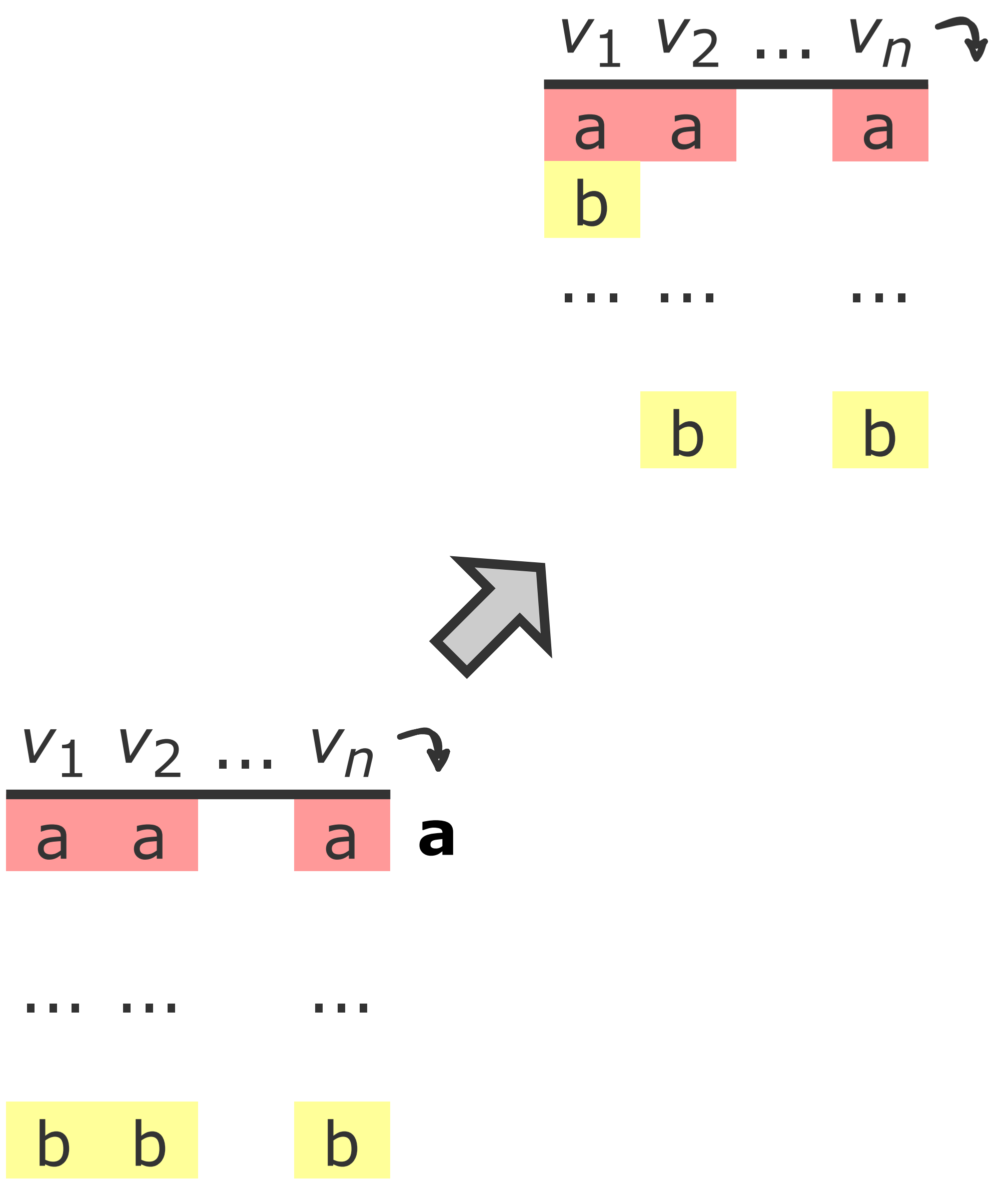

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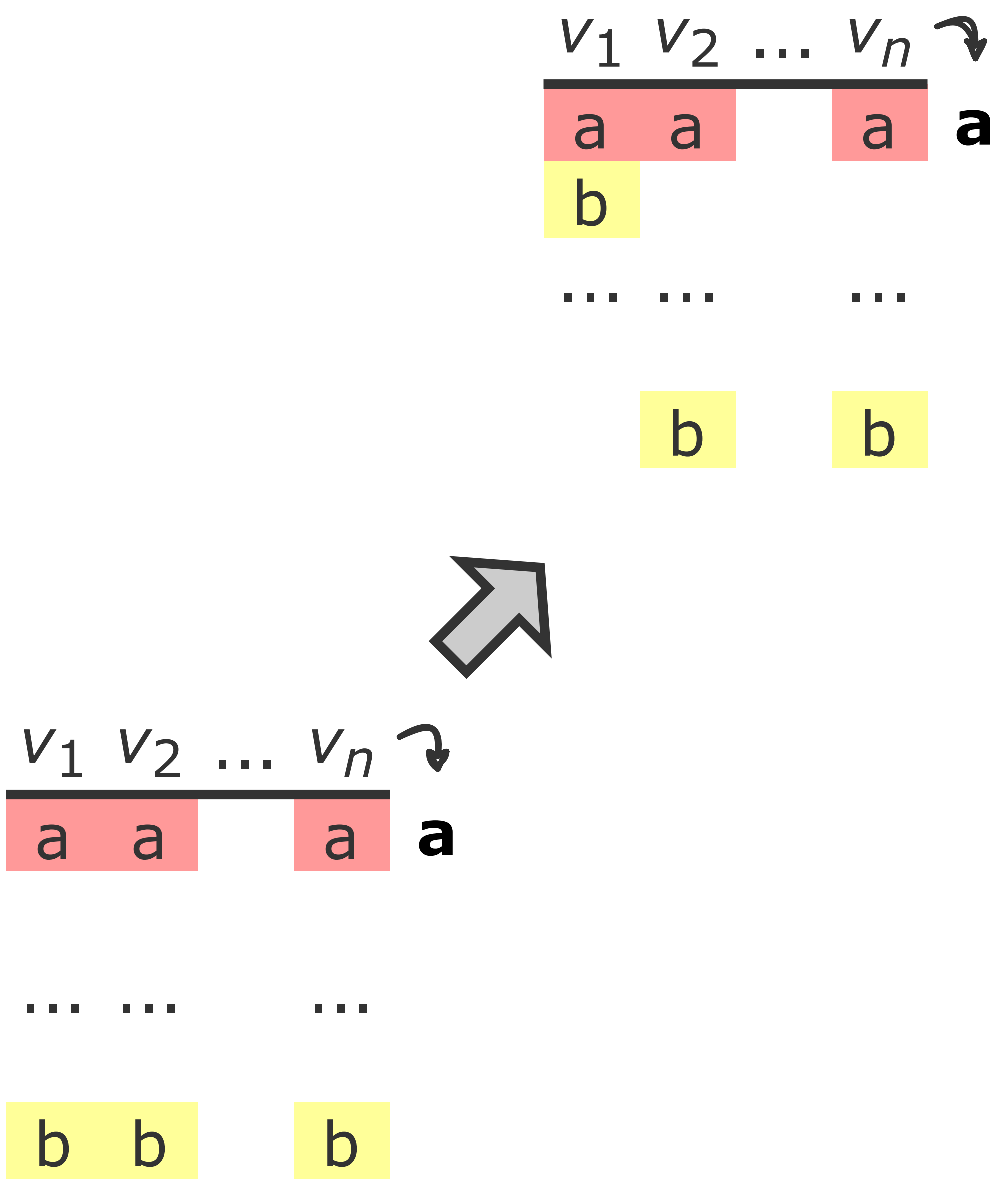

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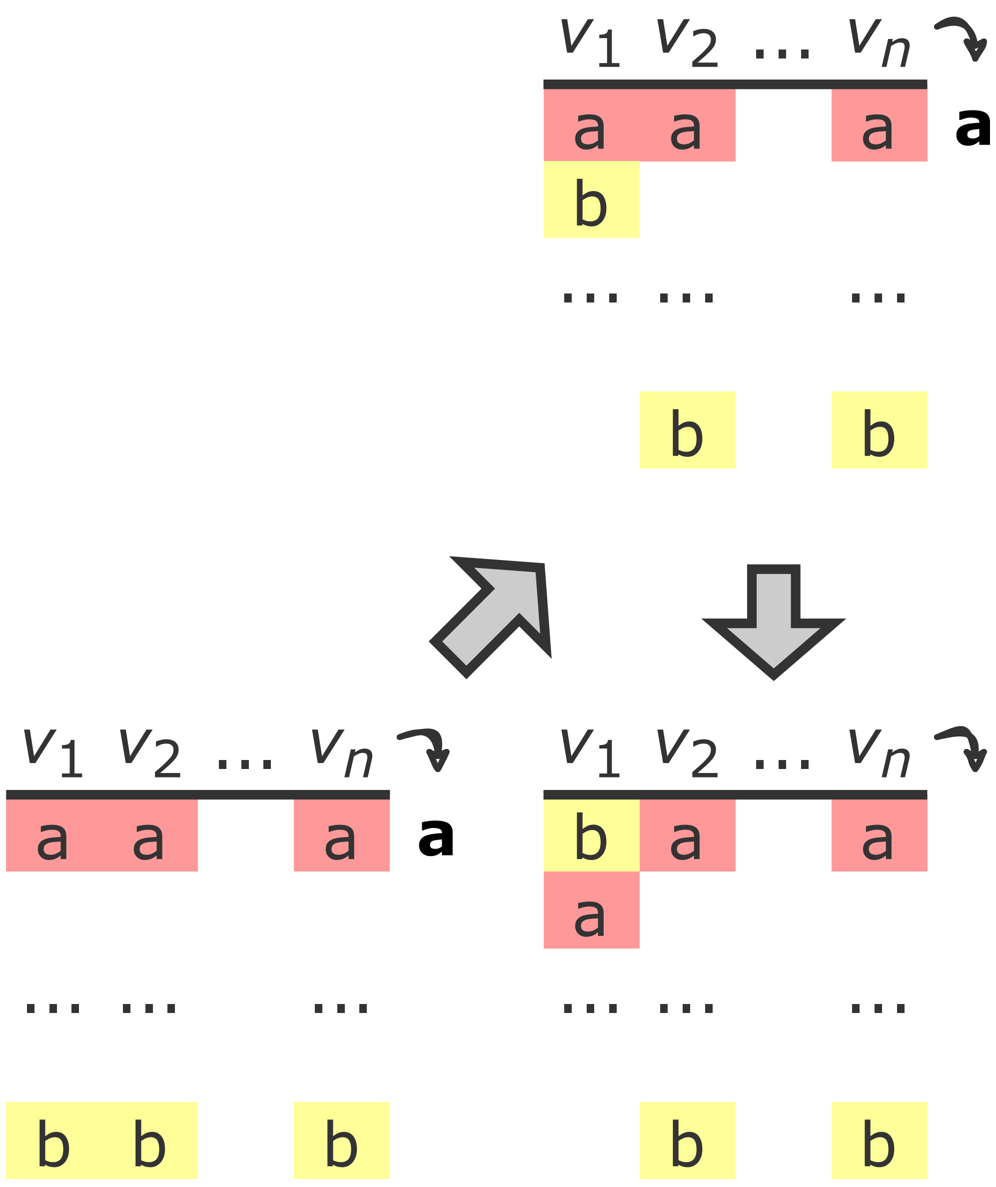

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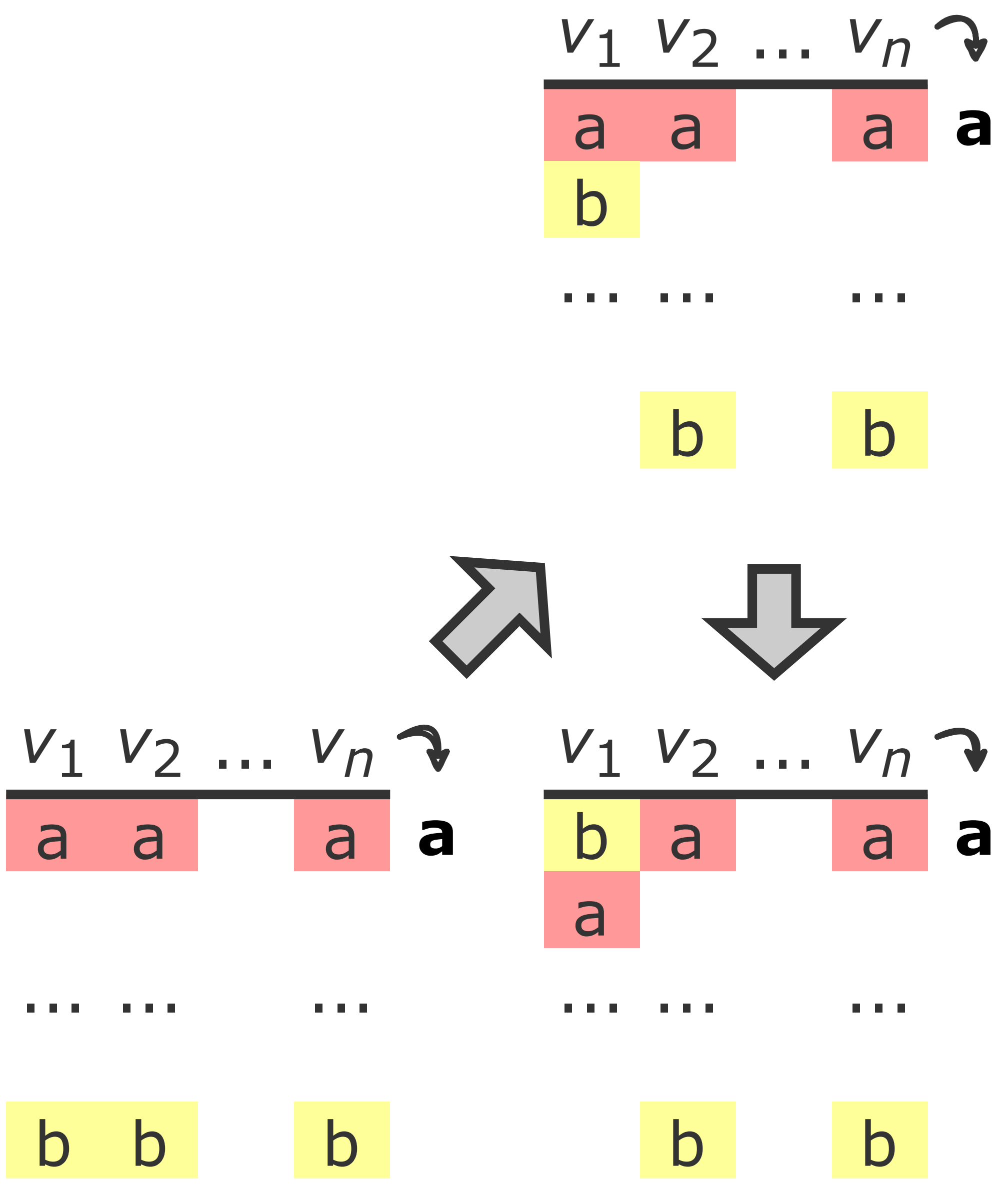

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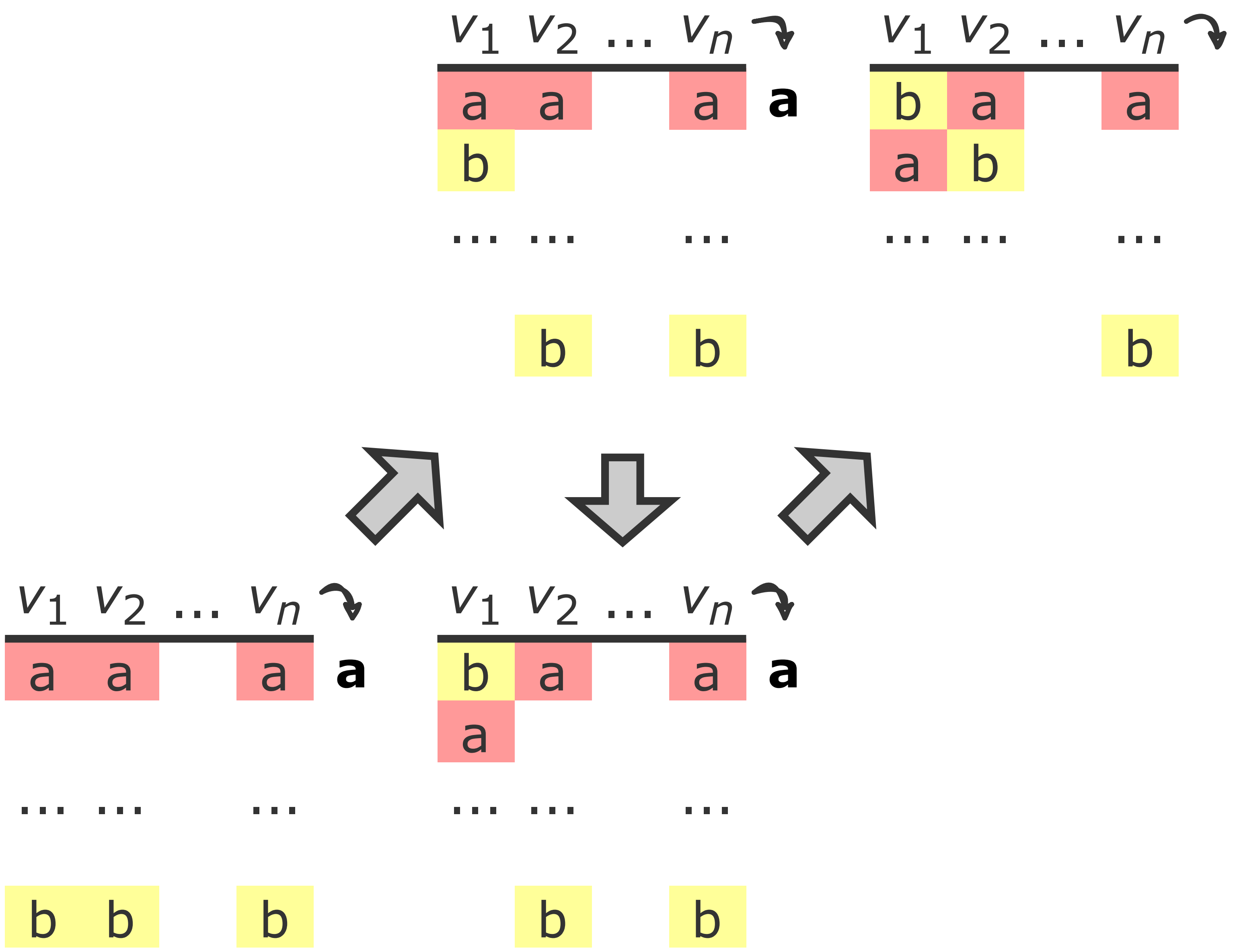

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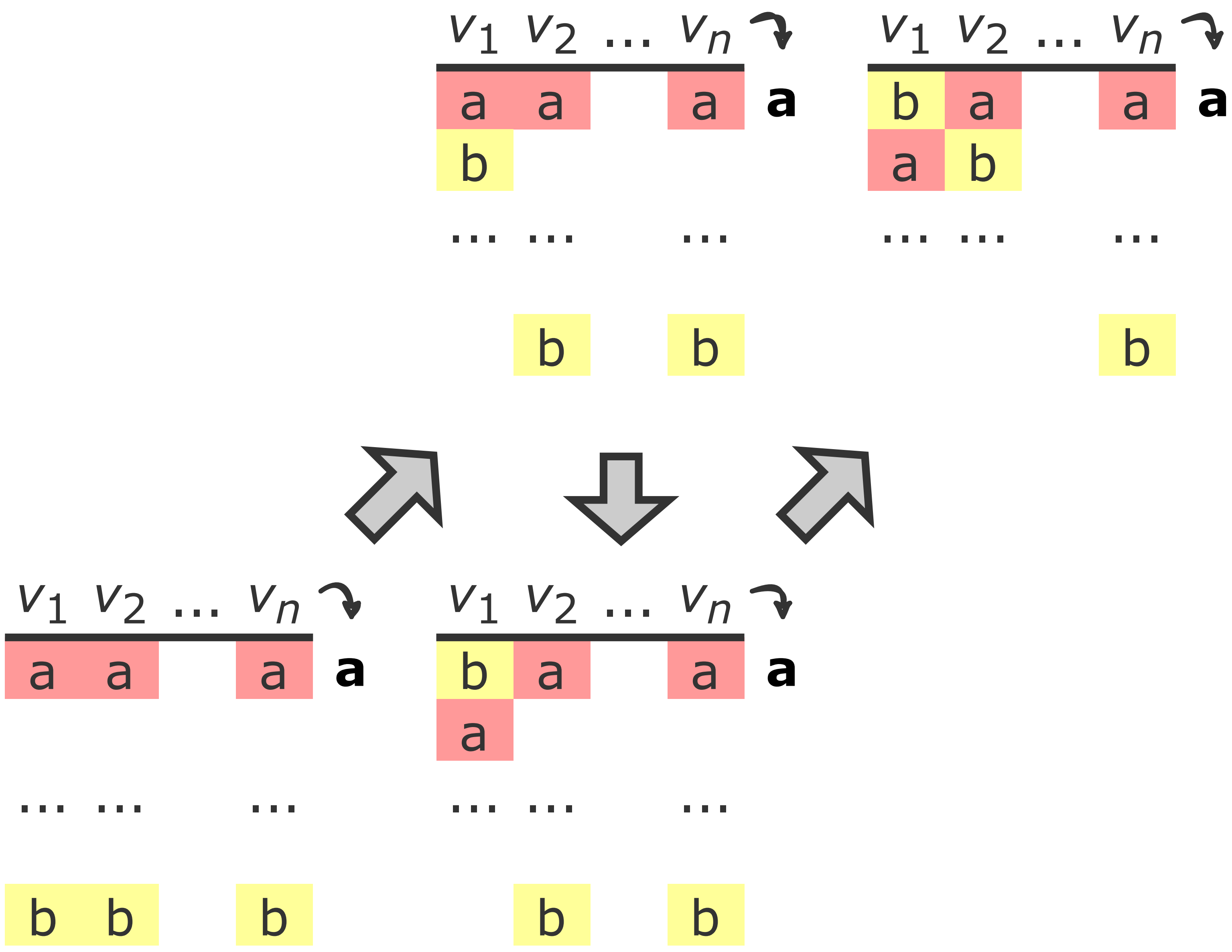

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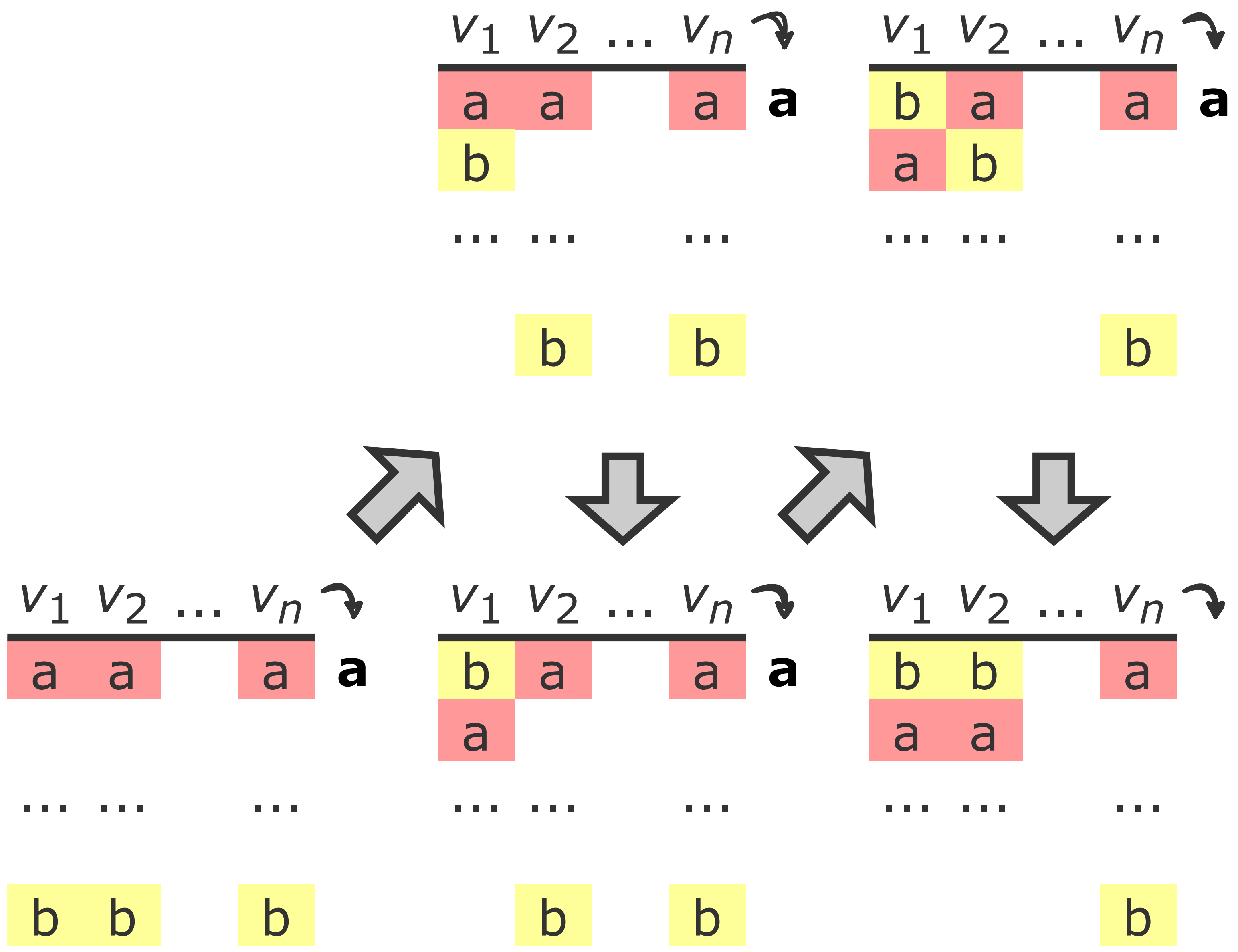

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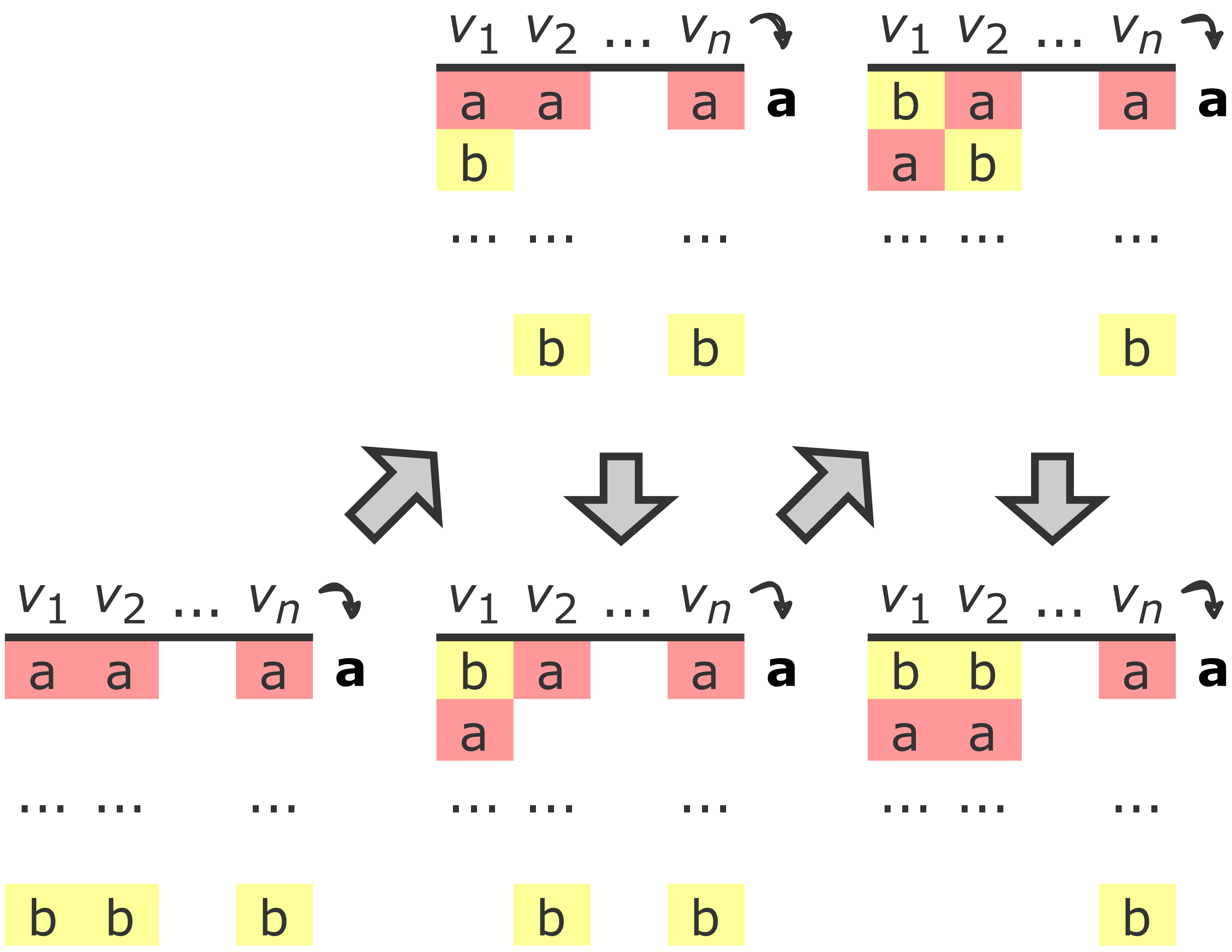

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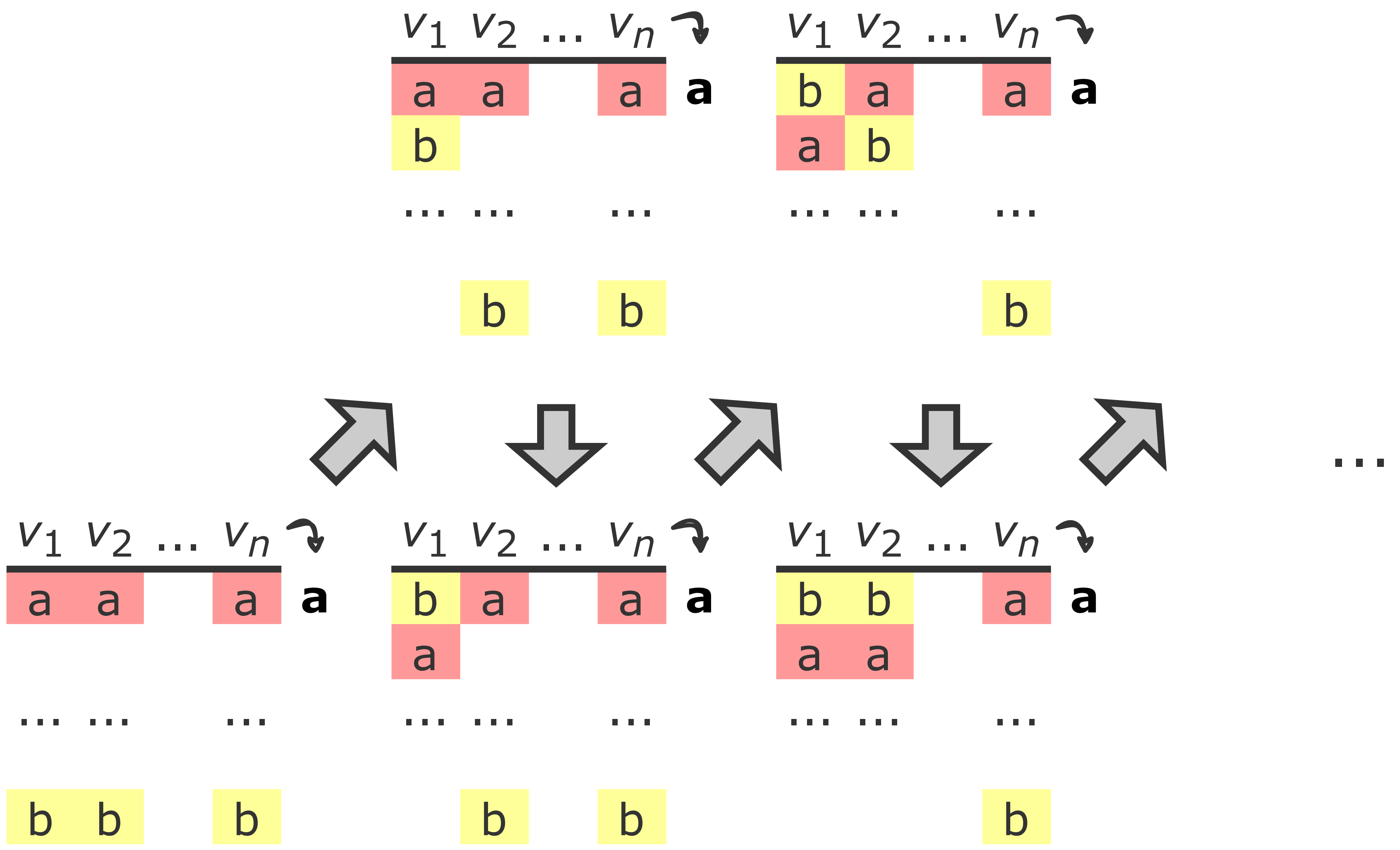

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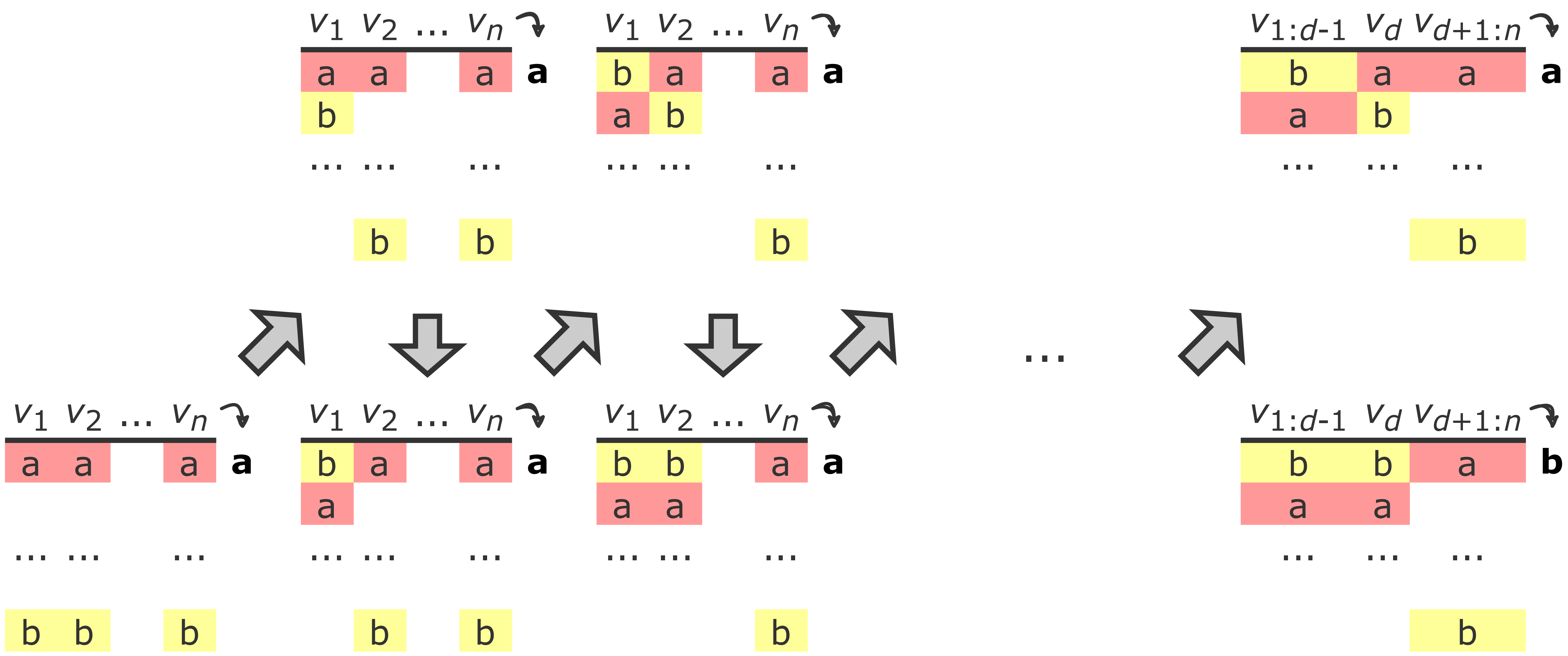
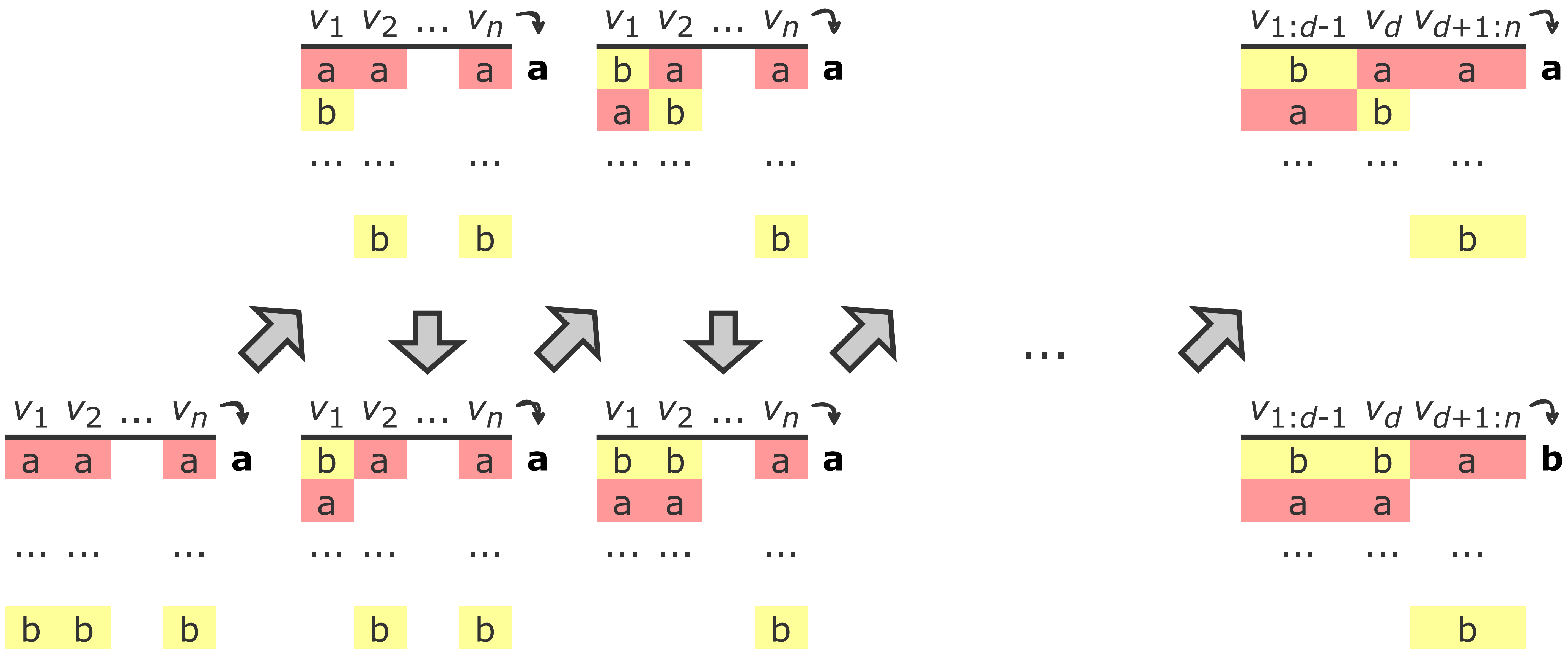


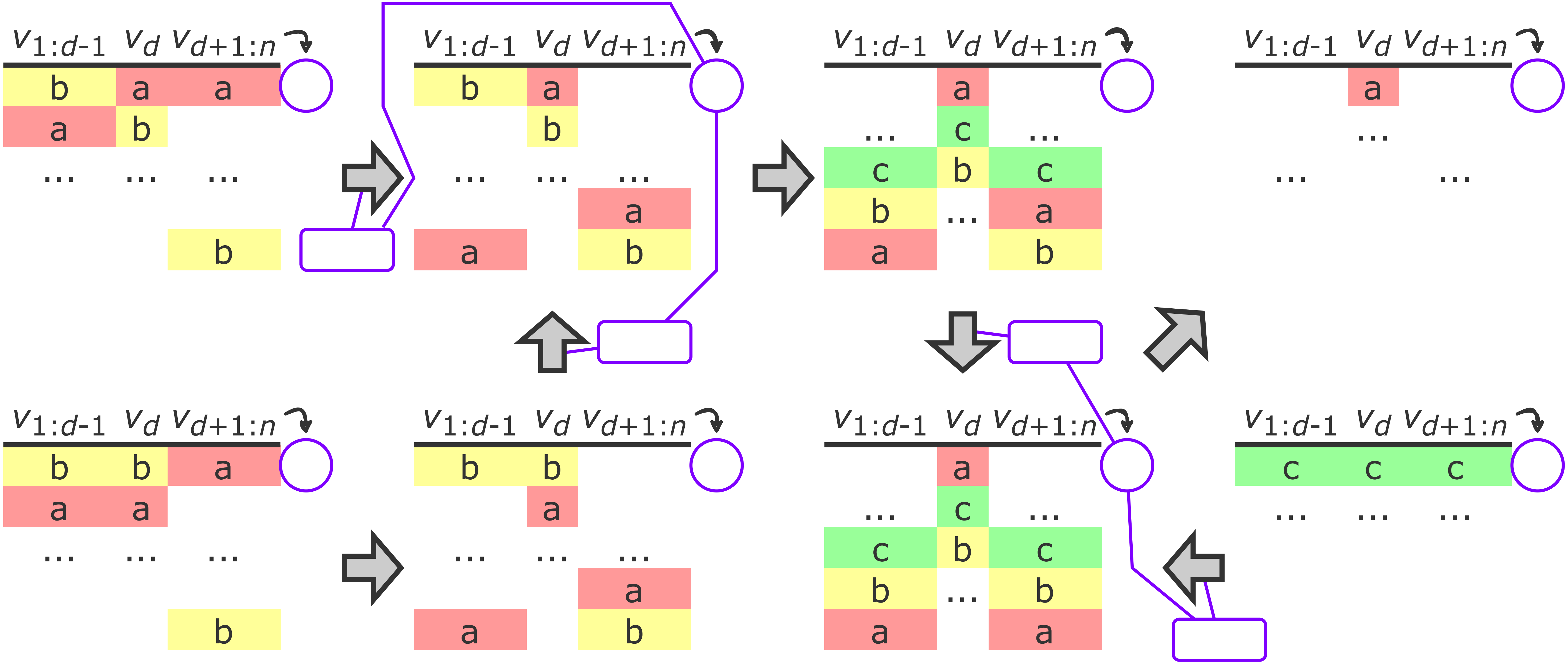
Proof of Muller-Satterthwaite, part 1

For any alternative a, there exists an index d and an alternative b such that:



Proof of Muller-Satterthwaite, part 2

For any alternative a, there exists an index d and alternatives b and c such that:



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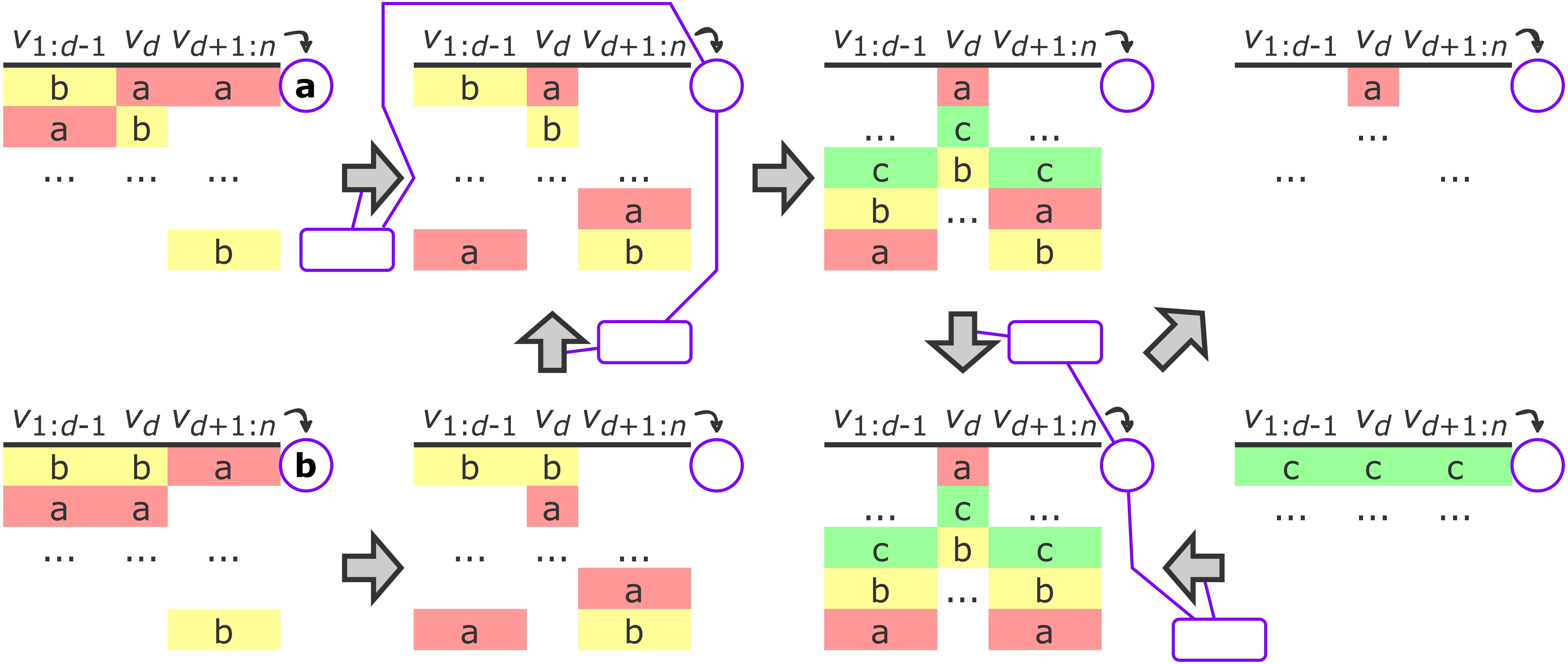

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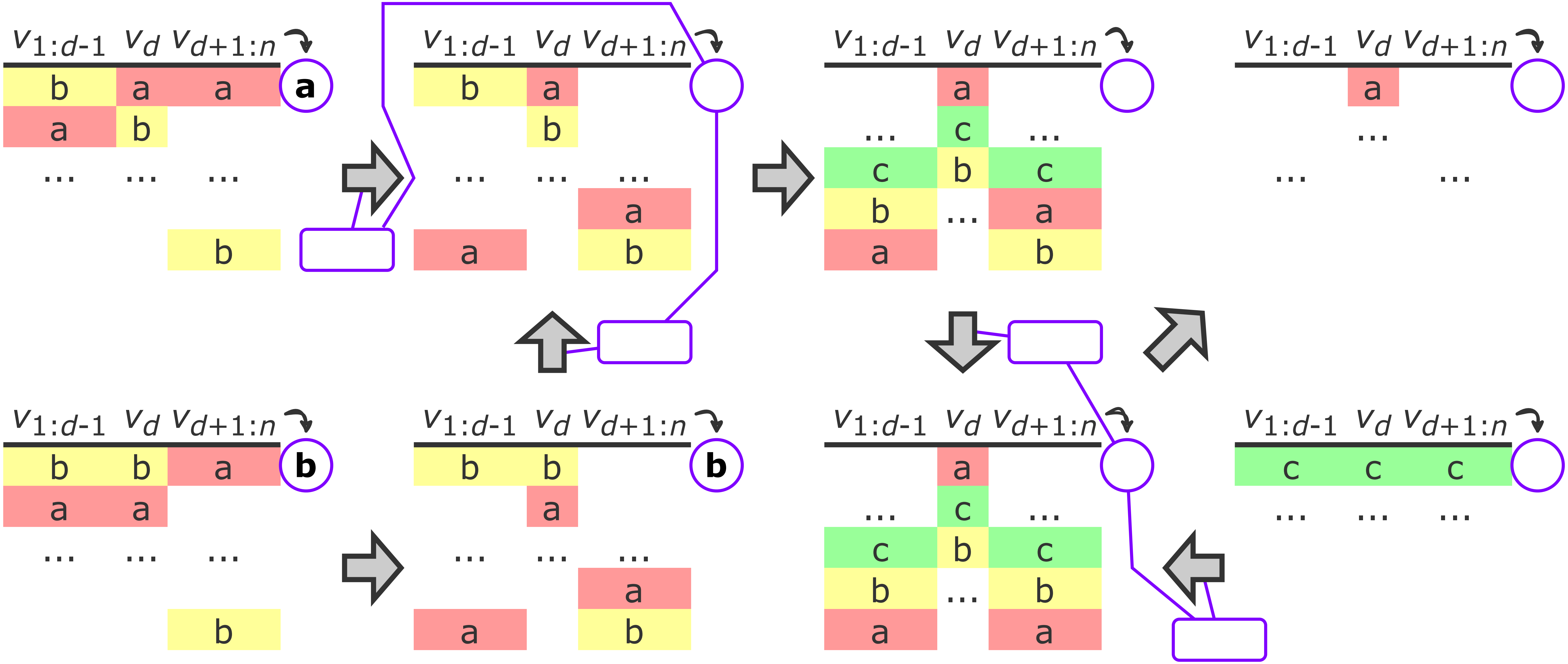

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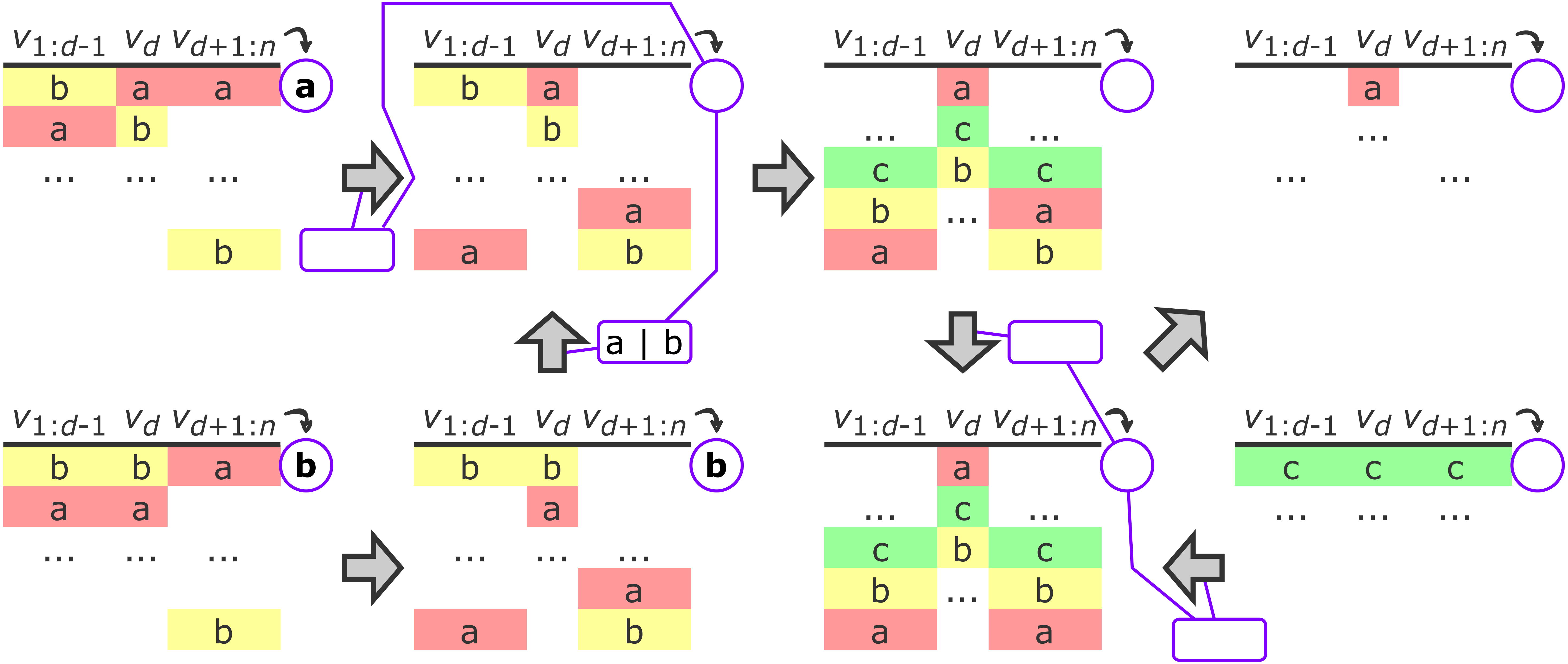

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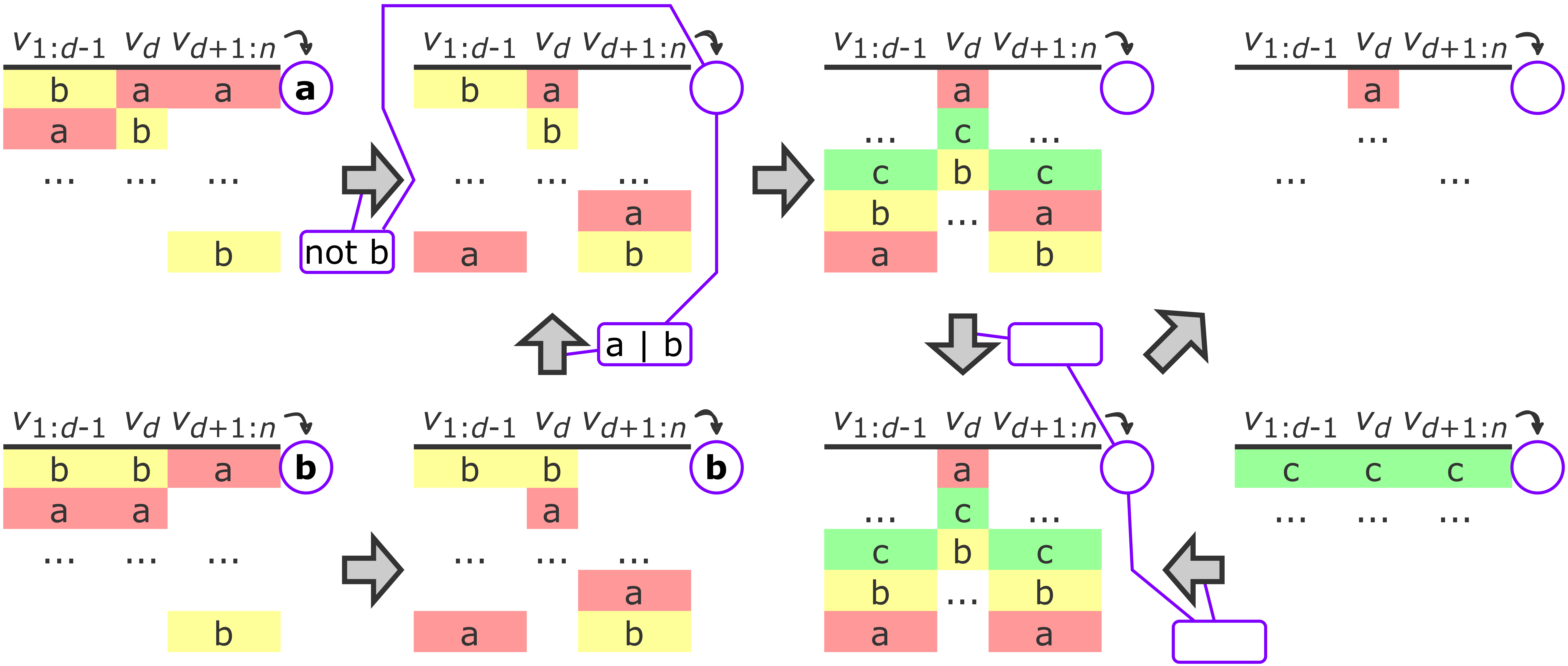

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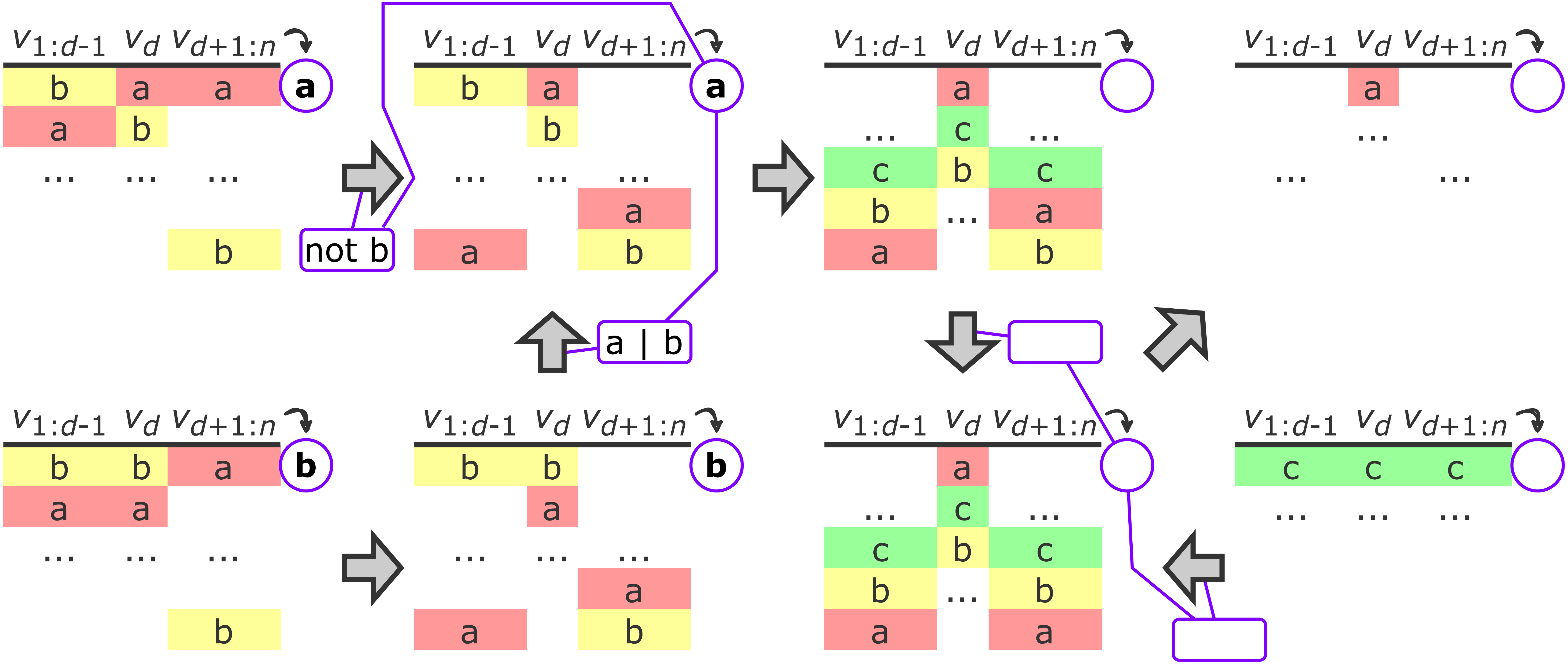

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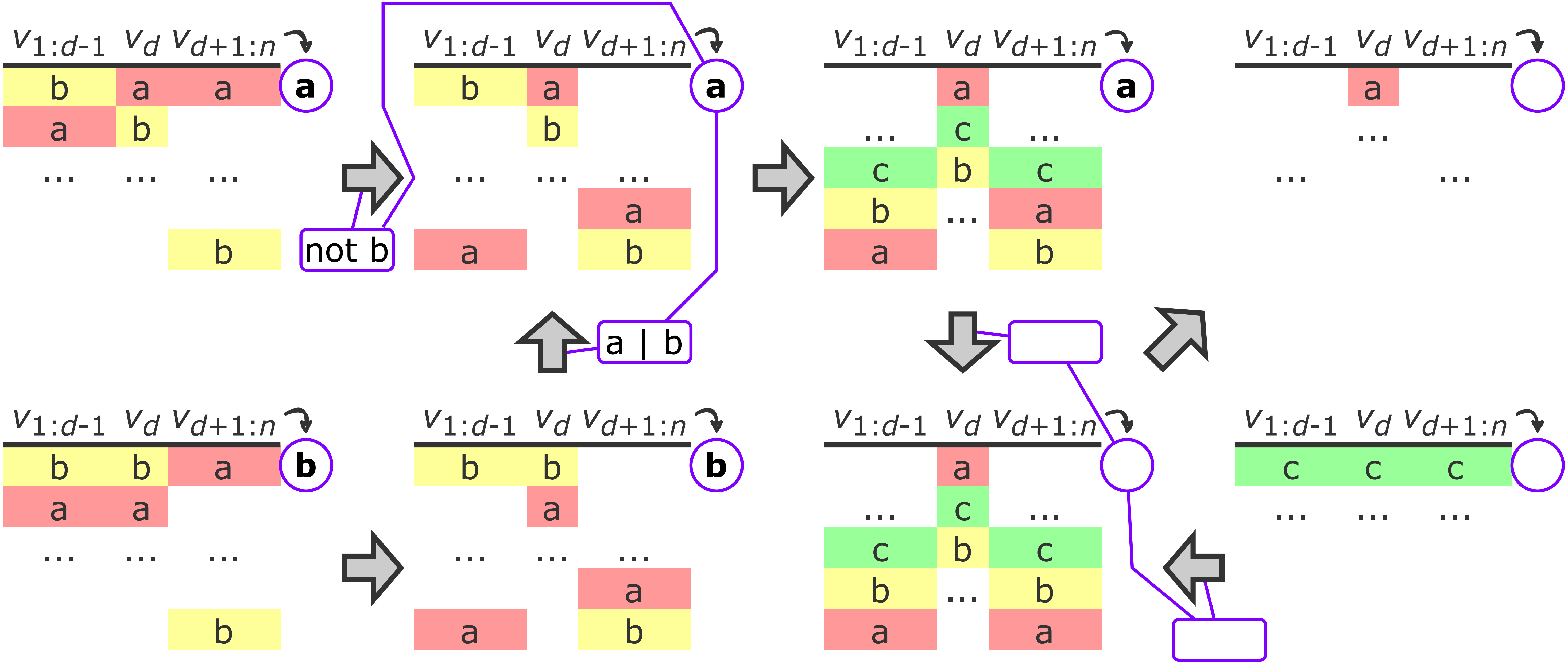

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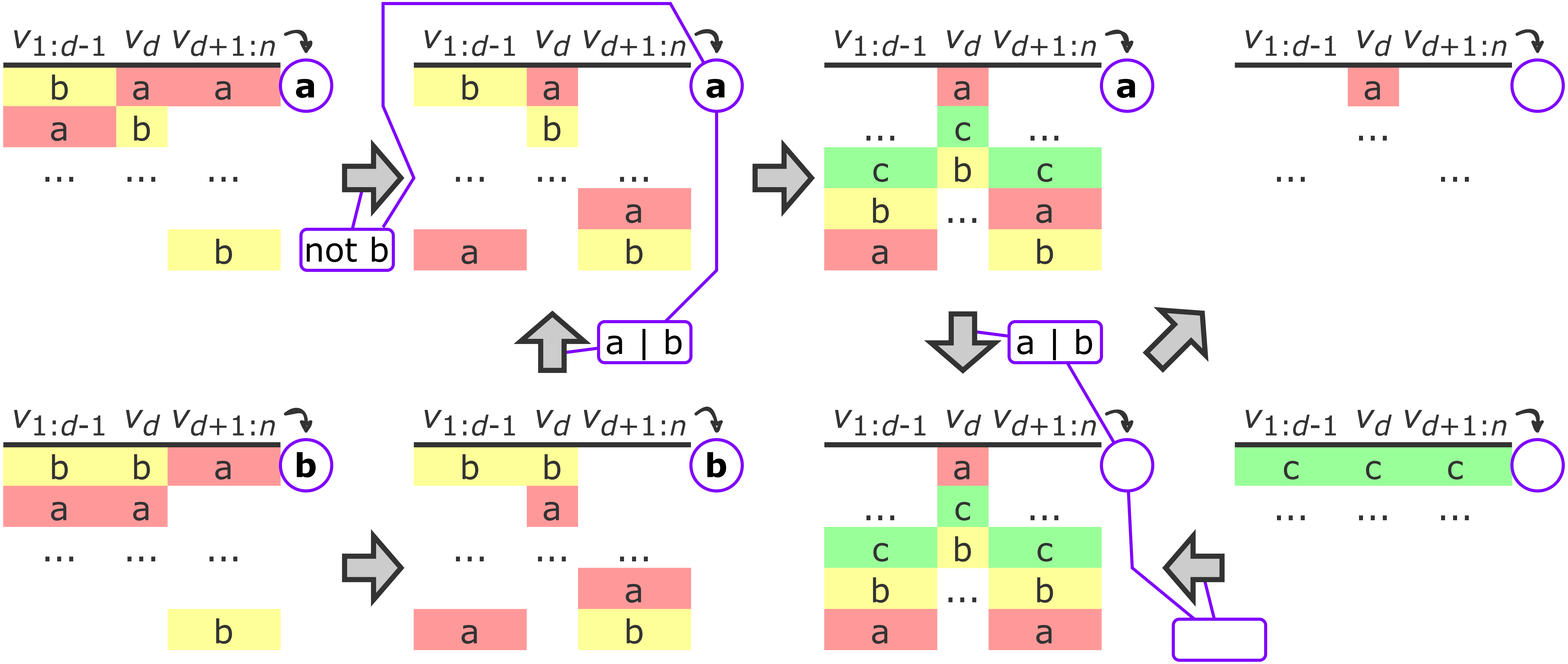

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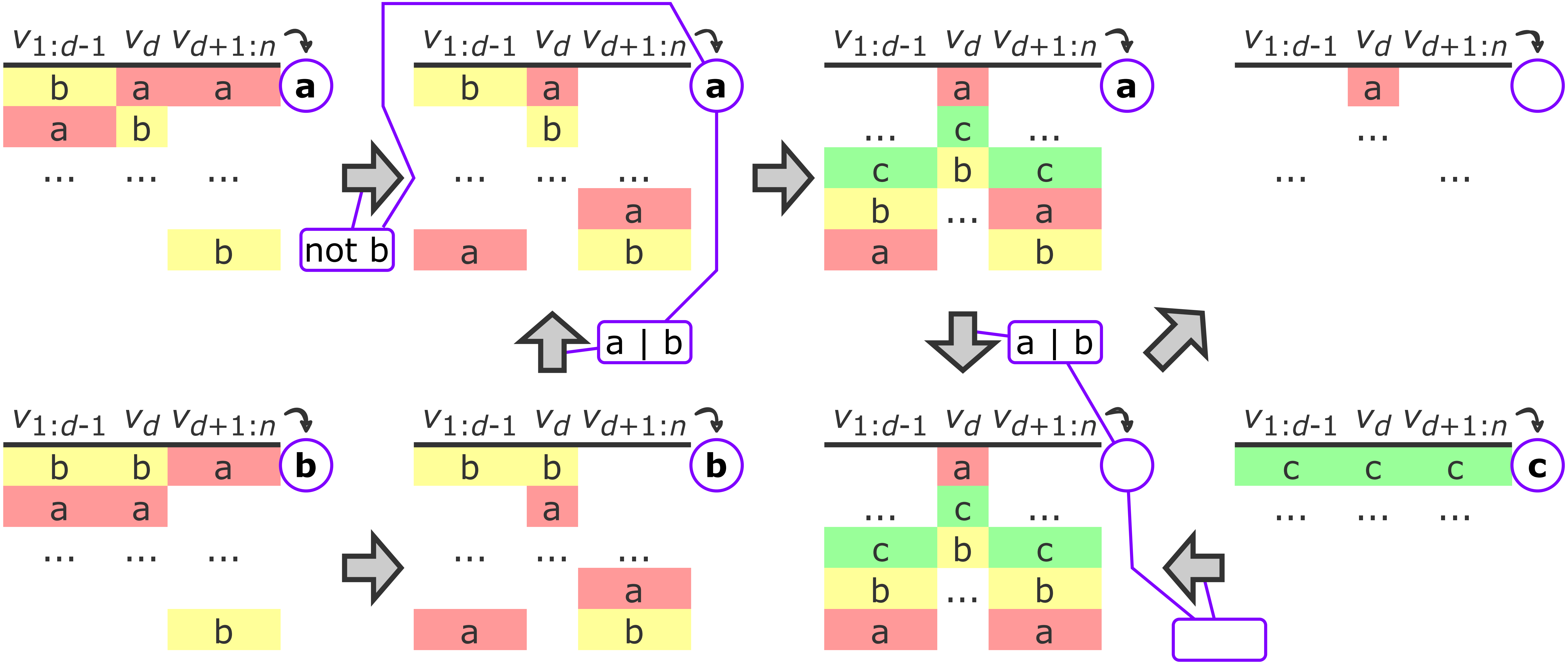

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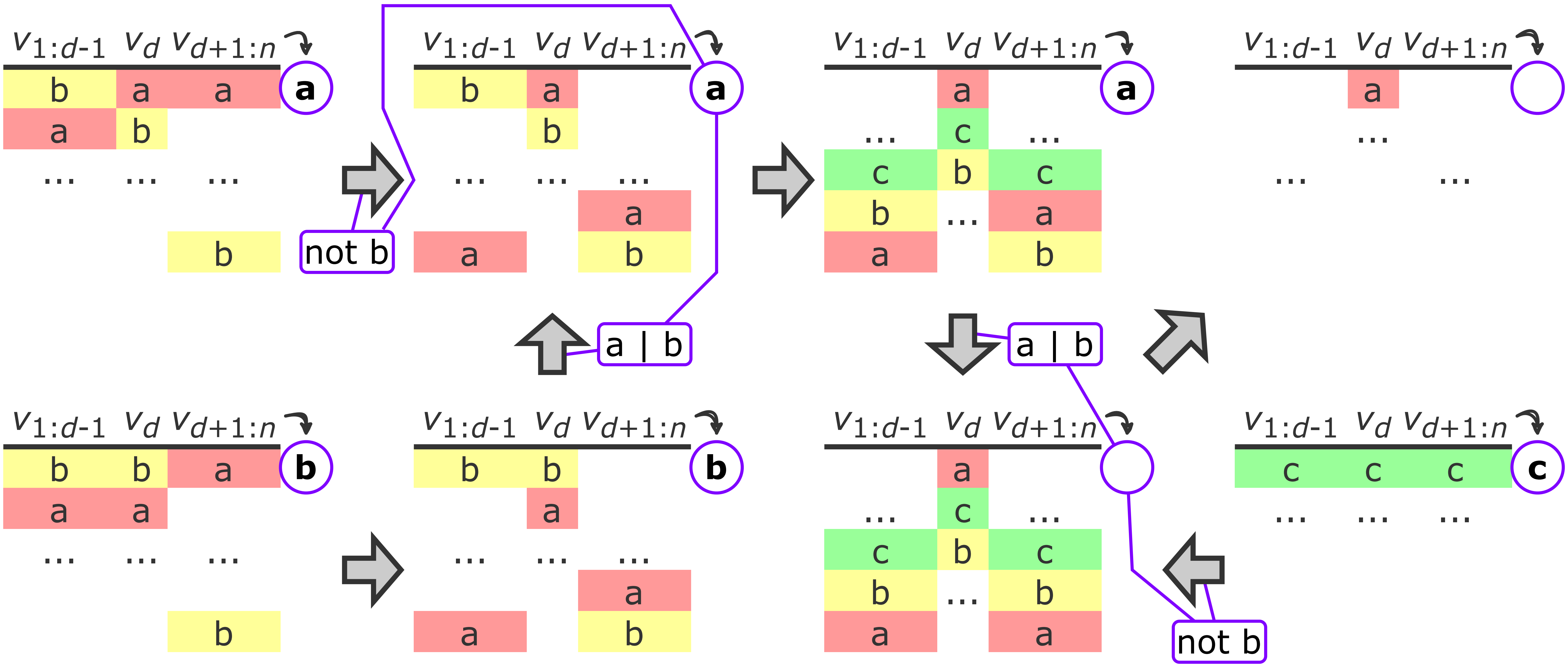

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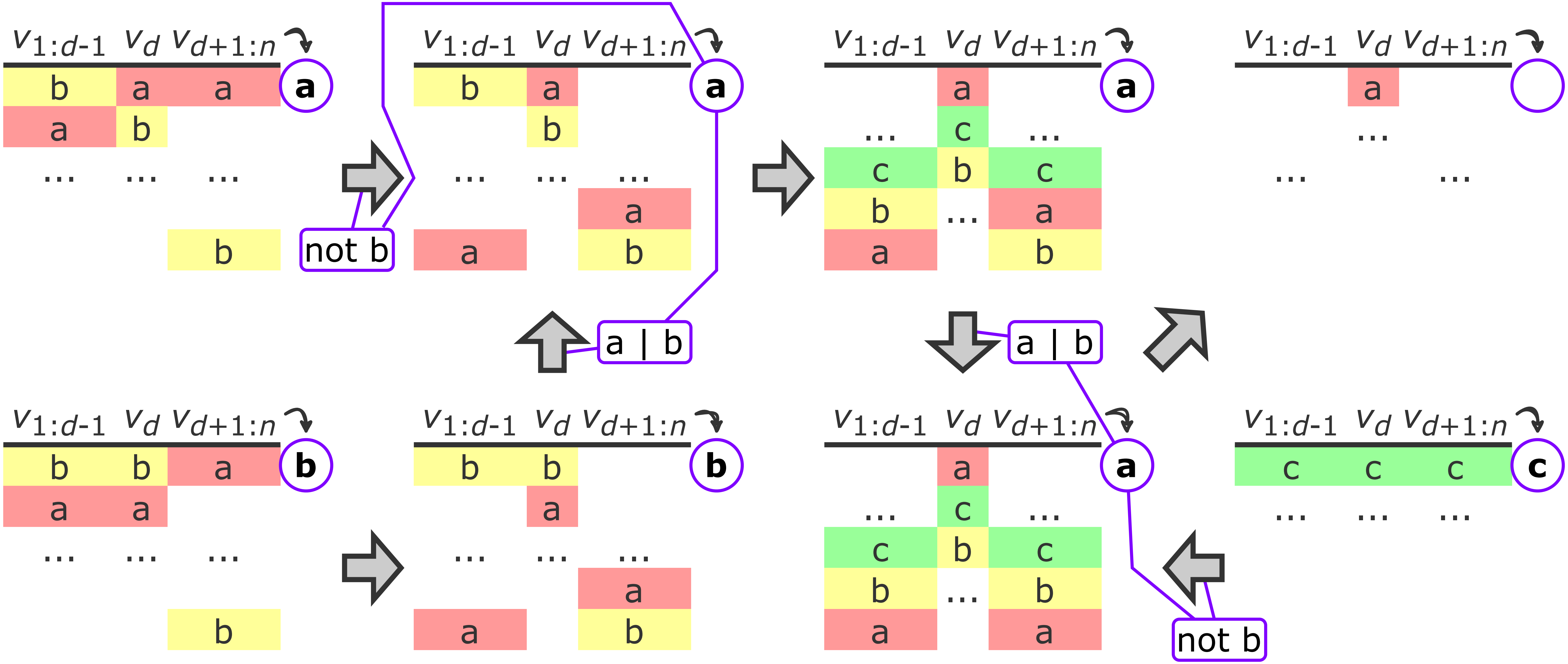

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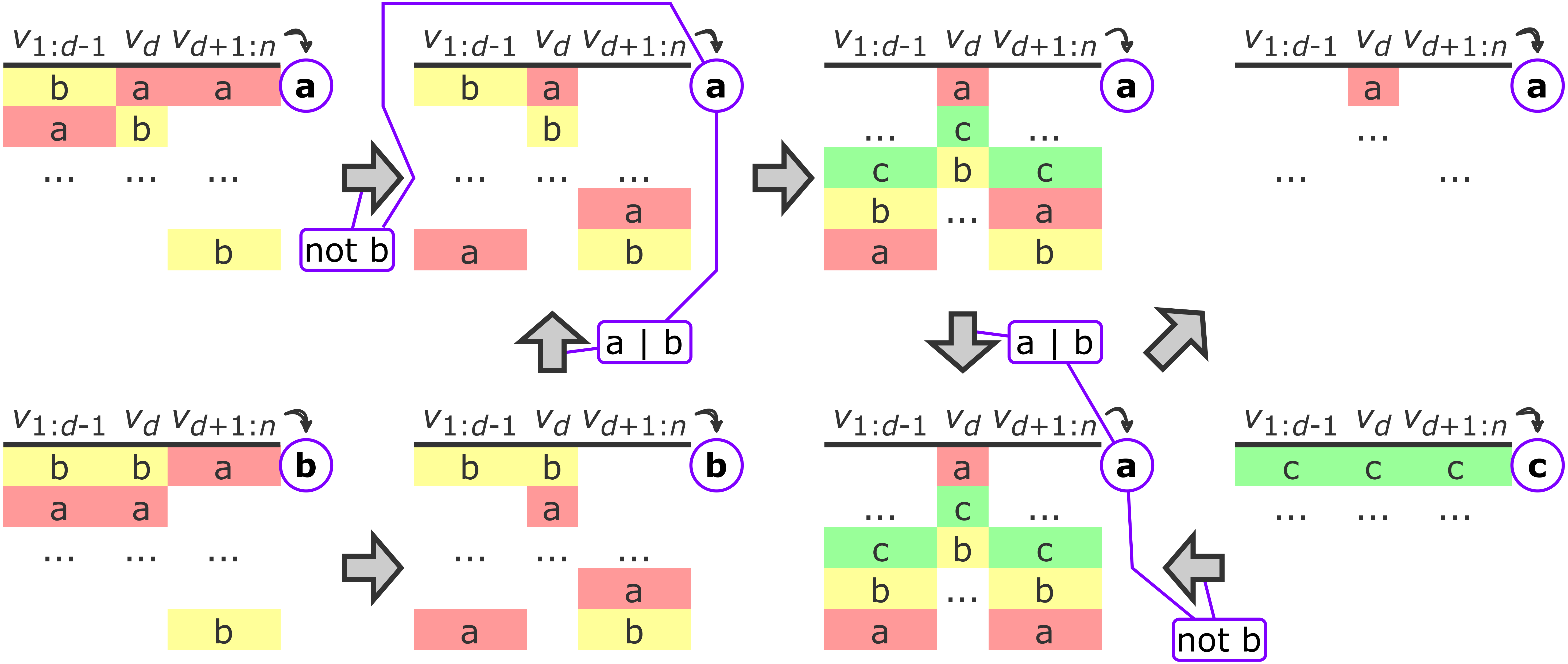

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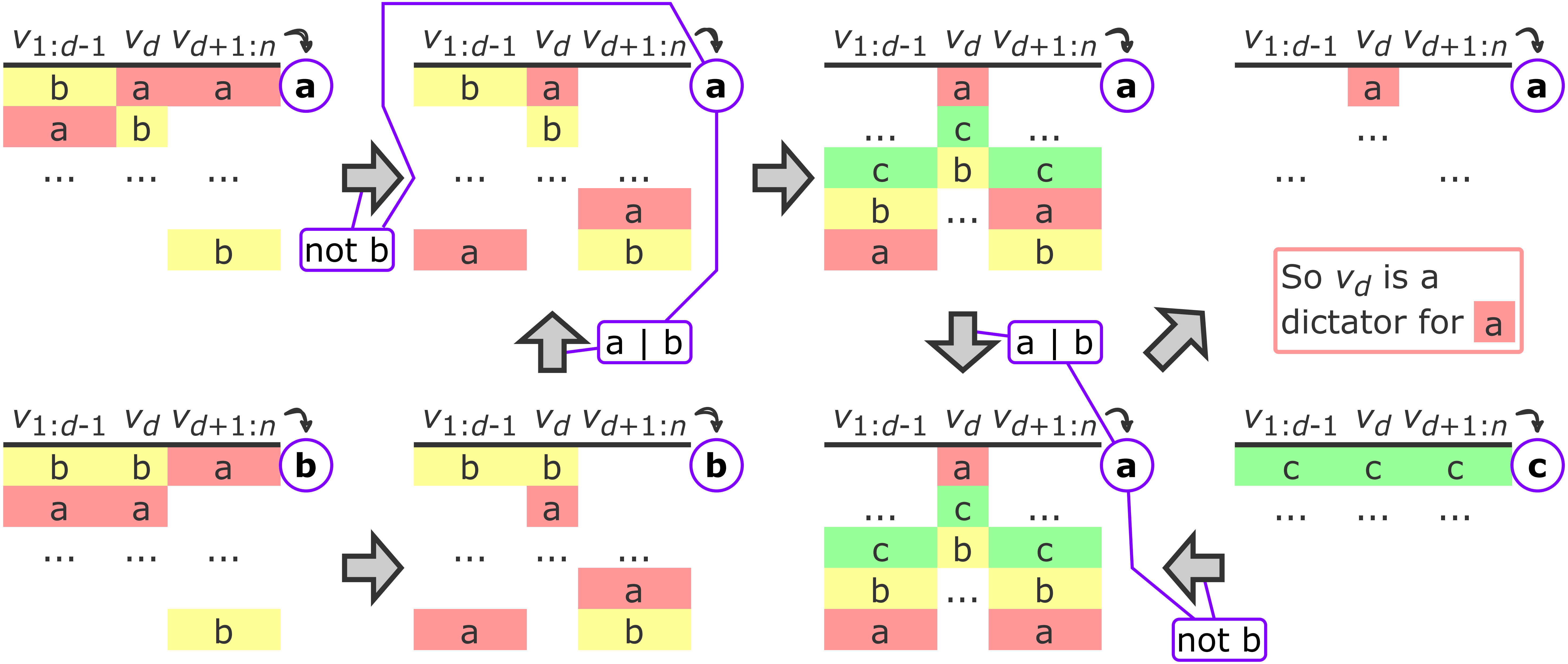

Proof of Muller-Satterthwaite, part 3

For any alternative a, there exists an index d such that v_d is a dictator for a.

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This can't happen!

Thus, any rule that is Pareto Efficient and Monotone has a dictator. ■

Arrow's Impossibility Theorem

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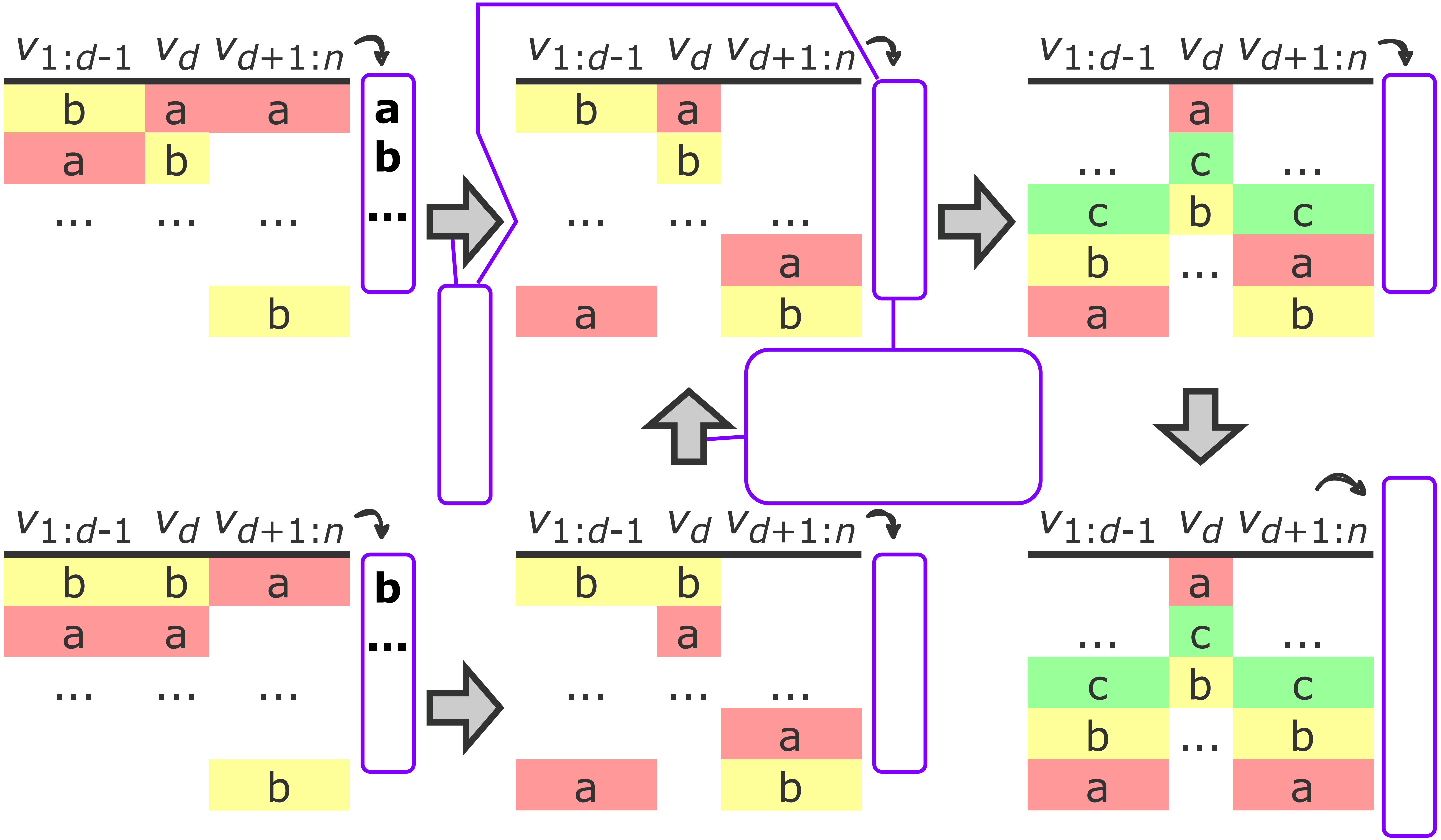
Proof. Part 1 is the same: pick the first index where a is not ranked *first* by society.

Proof of Arrow, part 2

For any alternative a, there exists an index *d* and alternatives b and c such that:



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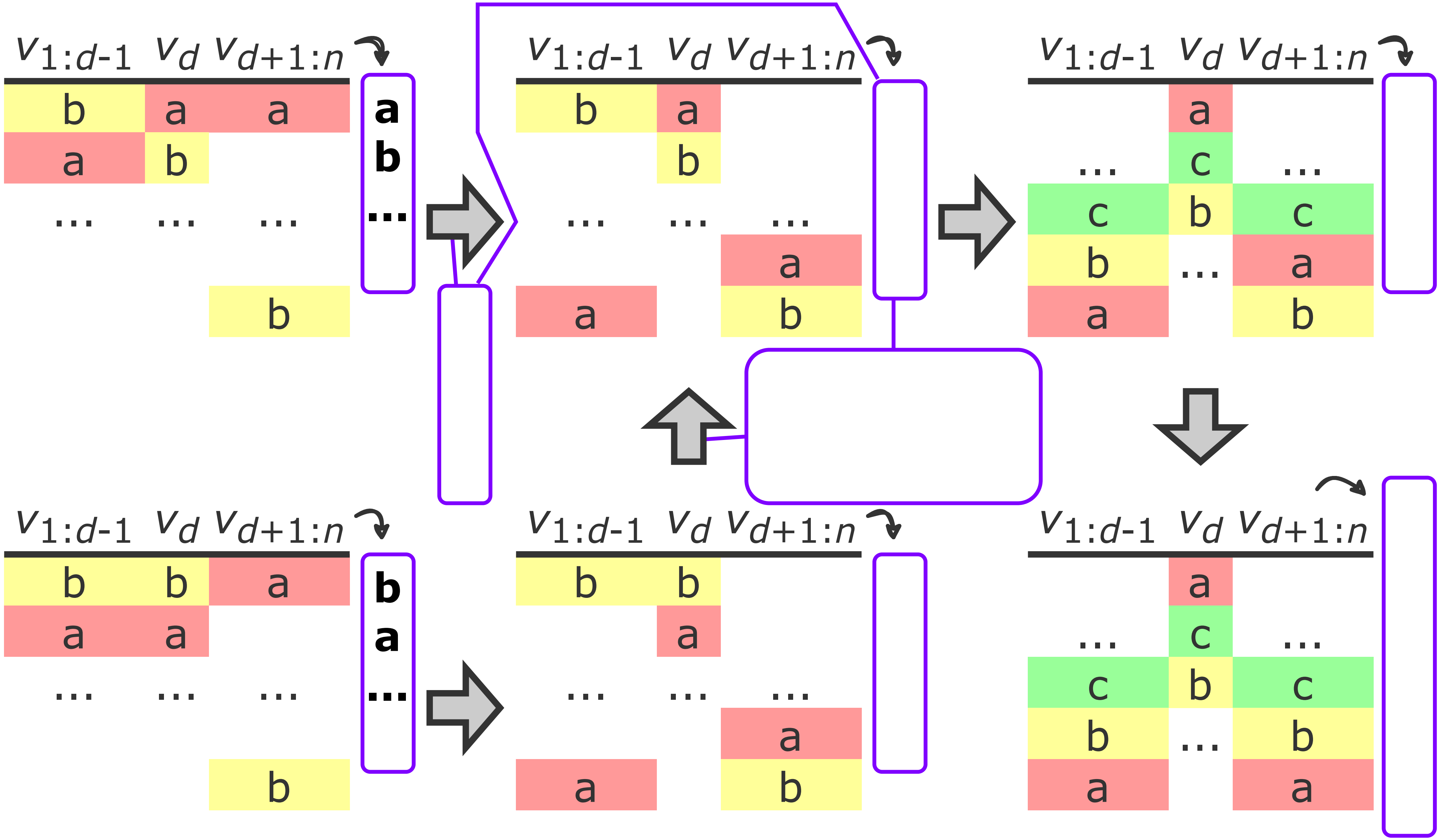

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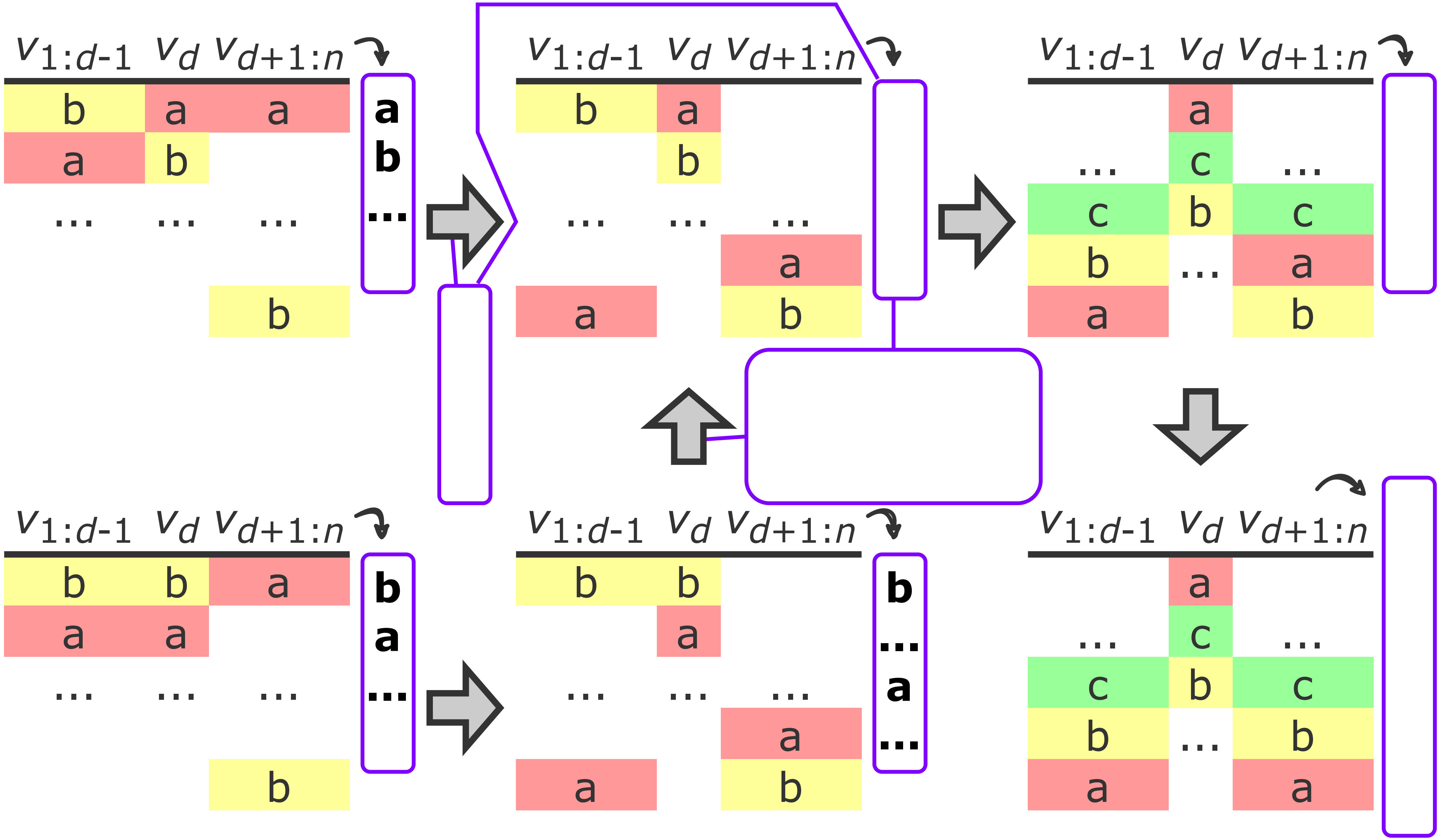

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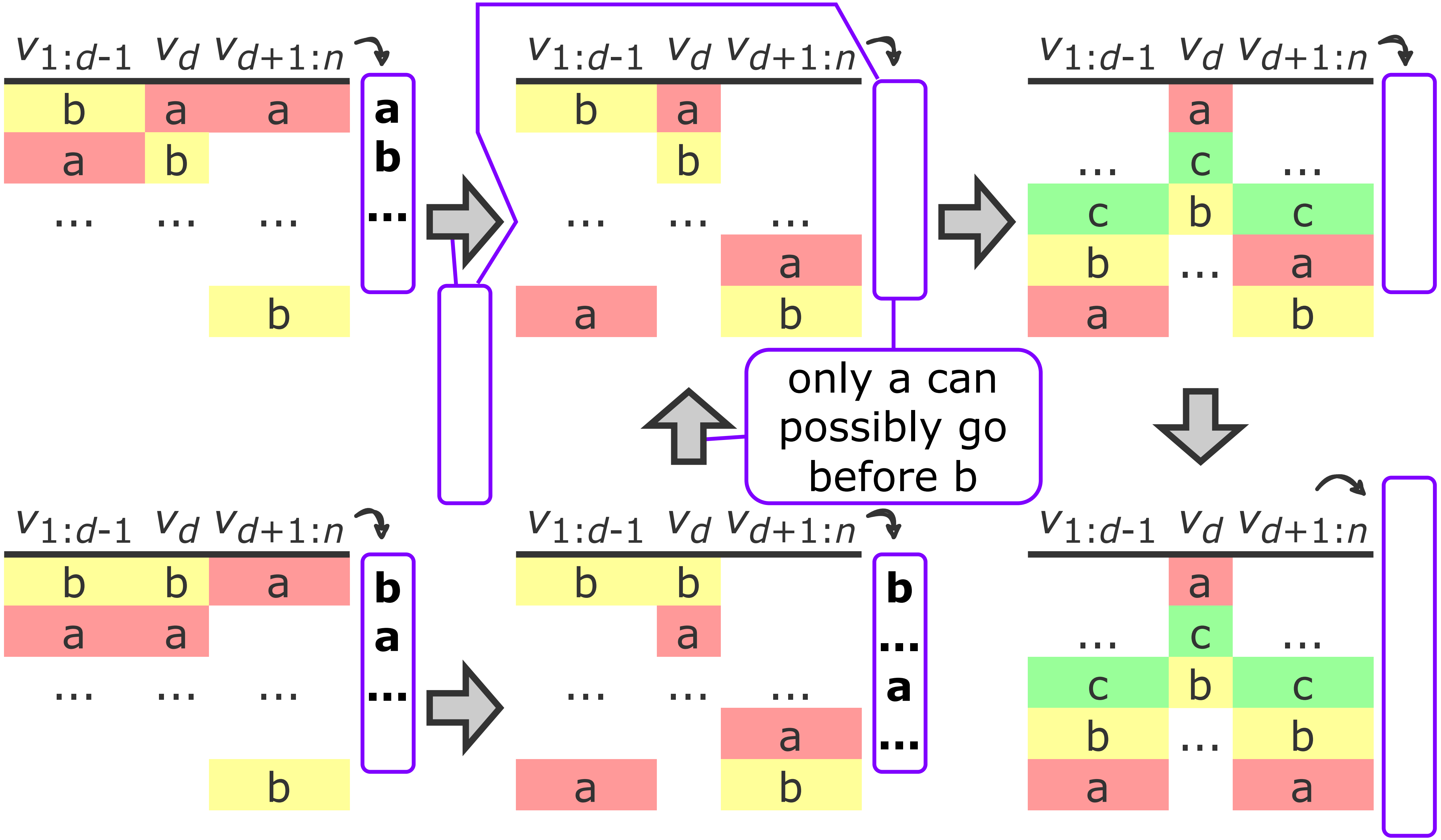

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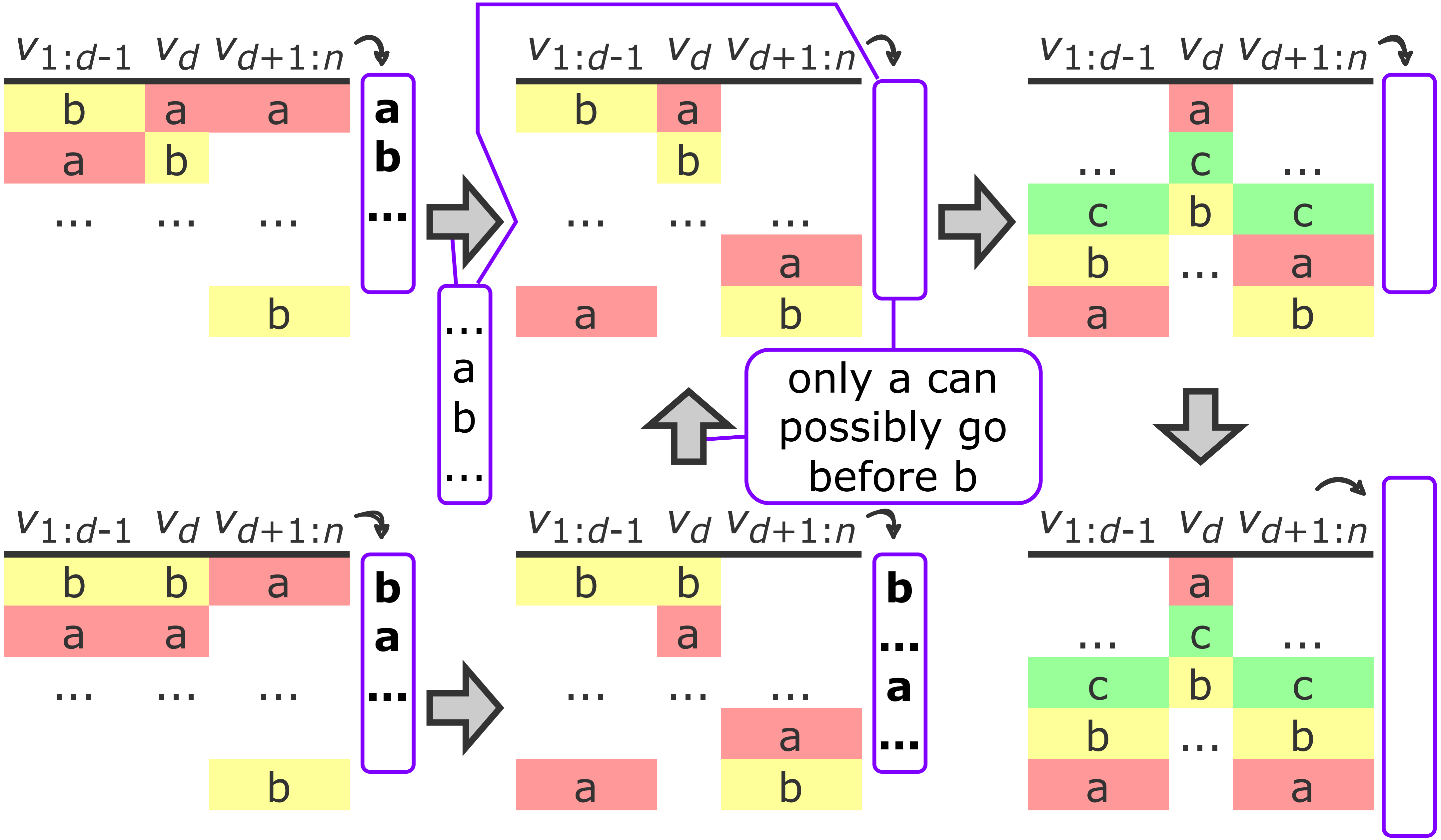

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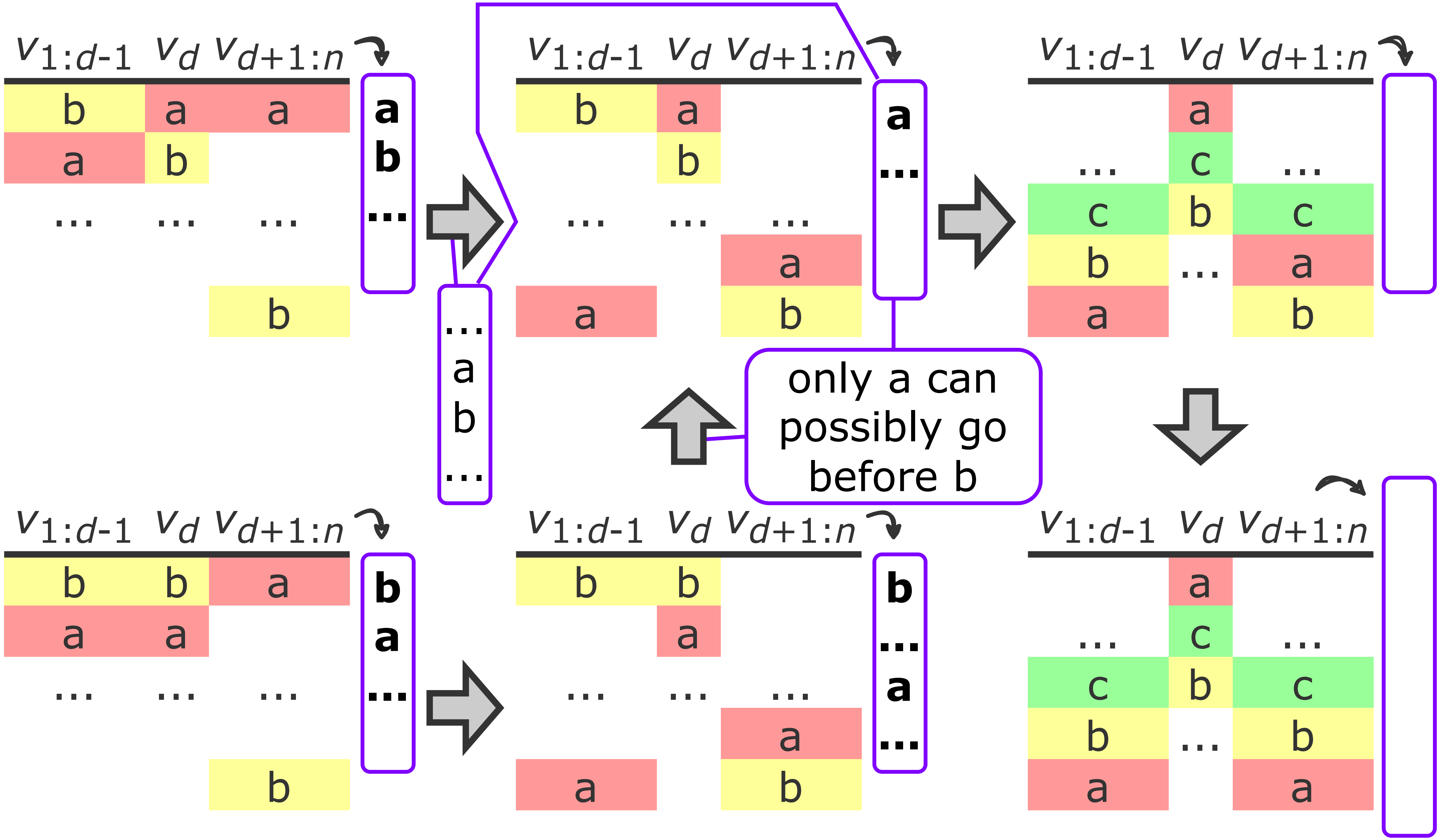

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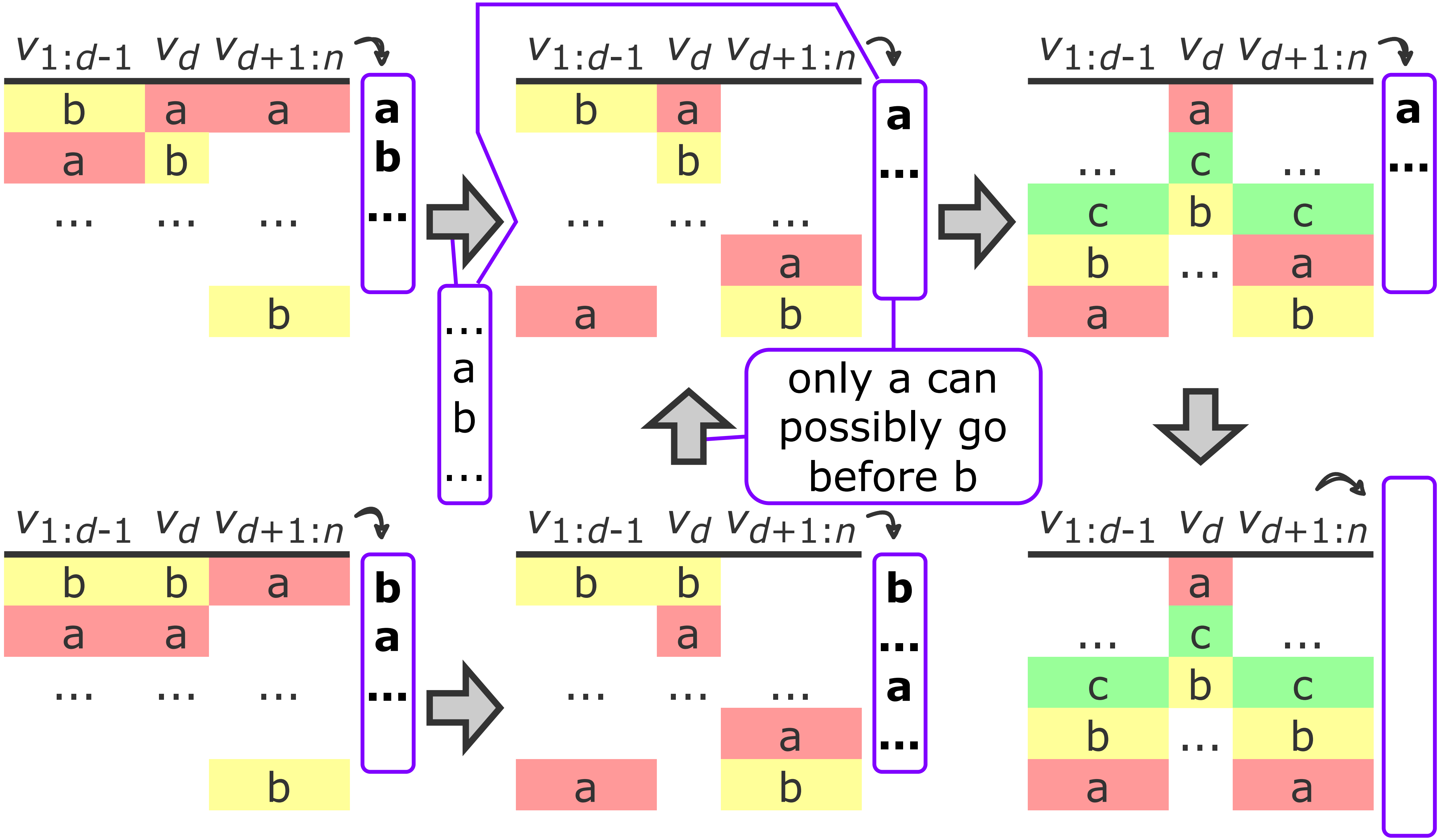

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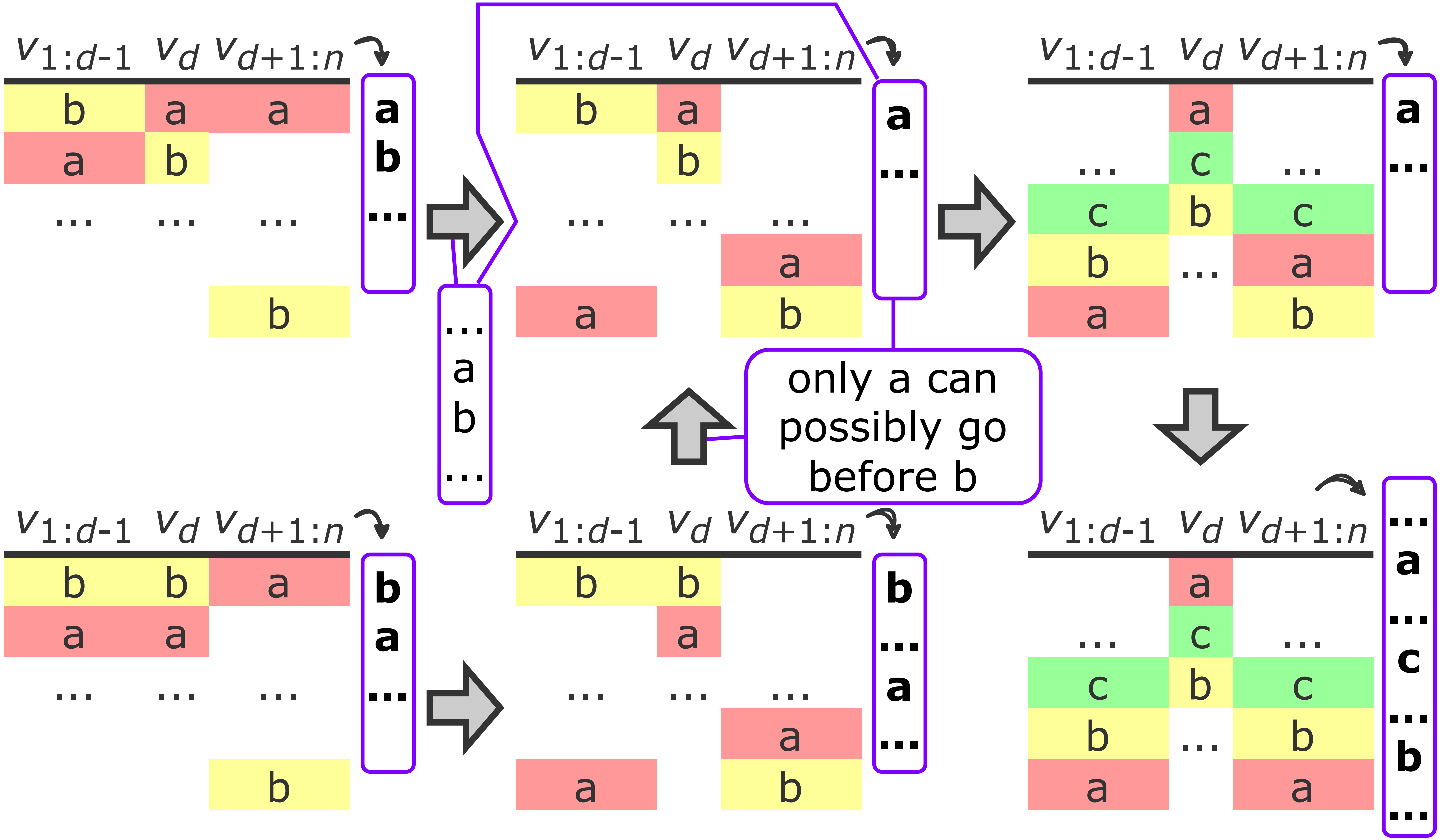

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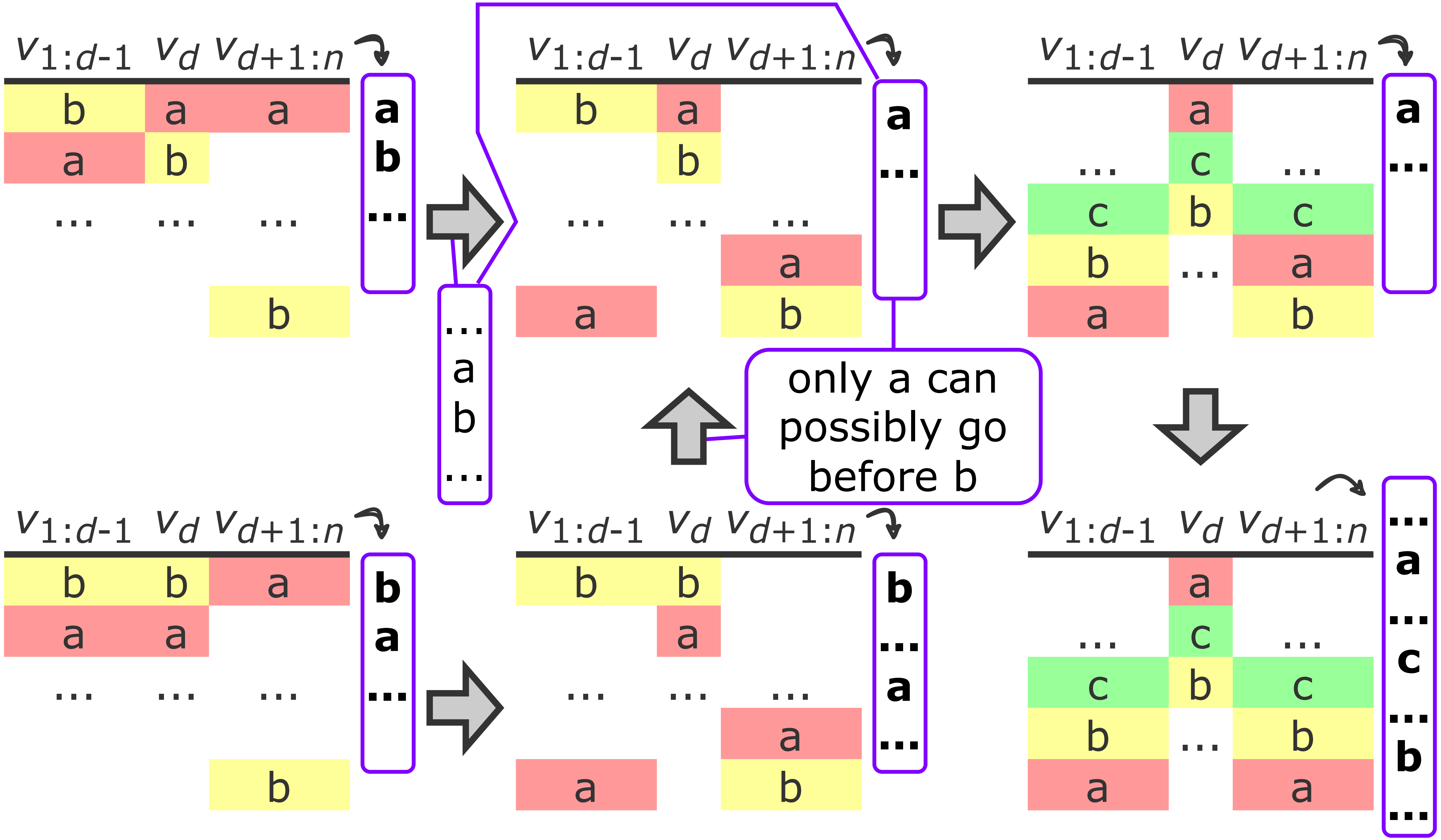

For any x ≠ a, pick y ∈ {b, c}\{x}.

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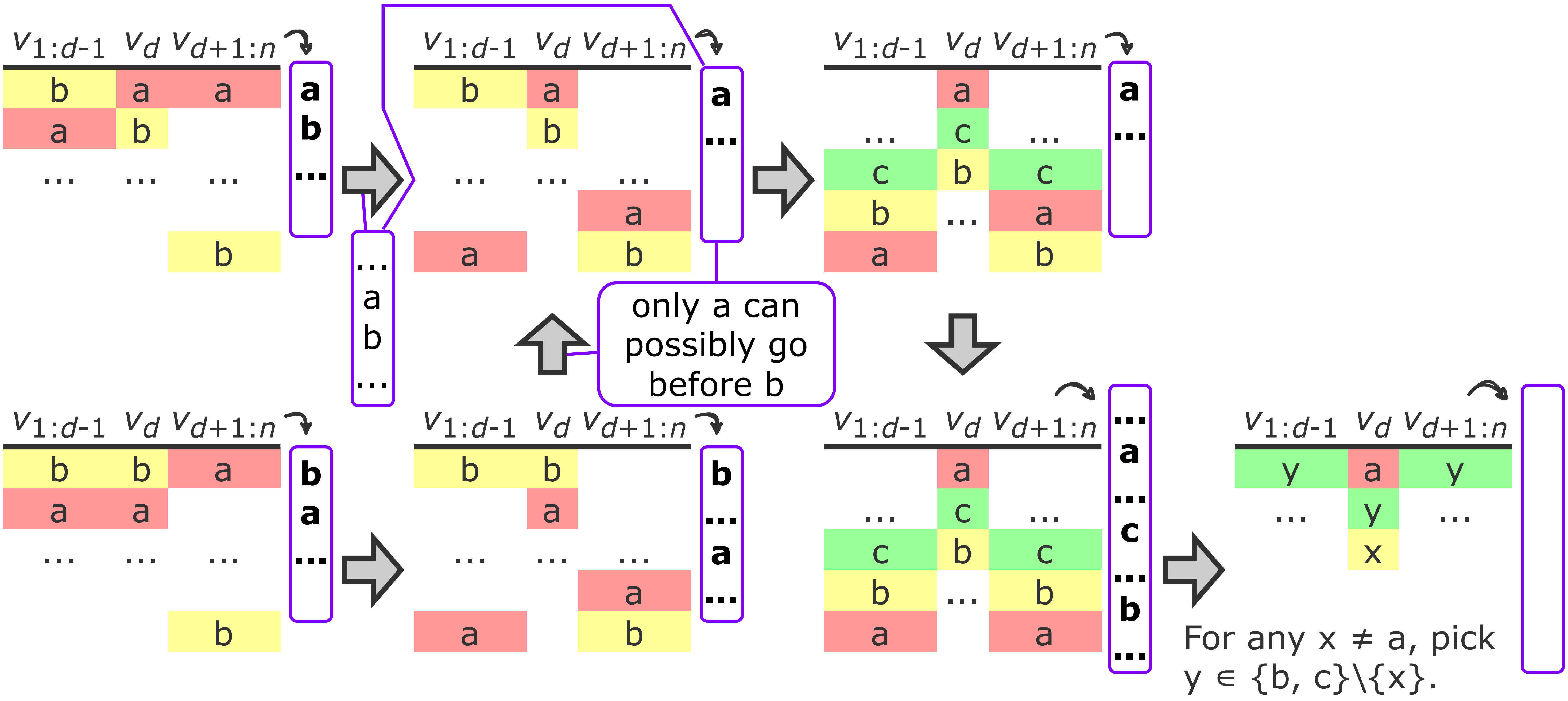

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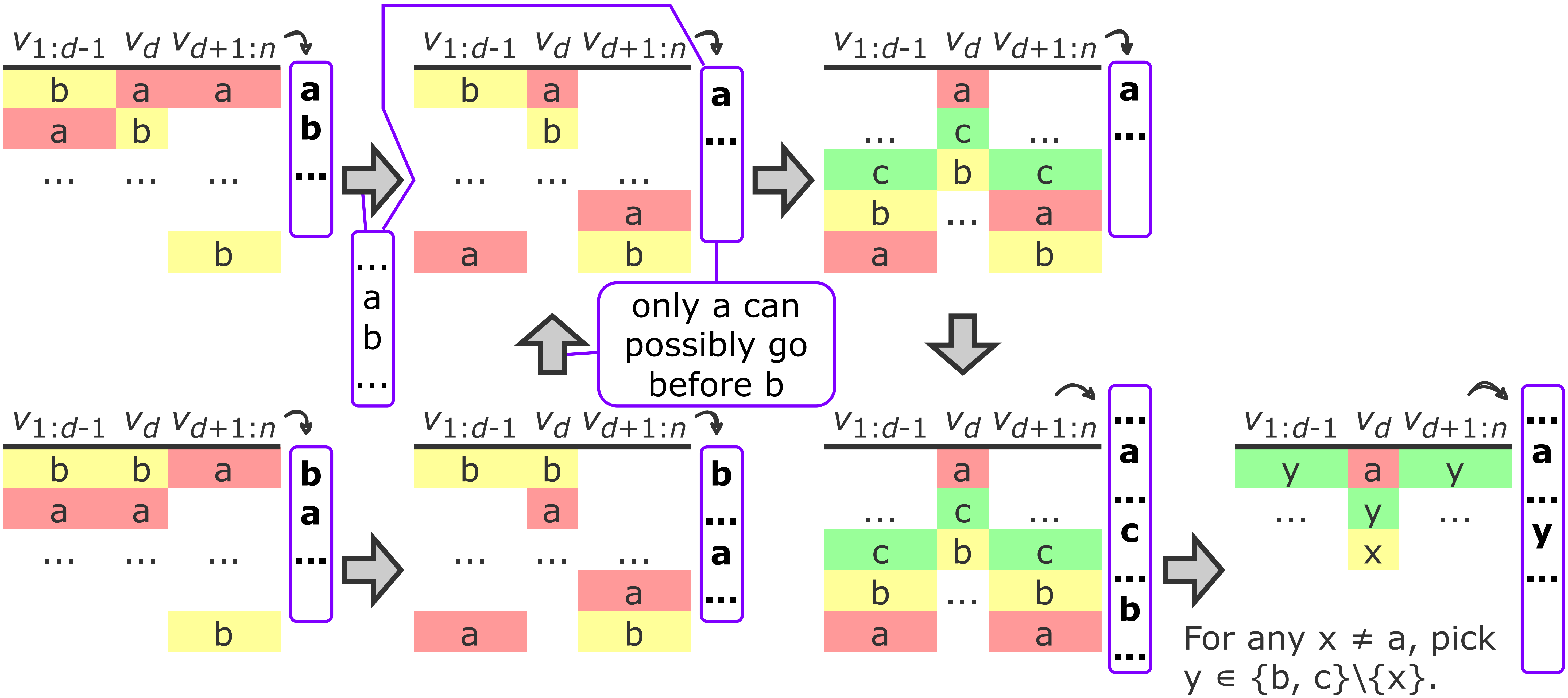

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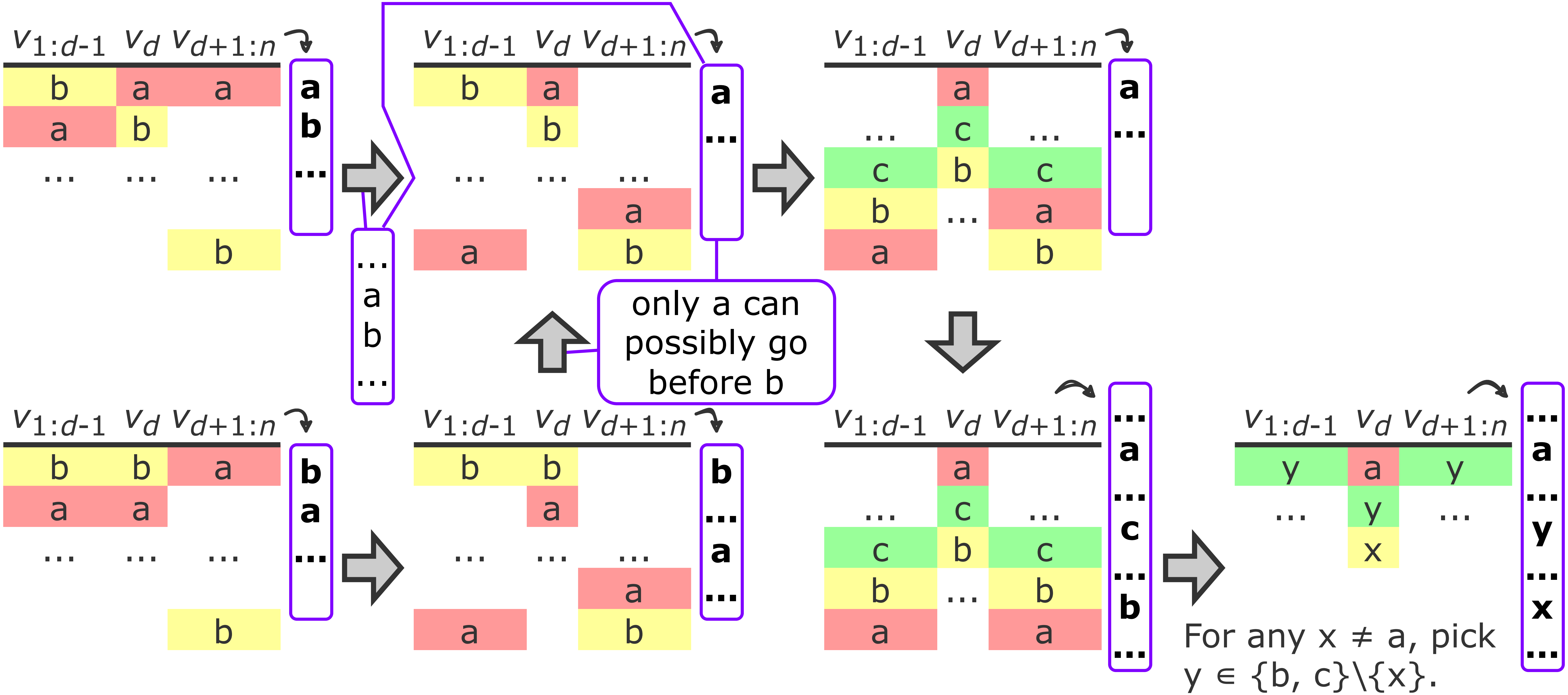

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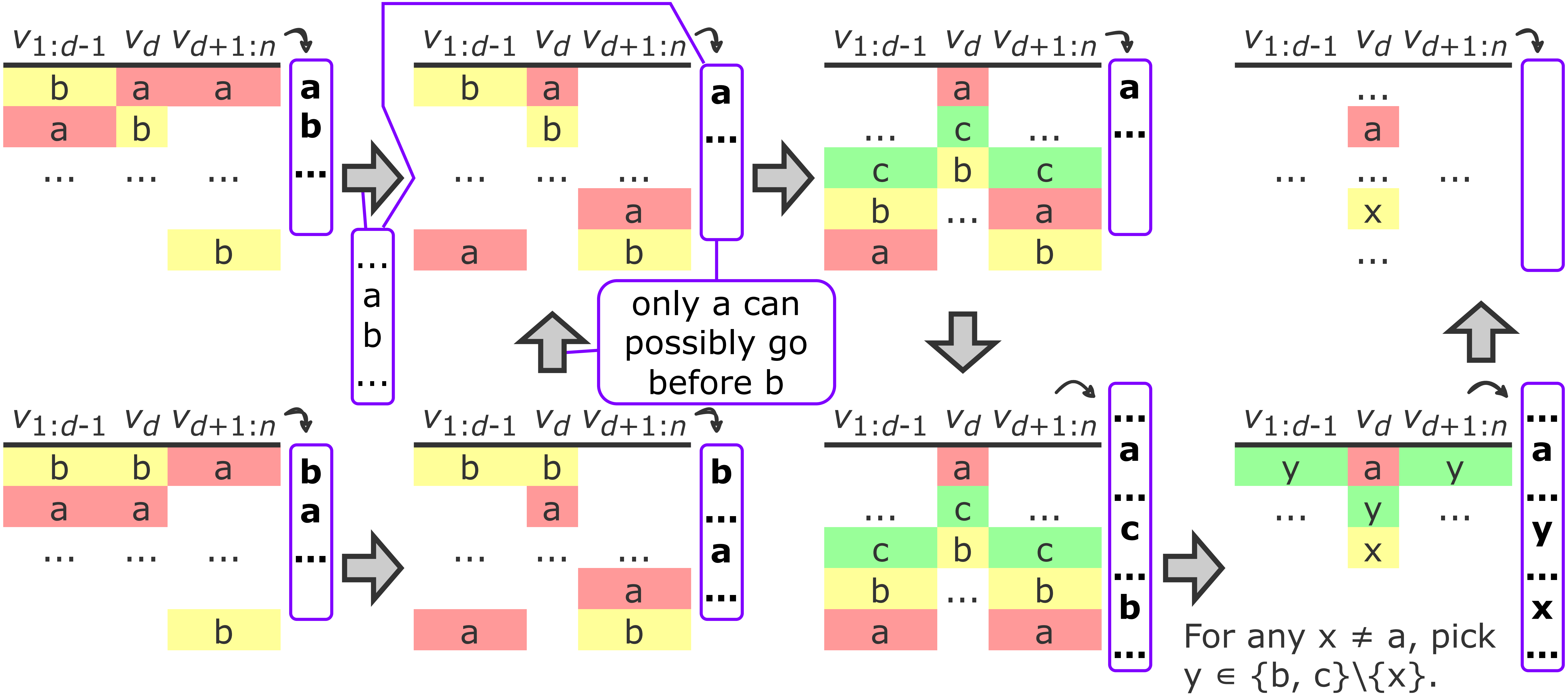

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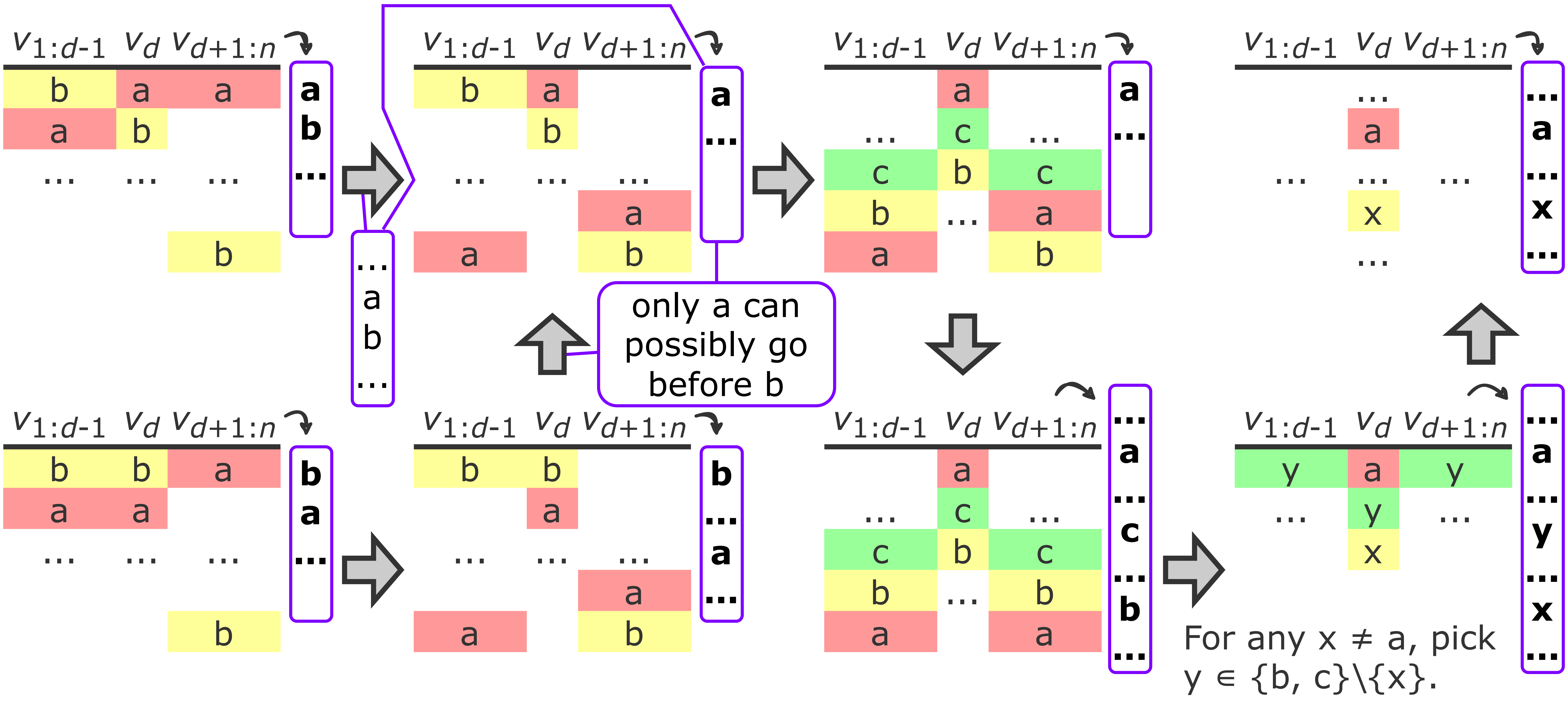

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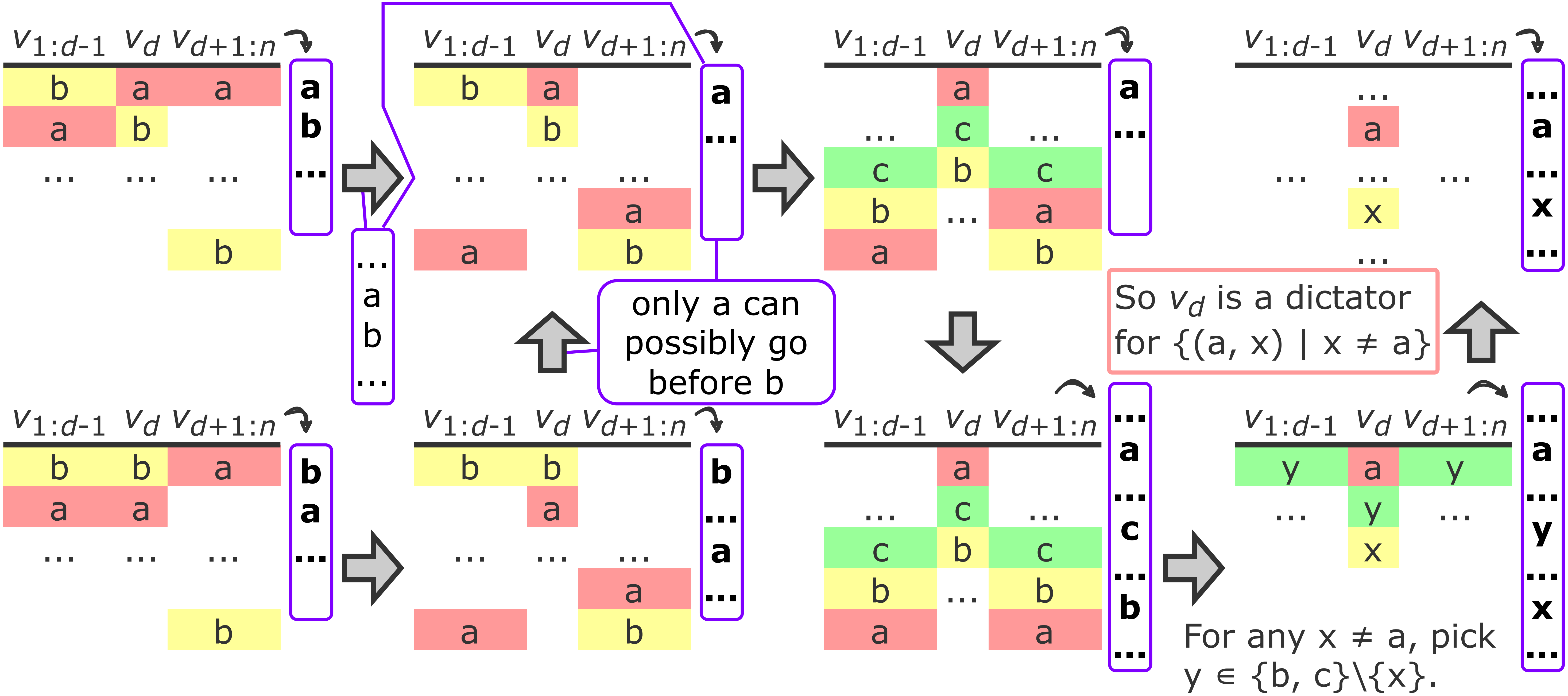

Proof of Arrow, part 3

For any alternative a, there exists an index d such that v_d is a dictator for making a win over any other alternative.

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This can't happen!

Thus, any rule that is Pareto Efficient and IIA has a dictator. ■