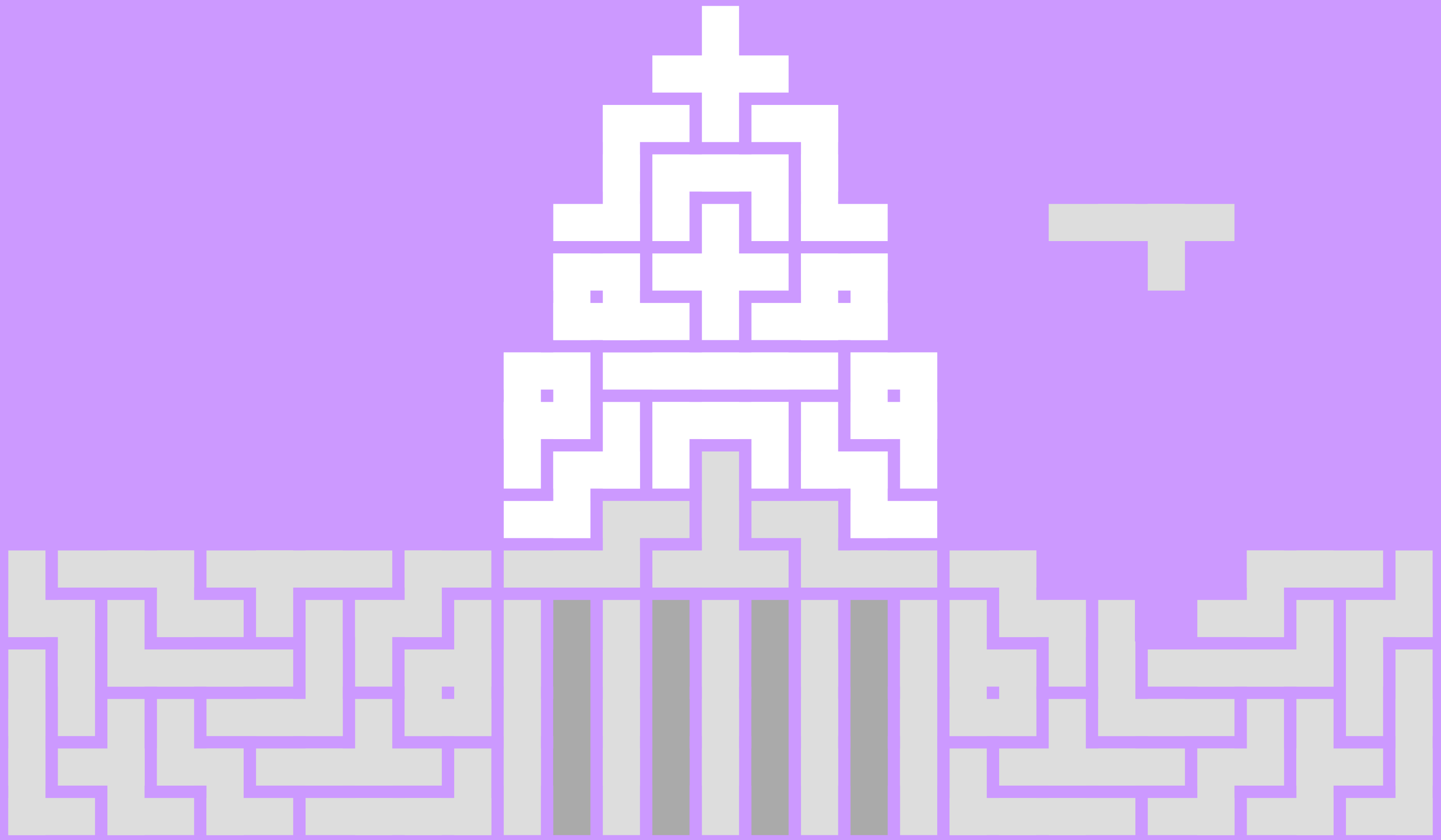


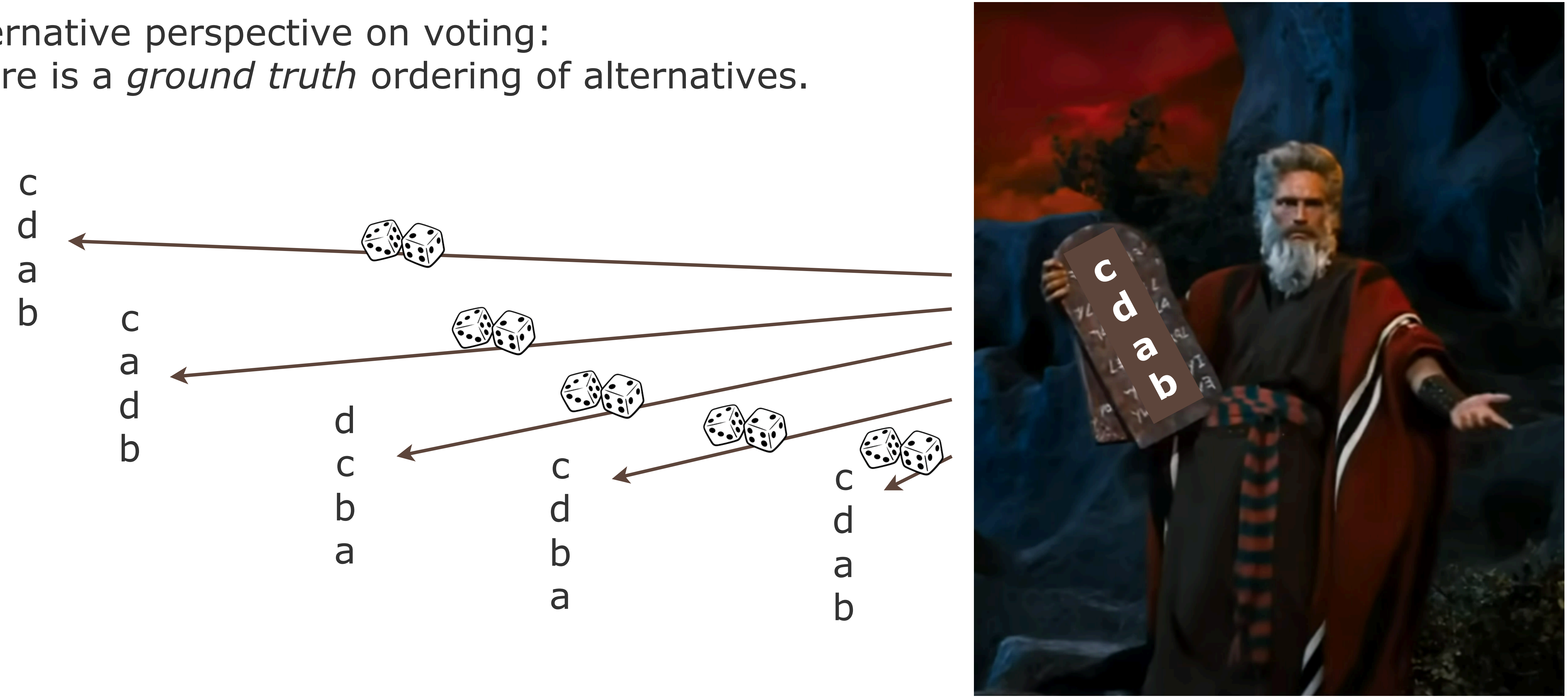
Algorithms For Democratic Decision-Making

Jamie Tucker-Foltz • Yale University • Spring 2026

Lecture 7: **The Epistemic Approach**

Introduction

Alternative perspective on voting:
There is a *ground truth* ordering of alternatives.



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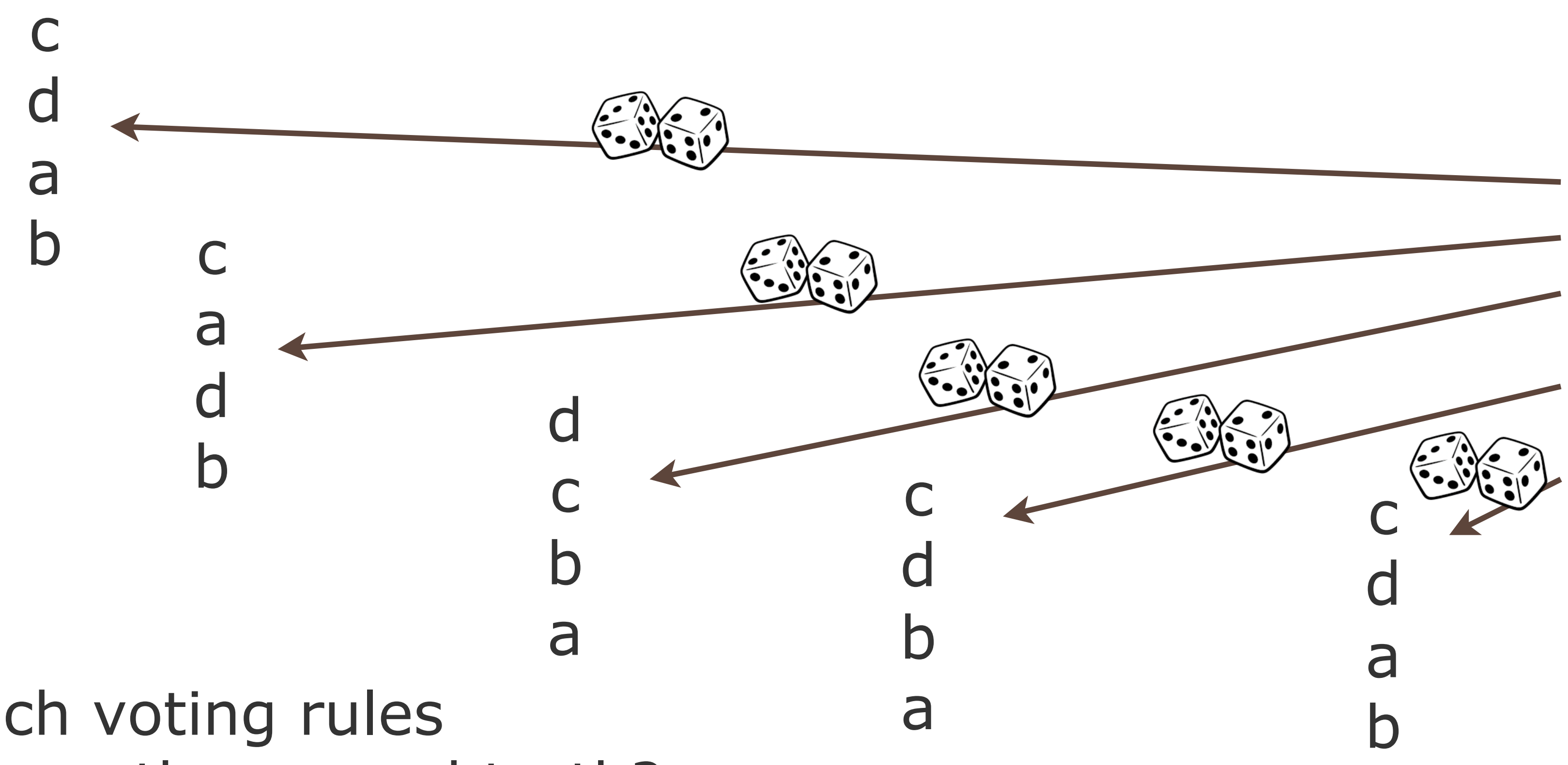
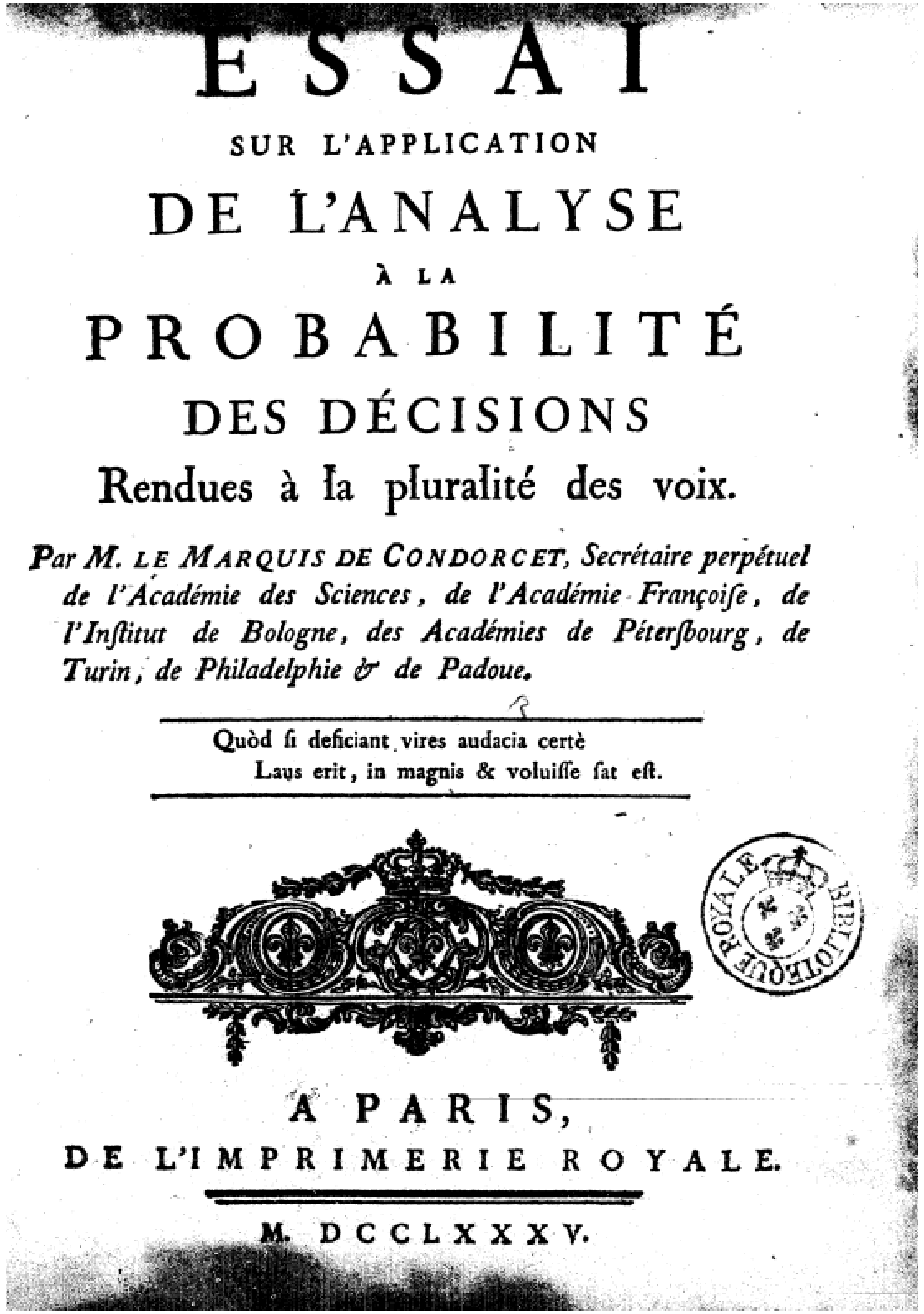


Which voting rules
recover the ground truth?

Condorcet's Jury Theorem



ESSAI

SUR L'APPLICATION

DE L'ANALYSE

À LA

PROBABILITÉ

DES DÉCISIONS

Rendues à la pluralité des voix.

Par M. LE MARQUIS DE CONDORCET, Secrétaire perpétuel de l'Académie des Sciences, de l'Académie Françoise, de l'Institut de Bologne, des Académies de Péterſbourg, de Turin, de Philadelphie & de Padoue.

Quòd ſi deficiant vires audacia certè
Laus erit, in magnis & voluiſſe ſat eſt.

A PARIS,

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M. DCCLXXXV.

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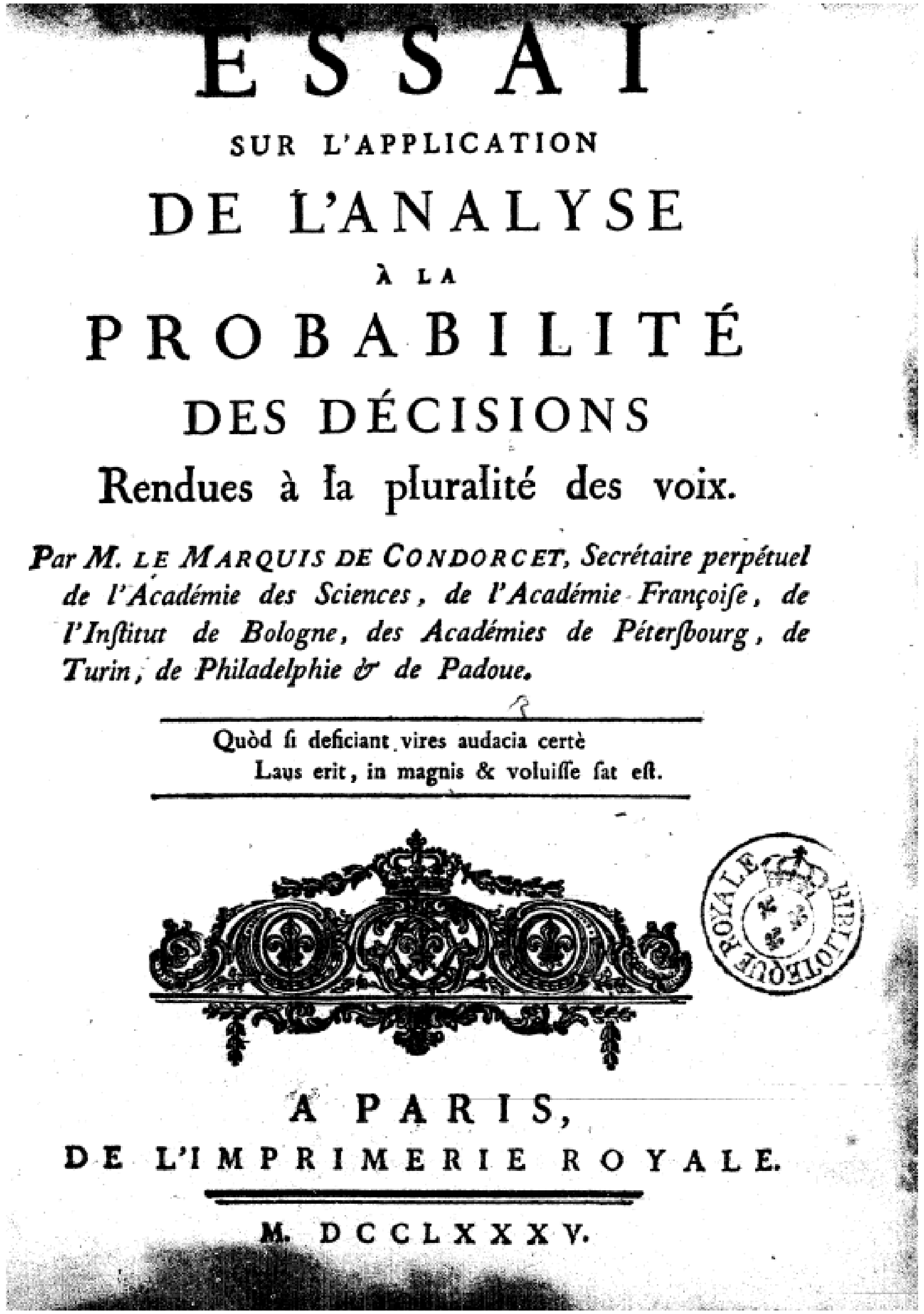

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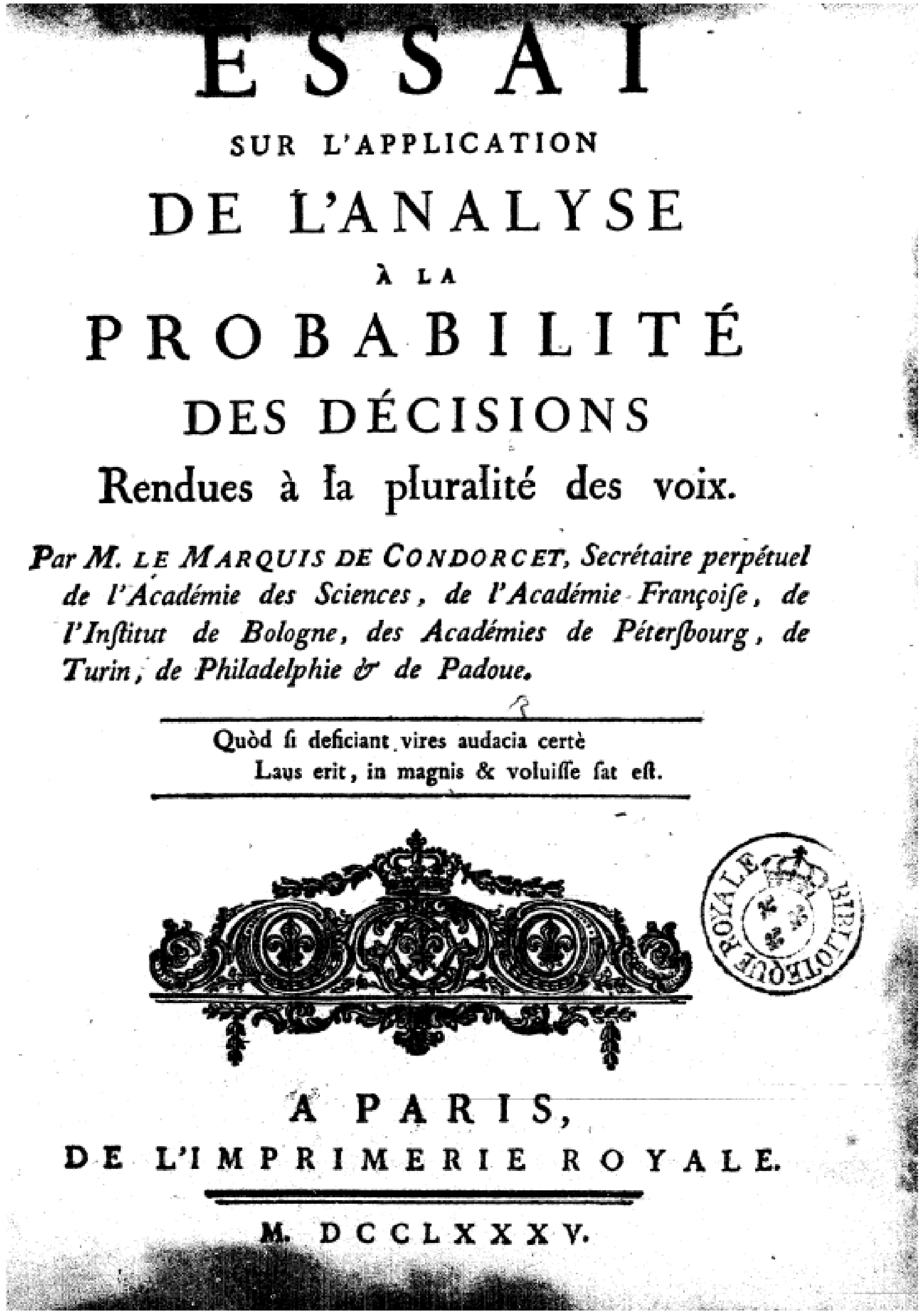

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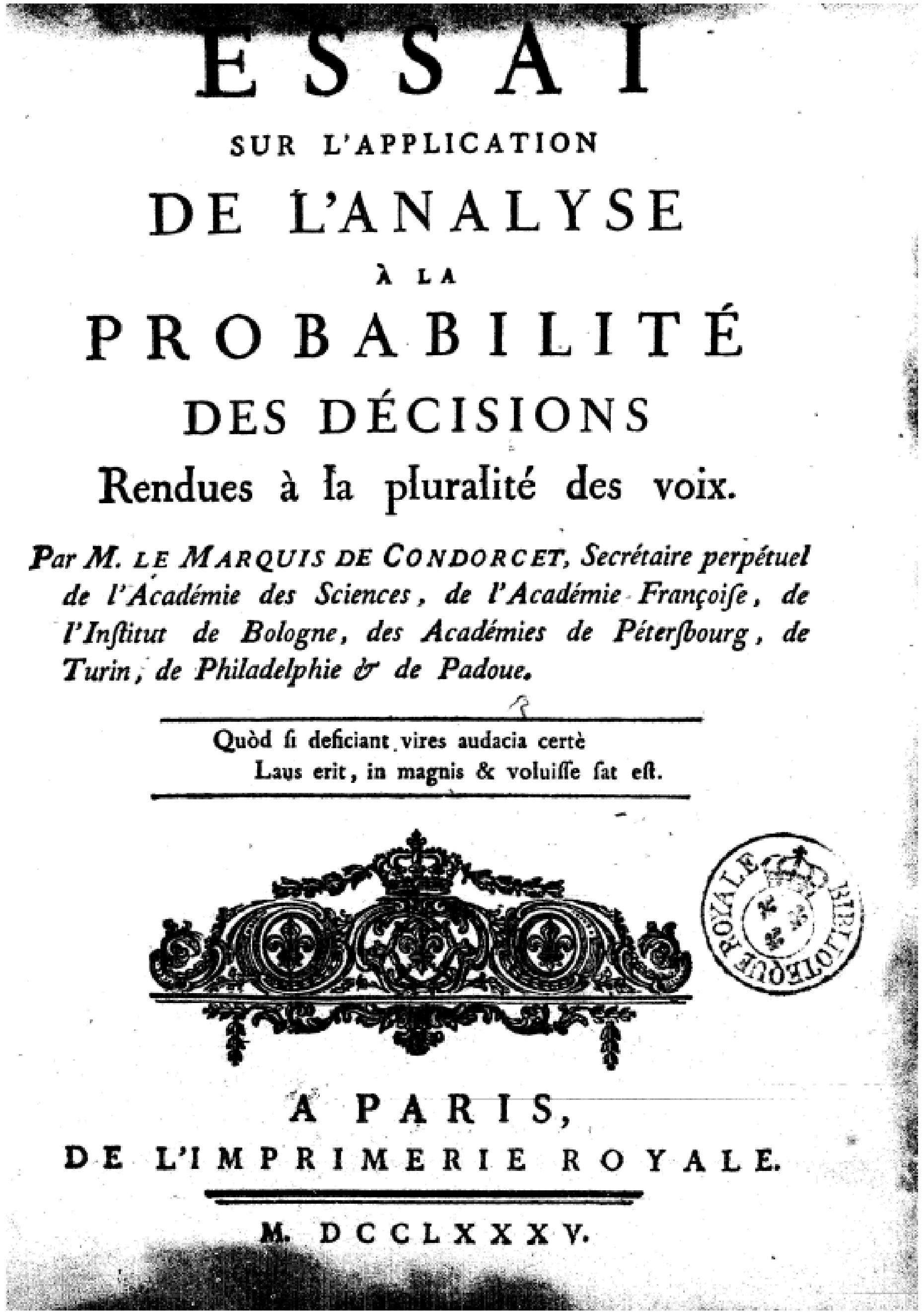

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Theorem (CJT)

The probability that majority rule elects the correct candidate is increasing in n and tends to 1 as n → ∞.

Proof of Condorcet's Jury Theorem (limiting case)

Let X_i be a random variable that is 0 or 1 depending on whether voter i votes incorrectly or correctly.

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Hoeffding's Inequality

The probability that the sum of indep. random variables $a_i \leq X_i \leq b_i$ deviates from its mean by t in one direction is

$$\leq \exp\left(\frac{-2t^2}{\sum_{i=1}^{n}(b_i - a_i)}\right).$$

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- Hoeffding $\Longrightarrow$ majority correct with probability ≥ 86.2%
- True probability is 97.8%

What about $m \geq 3$?

The *Kendall tau distance* between two rankings $\sigma_1, \sigma_2 \in \Sigma([m])$ is

$$d_{KT}(\sigma_1, \sigma_2) := |\{(a,b) | a \succ_{\sigma_1} b \wedge b \succ_{\sigma_2} a\}|.$$

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Kemeny rule: Output the Most Likely Explanation (MLE) for the ground truth, i.e., the π of maximum probability under the Mallows model.

Computing the MLE

$$\Pr[\sigma_1, \sigma_2, \ldots, \sigma_n] = \prod_{i=1}^{n} \frac{\phi^{d_{KT}(\sigma_i, \pi)}}{\sum_{\sigma' \in \Sigma([m])} \phi^{d_{KT}(\sigma', \pi)}}$$

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b	c	c	a	a	b	d	a	a	b
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▸ **How does the Kemeny ranking compare a vs b?**

Respond at:

pollev.com/jtuckerfoltz255 or

bit.ly/jtfpoll or

text jtuckerfoltz255 to 37607

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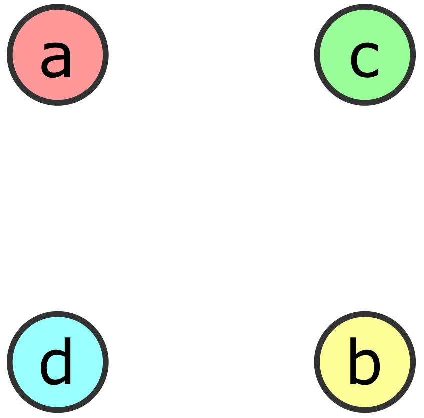

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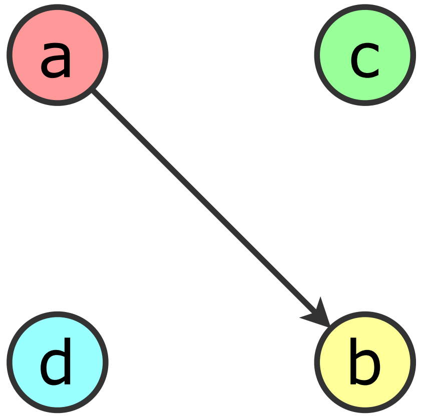

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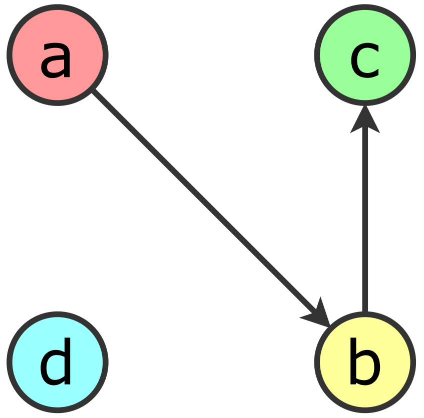

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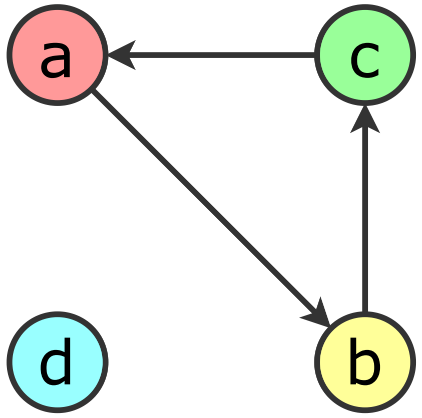

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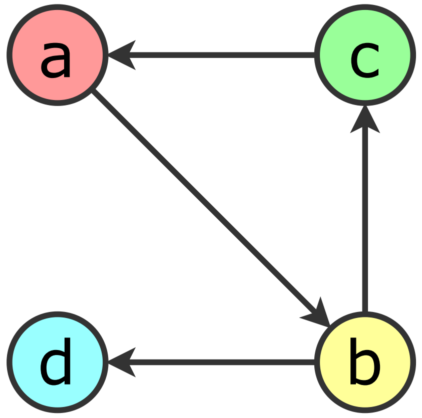

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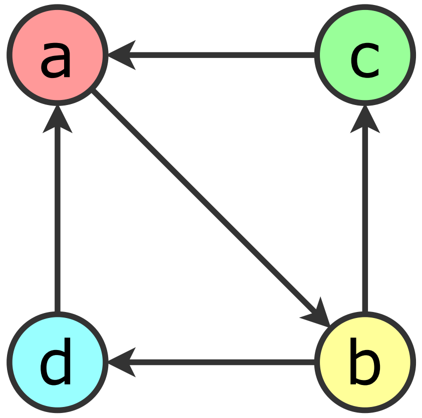
$$= \frac{1}{(\ldots)} \phi^{\sum_{i=1}^{n} d_{K,T}(\sigma_i, \pi)}$$



Kemeny outputs the ranking minimizing the sum of swap distances to all voters' rankings.

Example

1	1	1	1	1	1	1	1	1	1
a	d	b	d	c	d	b	c	d	c
b	c	c	a	a	b	d	a	a	b
c	a	a	b	b	c	a	b	b	d
d	b	d	c	d	a	c	d	c	a

▸ **How does the Kemeny ranking compare a vs b?**

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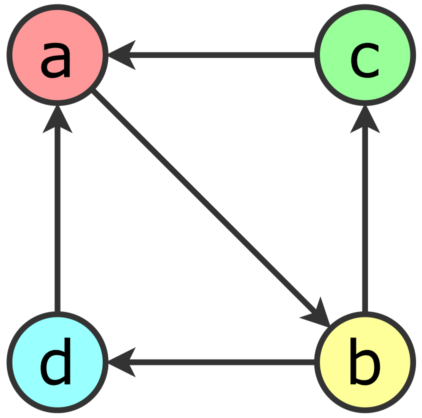

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▸ **How does the Kemeny ranking compare a vs b?**

Proposition

There is a ranking that disagrees with at most k edges

$\iff$

There is a set of at most k edges whose removal makes the graph acyclic.

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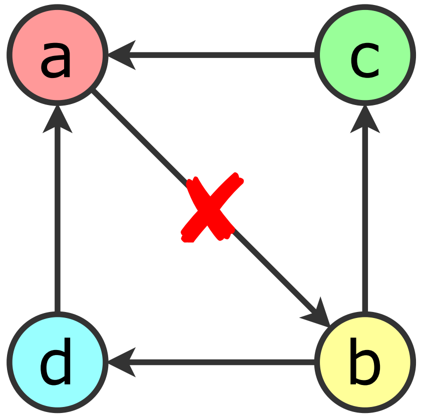

Solution: b ≻ {c, d} ≻ a

Example

1	1	1	1	1	1	1	1	1	1
a	d	b	d	c	d	b	c	d	c
b	c	c	a	a	b	d	a	a	b
c	a	a	b	b	c	a	b	b	d
d	b	d	c	d	a	c	d	c	a

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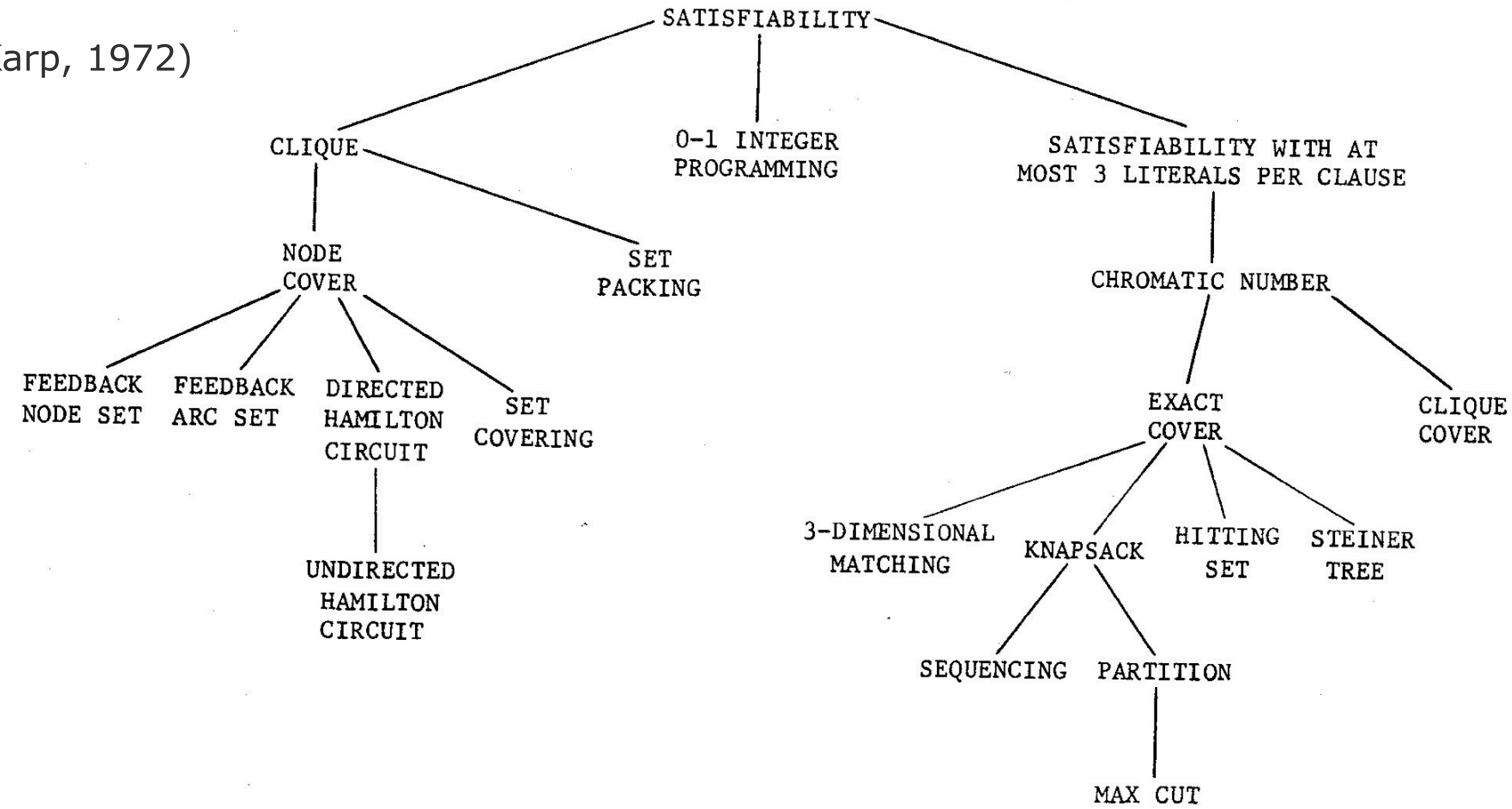
Kemeny is NP-complete

(Karp, 1972)



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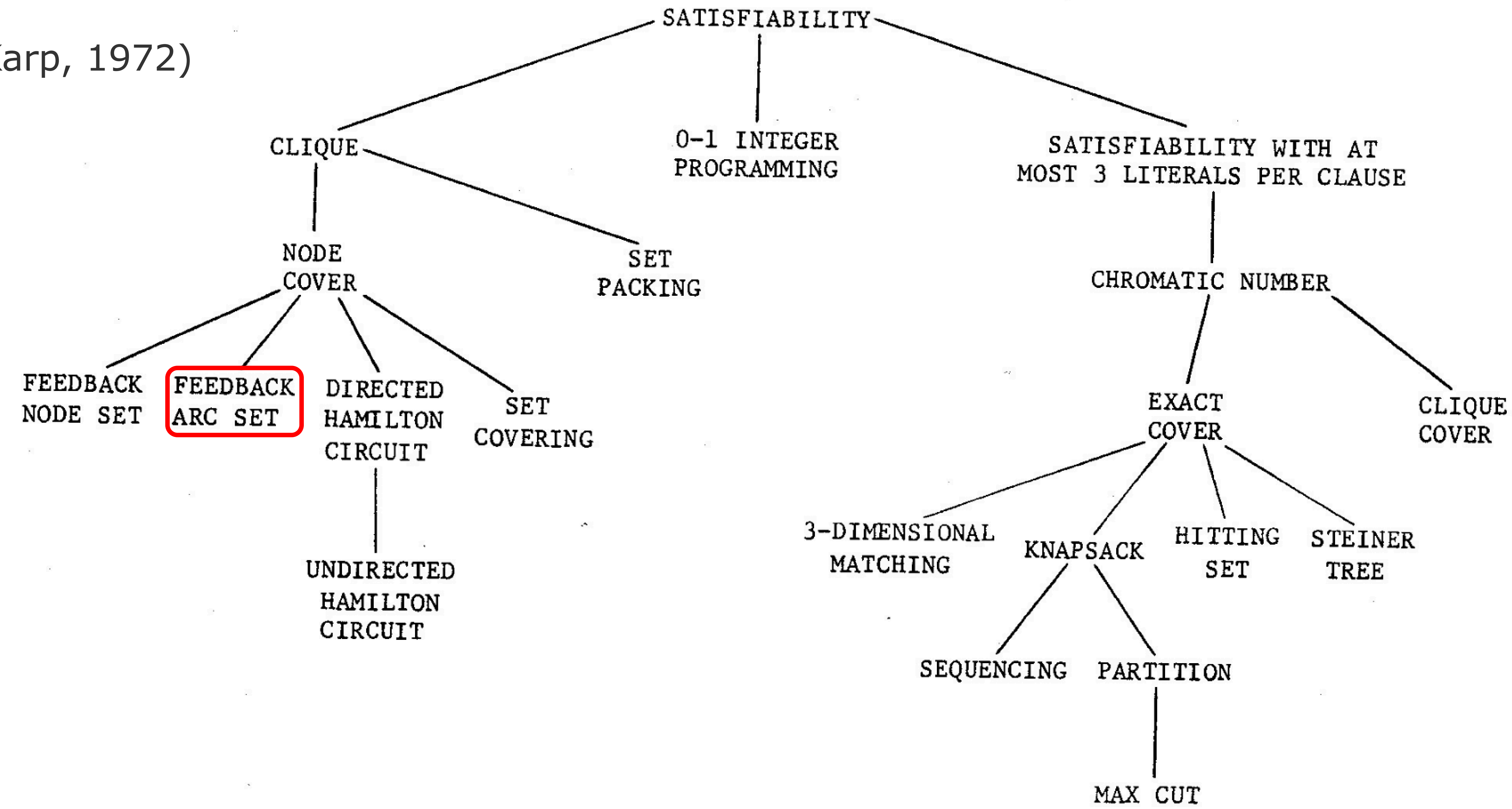

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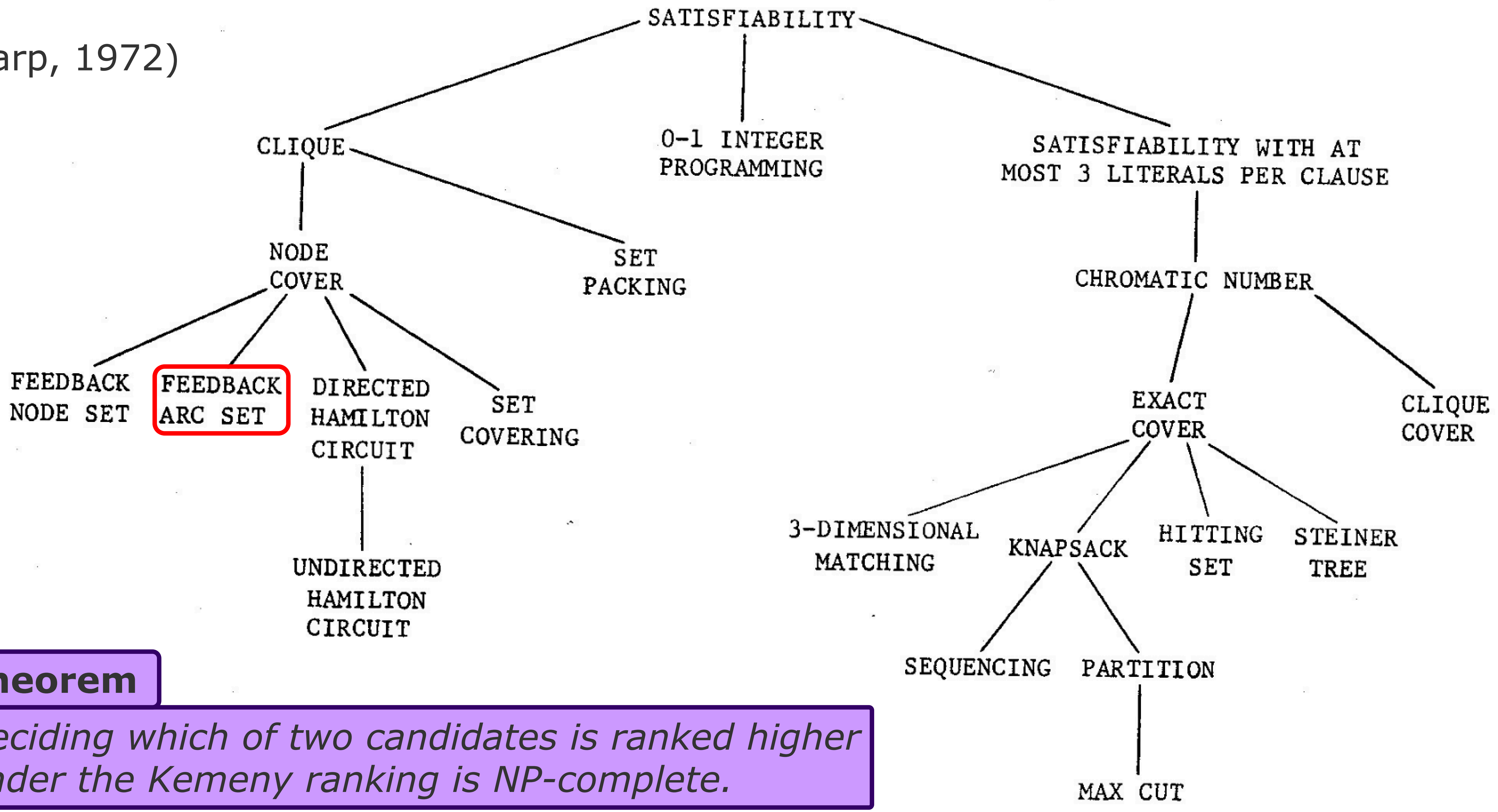

Theorem

Deciding which of two candidates is ranked higher under the Kemeny ranking is NP-complete.