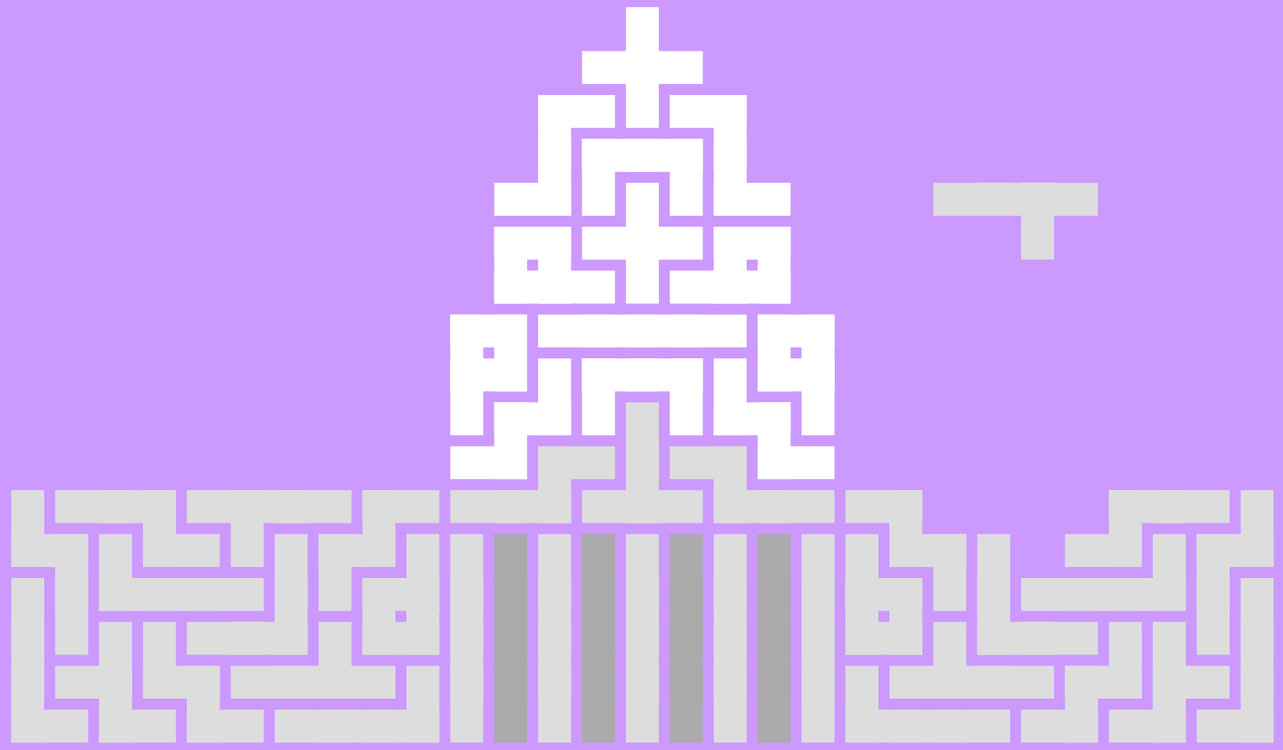

Algorithms For Democratic Decision-Making

Jamie Tucker-Foltz • Yale University • Spring 2026

Lecture 16: **Liquid Democracy**

Outline for the next 4 weeks

How do you fairly
allocate political power?

Outline for the next 4 weeks

2011 Berlin state election

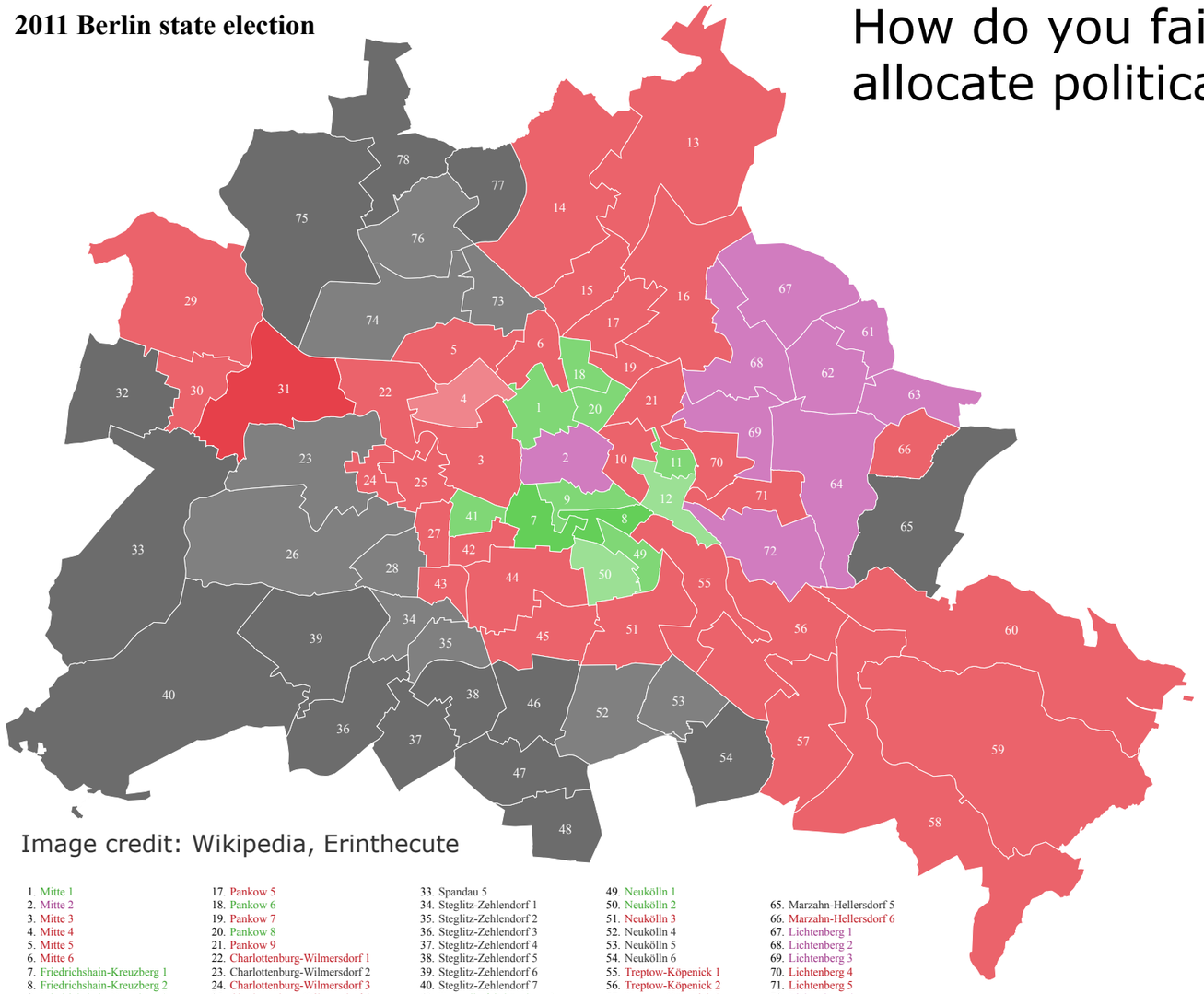

Image credit: Wikipedia, Erinthecute

How do you fairly allocate political power?

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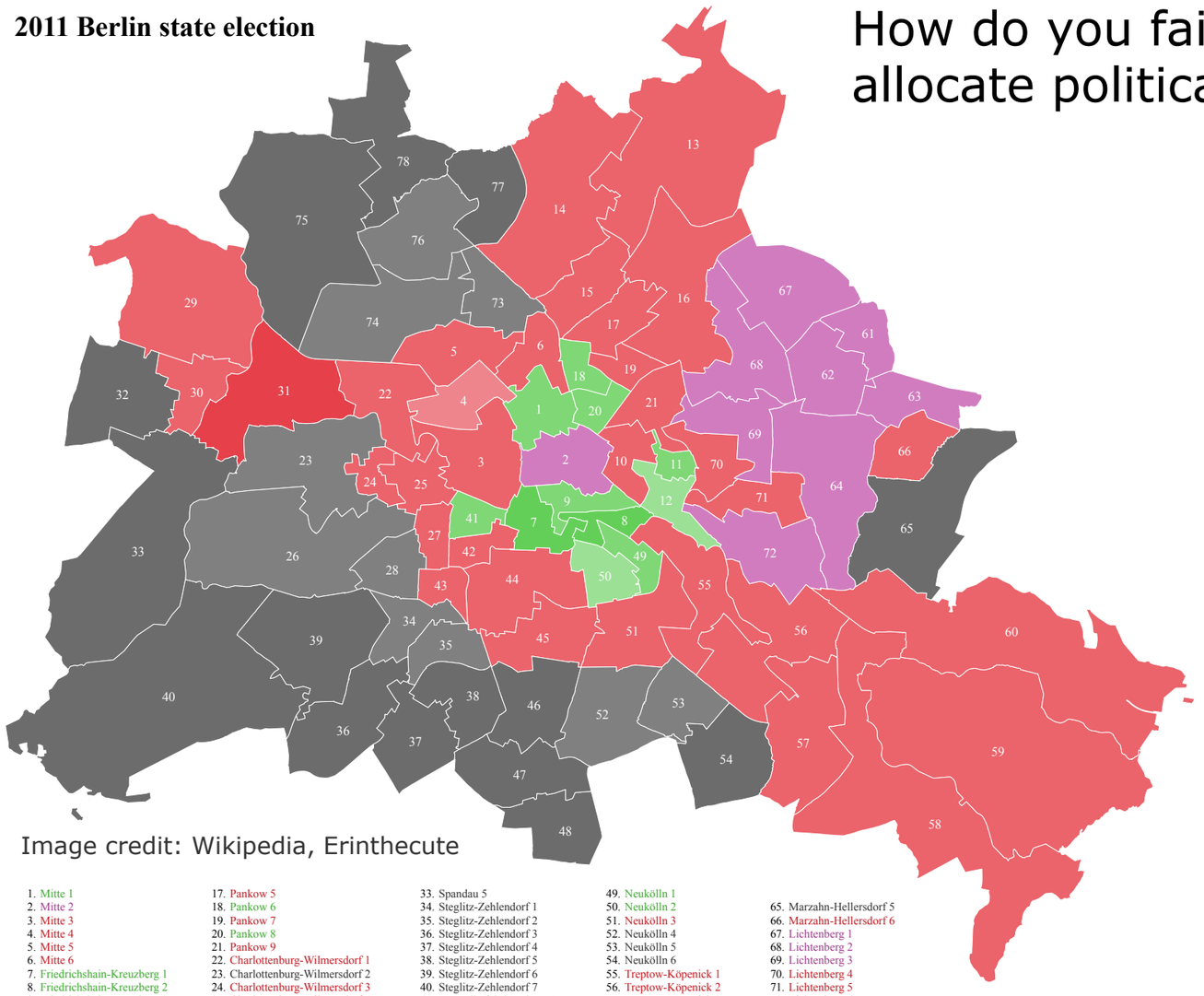
Outline for the next 4 weeks

2011 Berlin state election



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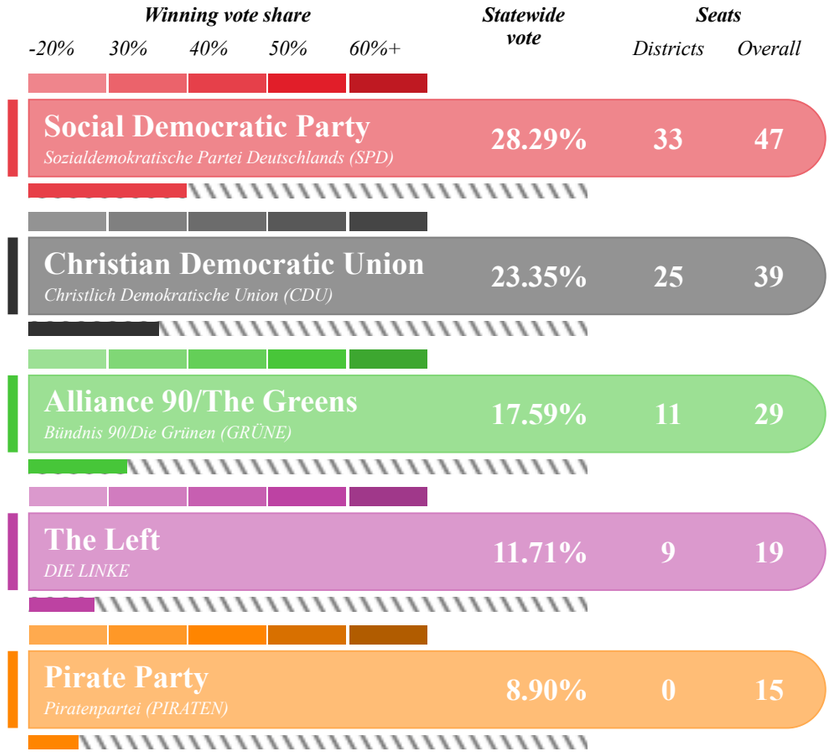

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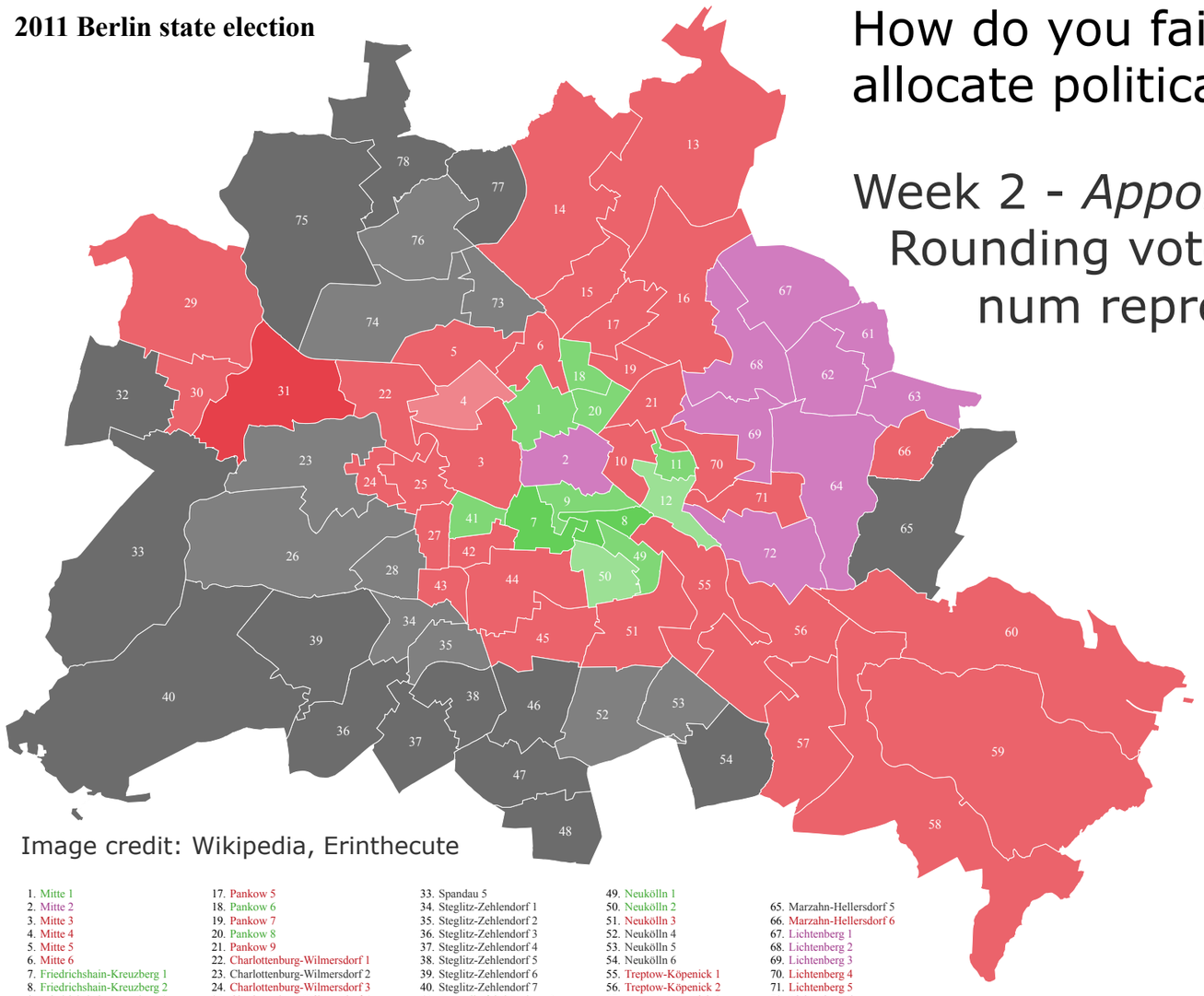

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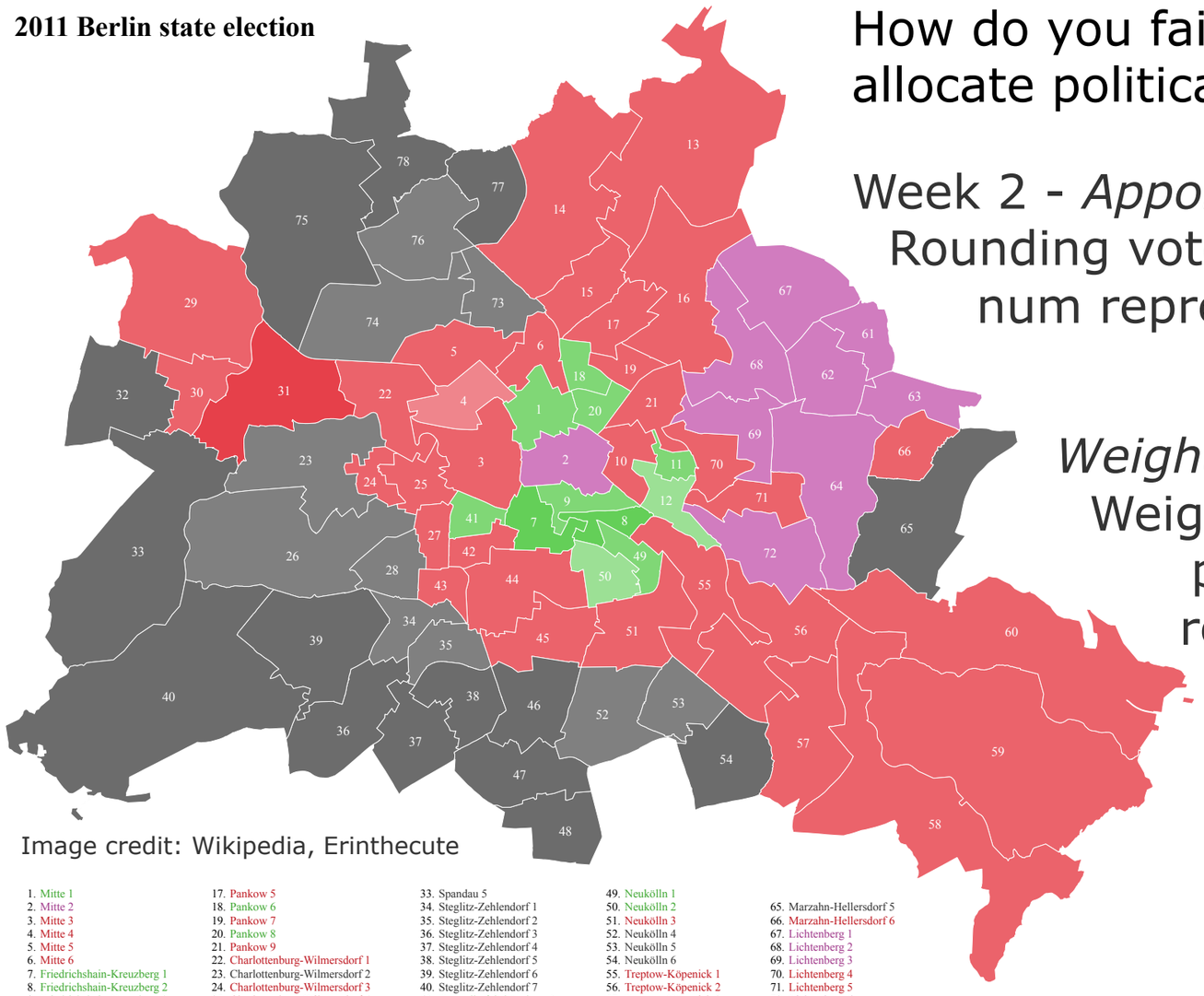
How do you fairly allocate political power?

Week 2 - *Apportionment*: Rounding vote shares to num representatives

	Winning vote share (-20% 30% 40% 50% 60%+)	Statewide vote	Seats: Districts	Seats: Overall
Social Democratic Party *Sozialdemokratische Partei Deutschlands (SPD)*		28.29%	33	47
Christian Democratic Union *Christlich Demokratische Union (CDU)*		23.35%	25	39
Alliance 90/The Greens *Bündnis 90/Die Grünen (GRÜNE)*		17.59%	11	29
The Left *DIE LINKE*		11.71%	9	19
Pirate Party *Piratenpartei (PIRATEN)*		8.90%	0	15

Outline for the next 4 weeks

2011 Berlin state election



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Week 2 - *Apportionment*: Rounding vote shares to num representatives

Week 3 - *Weighted voting*: Weights are the populations represented

	Winning vote share (-20% / 30% / 40% / 50% / 60%+)	*Statewide vote*	*Seats* Districts	*Seats* Overall
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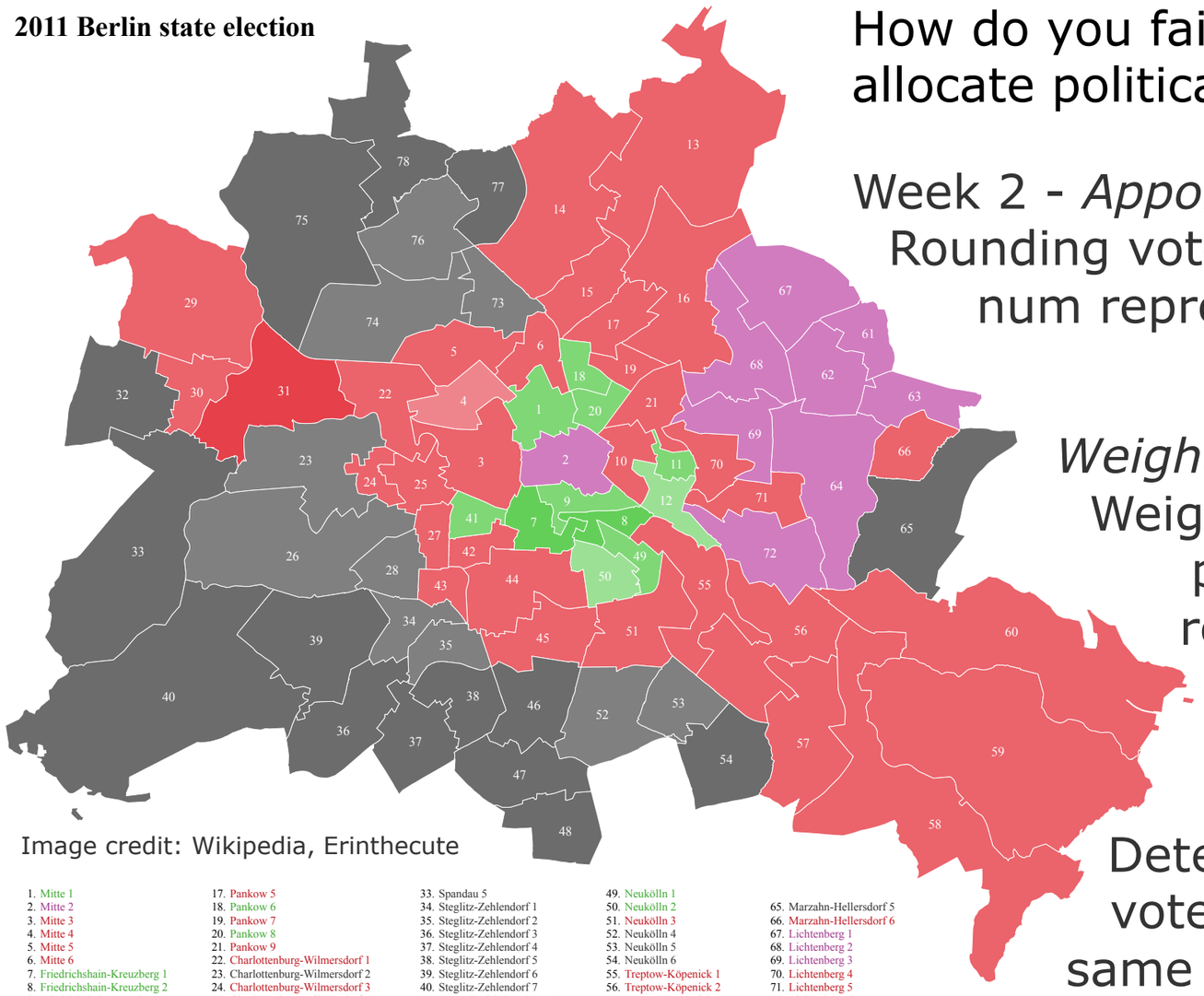

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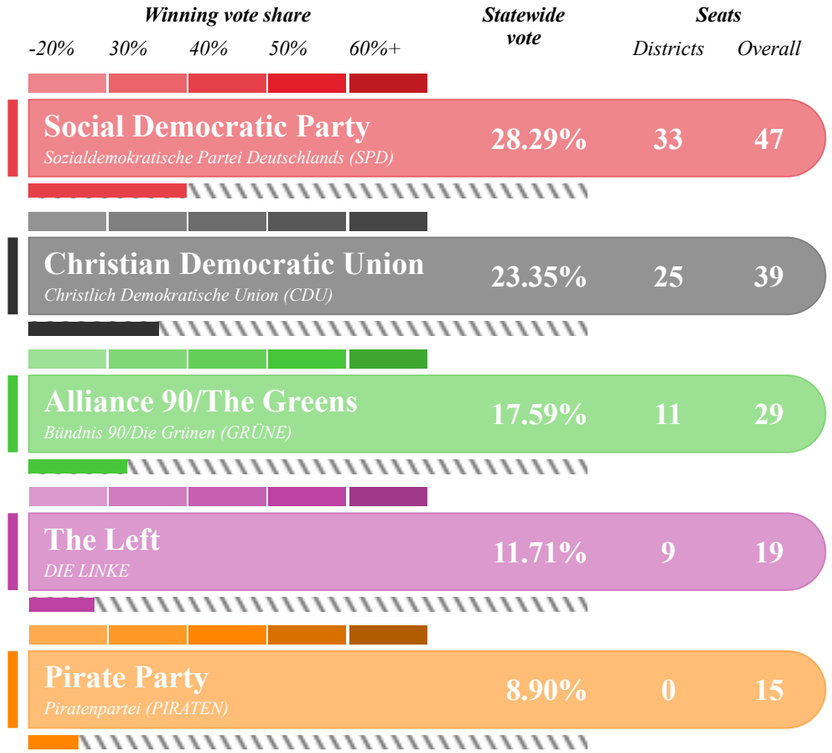
Week 3 - *Weighted voting*: Weights are the populations represented

Week 4 - *Redistricting*: Determining which voters vote for the same representative



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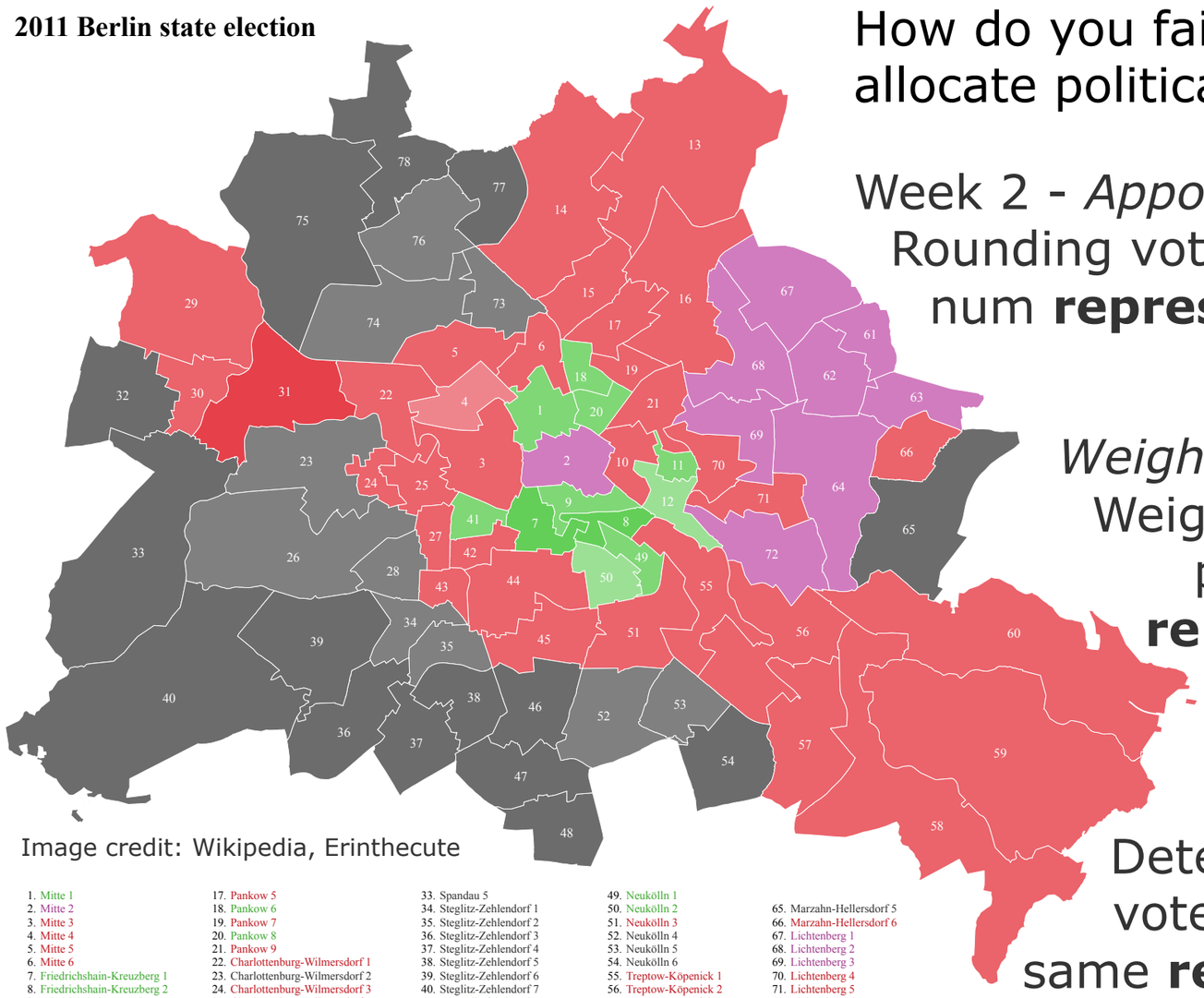

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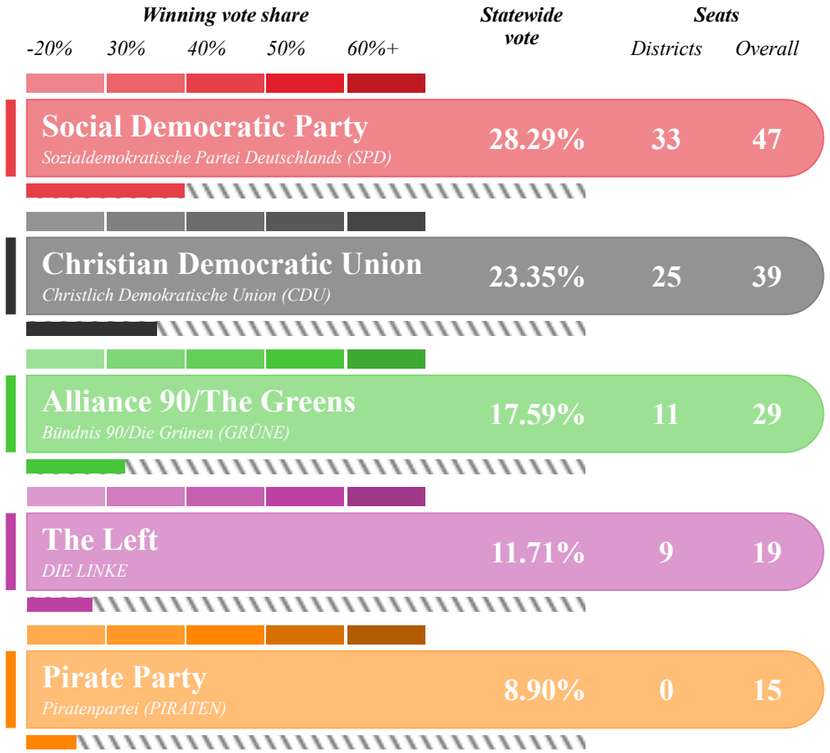
How do you fairly allocate political power?

Week 2 - *Apportionment*: Rounding vote shares to num **representatives**

Week 3 - *Weighted voting*: Weights are the populations **represented**

Week 4 - *Redistricting*: Determining which voters vote for the same **representative**



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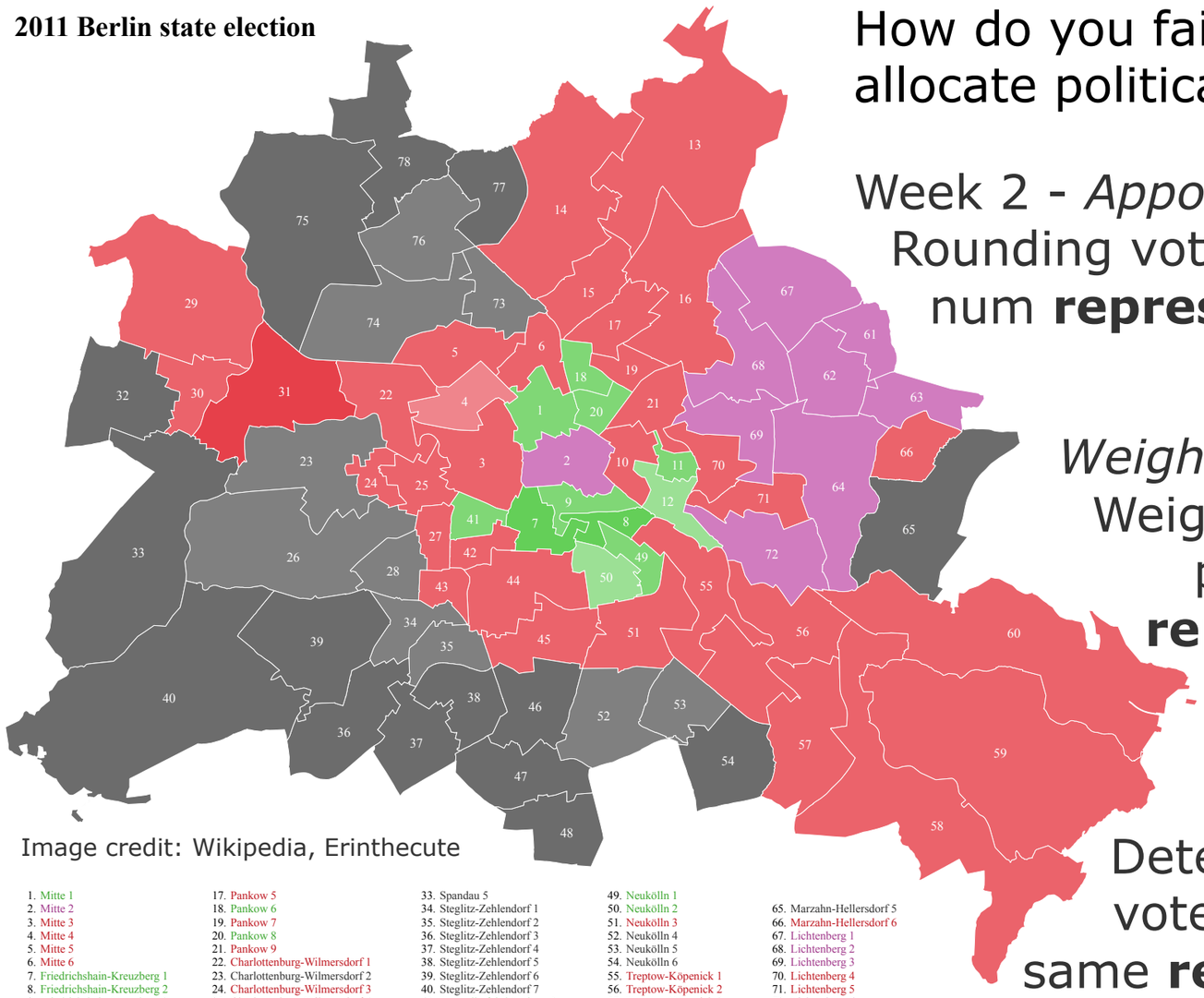

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Week 2 - *Apportionment*: Rounding vote shares to num **representatives**

Week 3 - *Weighted voting*: Weights are the populations **represented**

Week 4 - *Redistricting*: Determining which voters vote for the same **representative**

Week 1: **Alternatives** to representative democracy

	Winning vote share (-20% 30% 40% 50% 60%+)	*Statewide vote*	*Seats* Districts	Overall
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Outline for the next 4 weeks

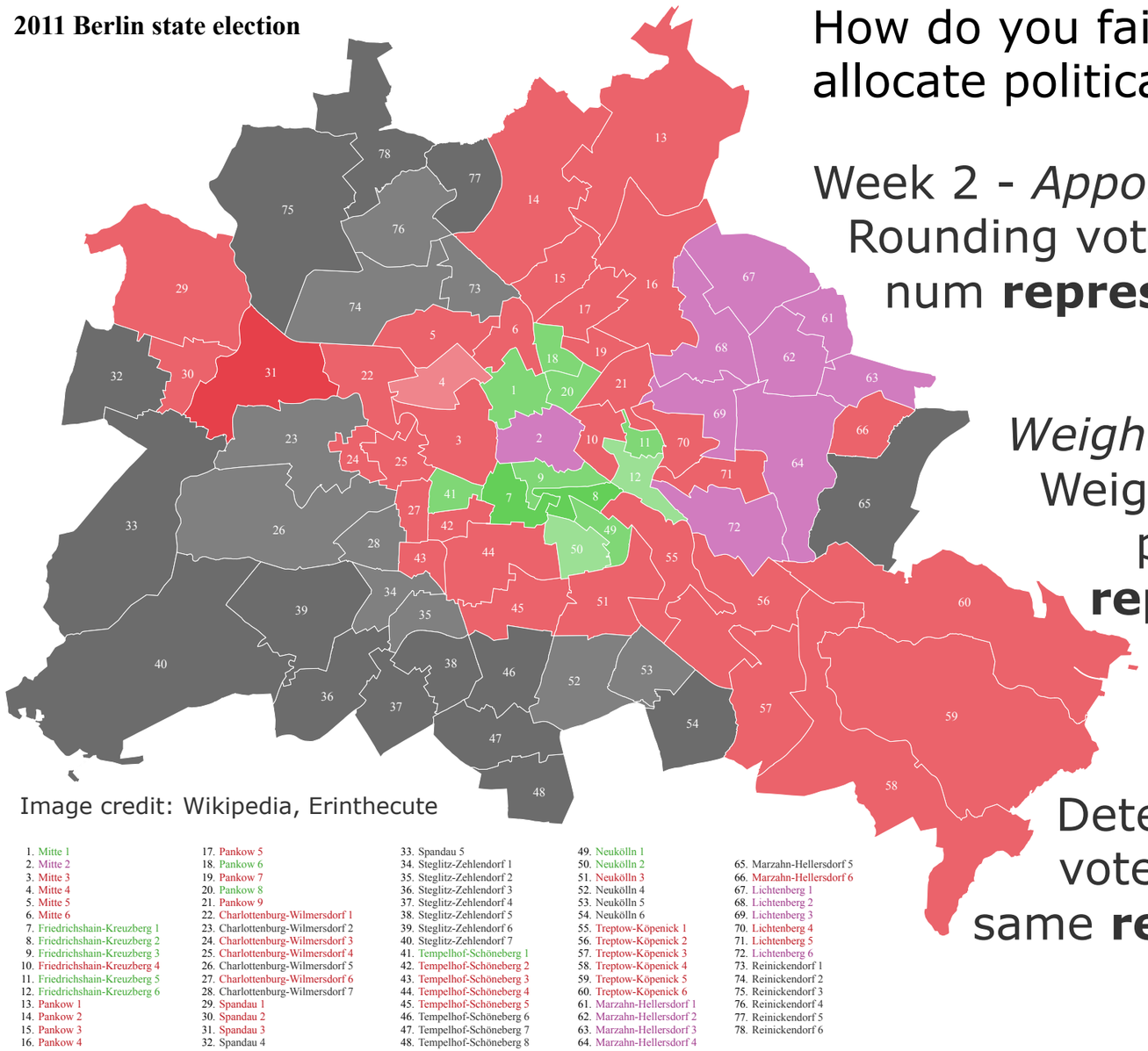

Image credit: Wikipedia, Erinthecute

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13. Pankow 1
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22. Charlottenburg-Wilmersdorf 1
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How do you fairly allocate political power?

Week 2 - *Apportionment*: Rounding vote shares to num **representatives**

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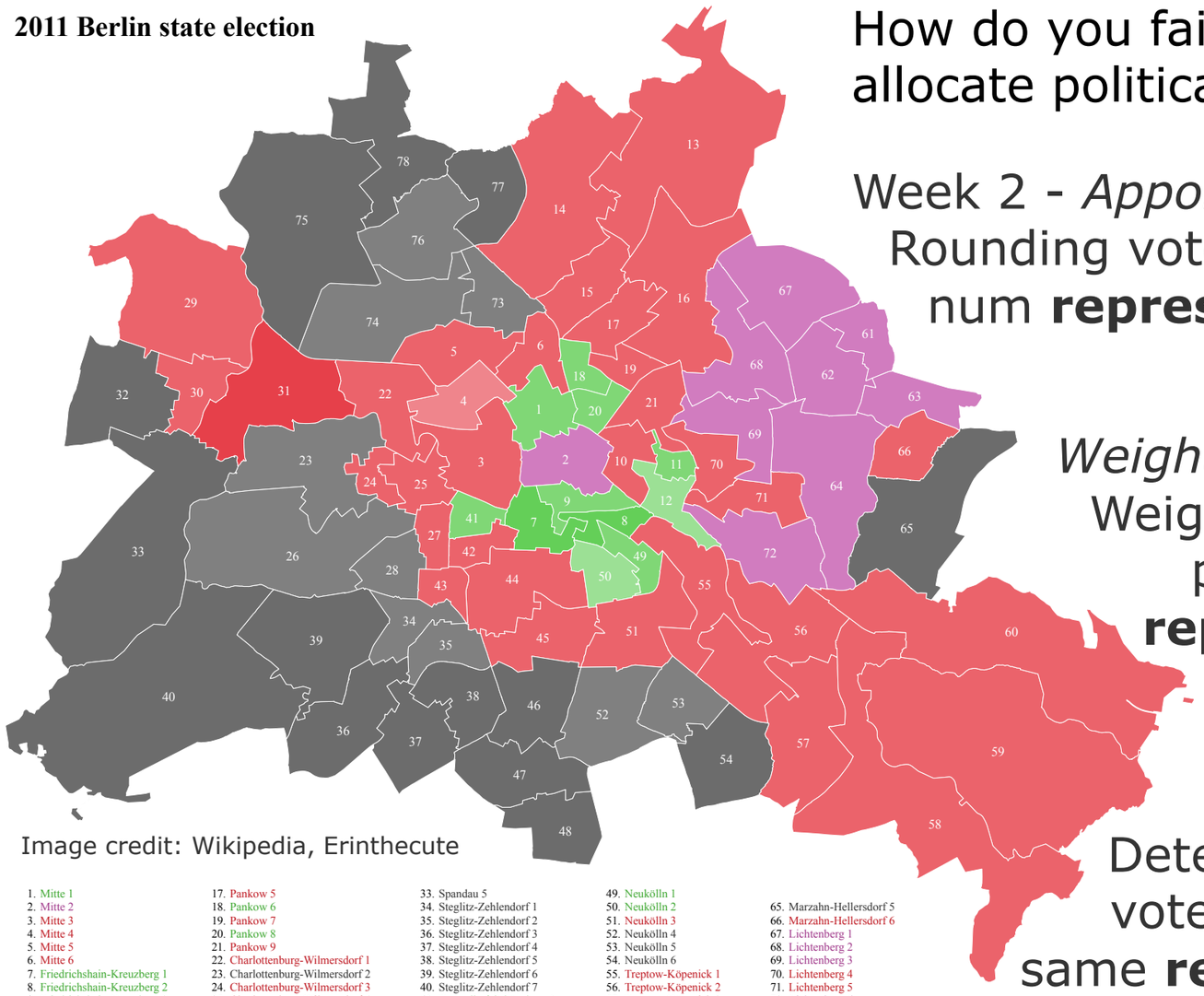
WTF?

Week 1: **Alternatives** to representative democracy

Outline for the next 4 weeks



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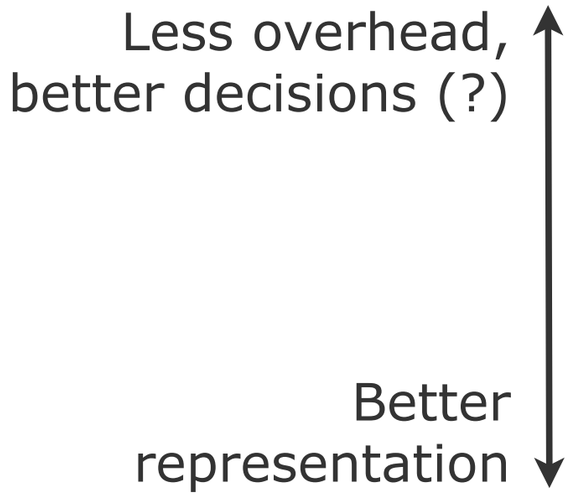
youtu.be/tvPCdoaxZdM?si=Nqzgx5iCTIMxemc4&t=112

Liquid democracy

Most widely-used system is *representative democracy*: voters elect representatives, then representatives make decisions



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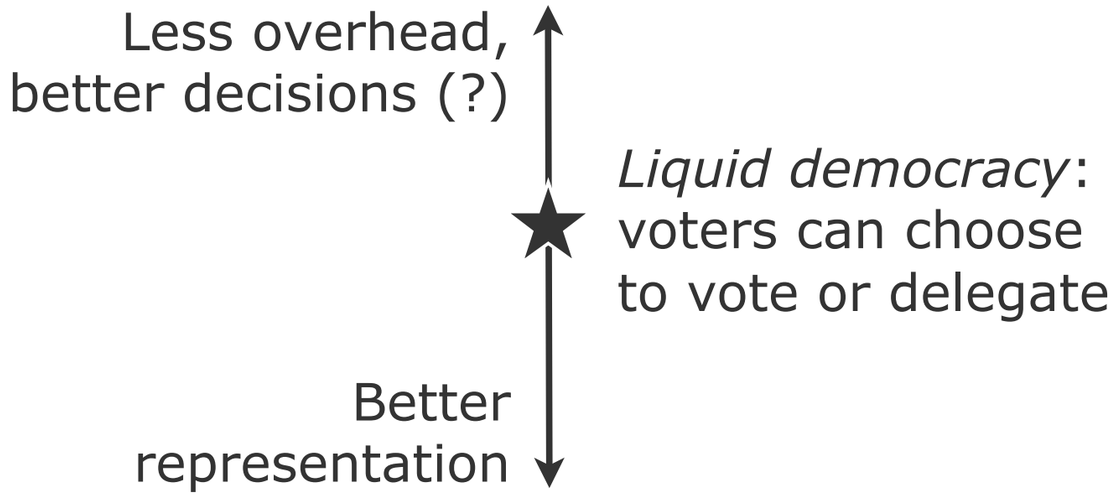

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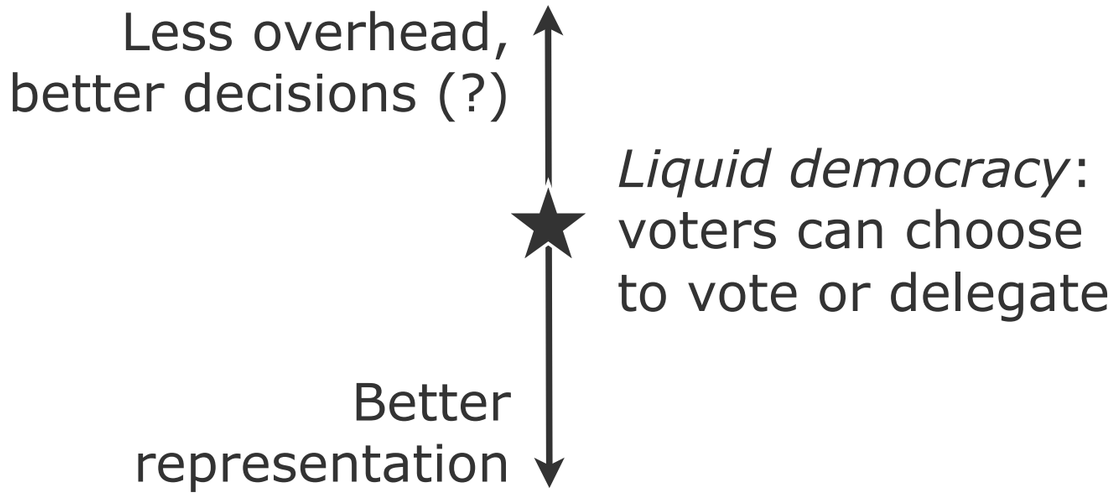

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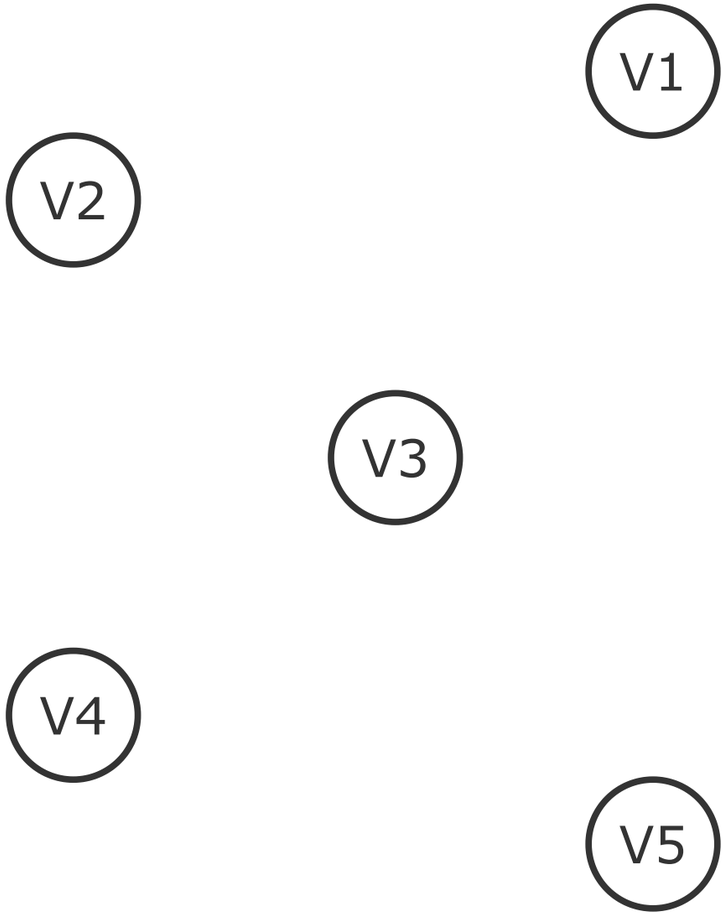

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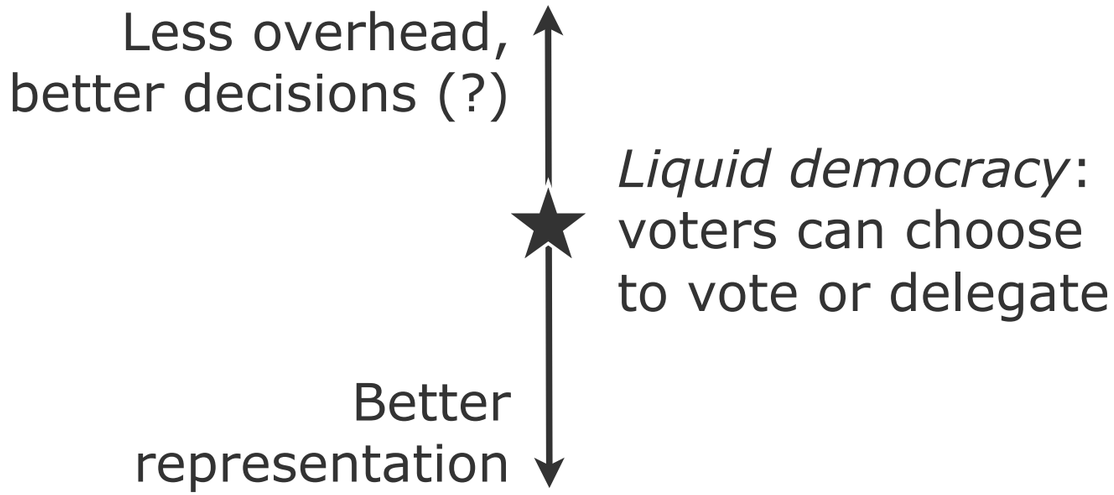

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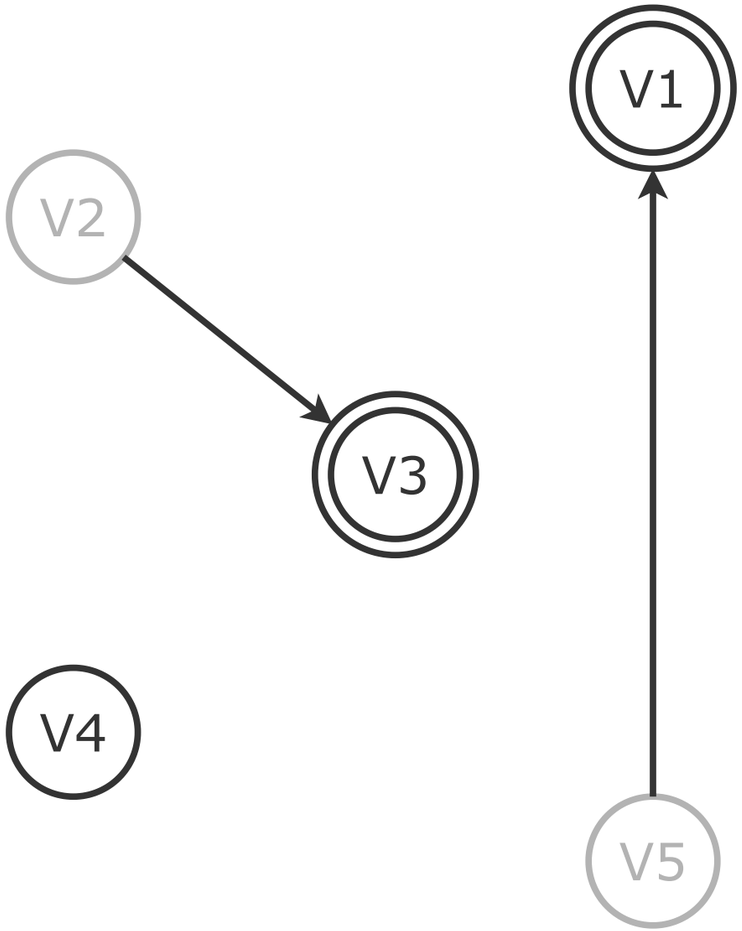

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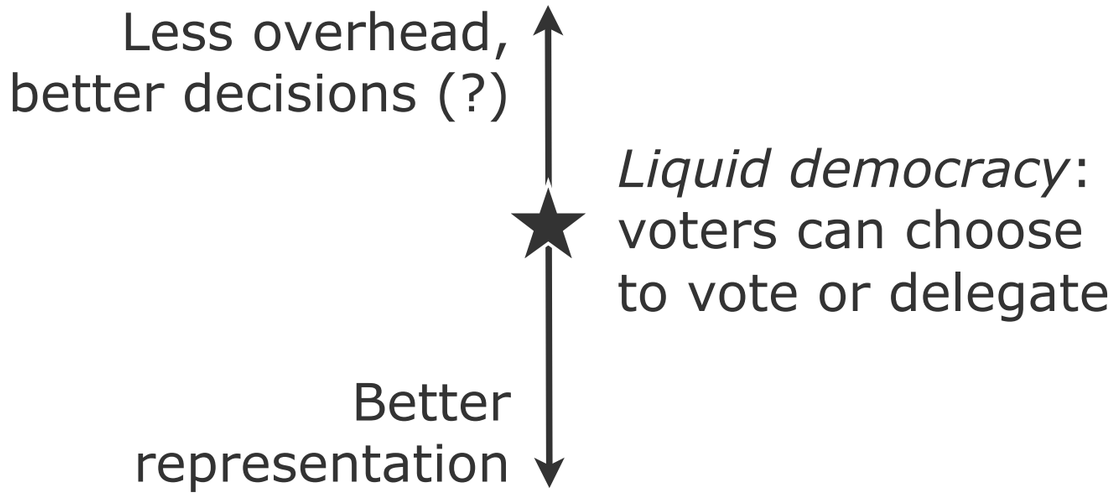

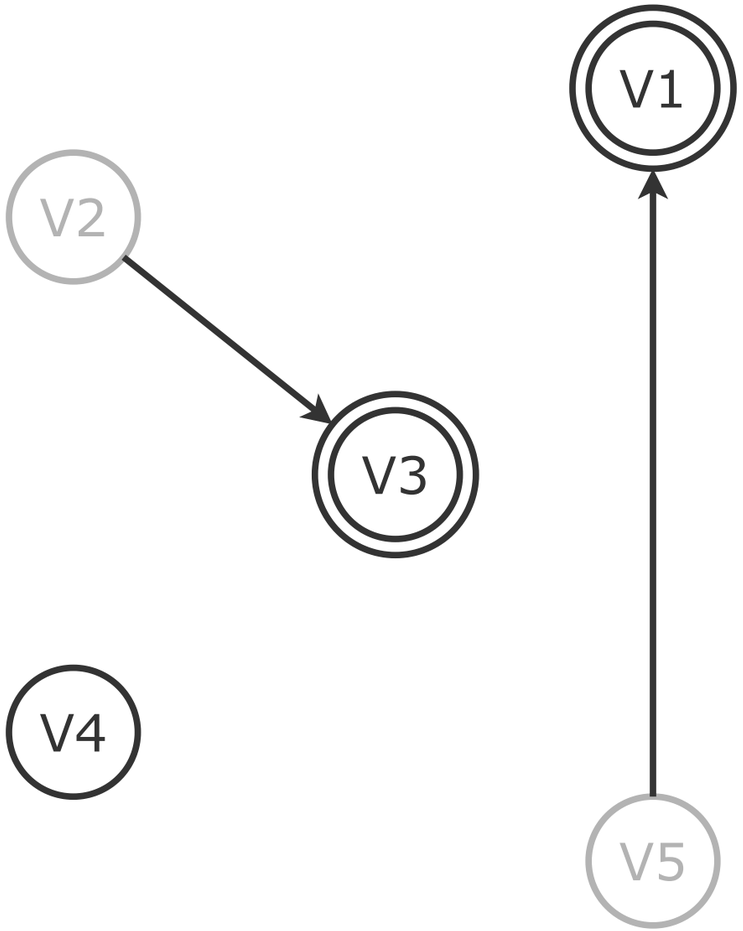

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Epistemic approach: Assume that each voter *i* has a competence level p_i, which is the probabiliy with which they will vote for the correct of two alternatives.

Optimal delegation

Suppose there is one voter with competence $p_1 = 0.8$
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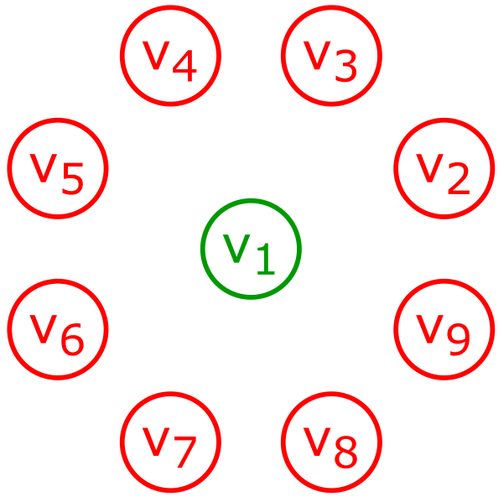

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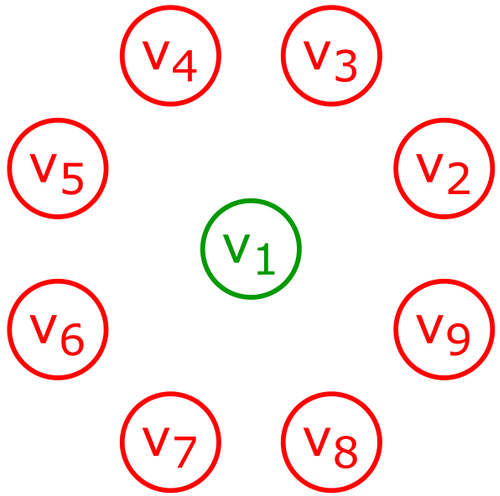

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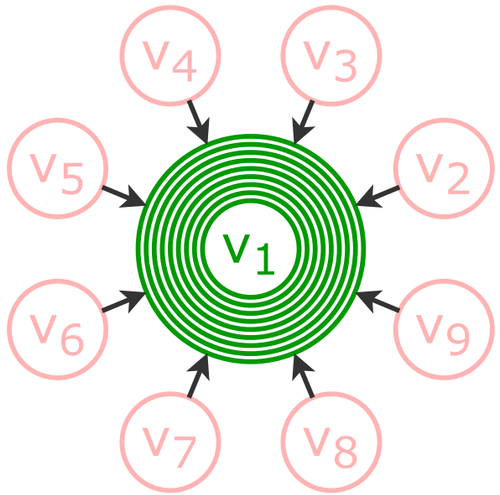

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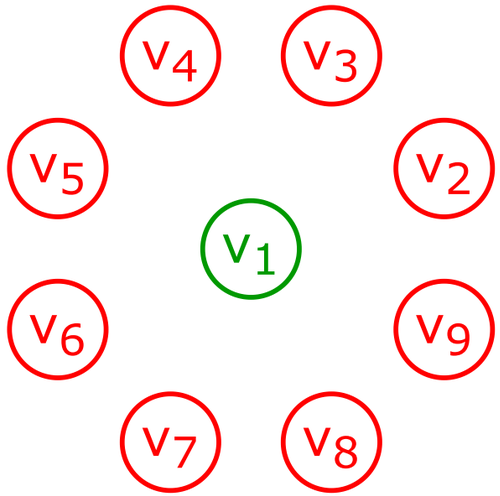

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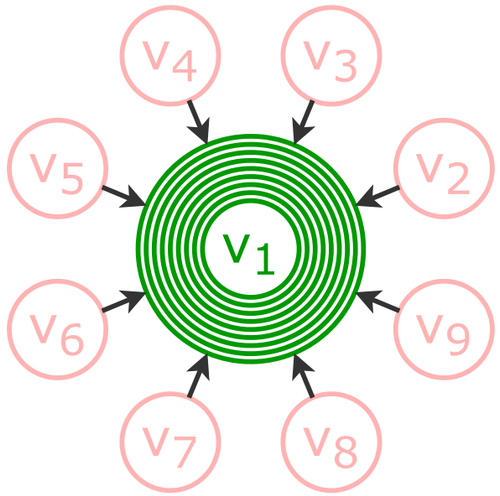

"Epistocracy"

- **What is the optimal delegation graph?**
  - **Direct democracy**
  - **Epistocracy**
  - **Something else**

Respond at:

pollev.com/jtuckerfoltz255 or

bit.ly/jtfpoll or

text jtuckerfoltz255 to 37607

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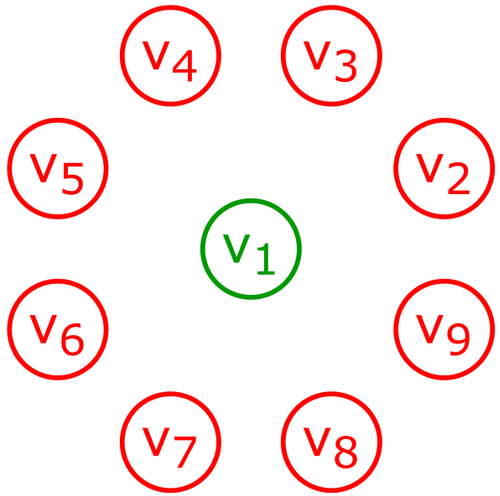

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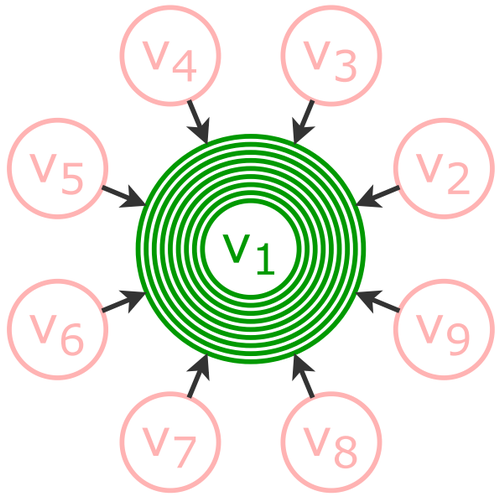

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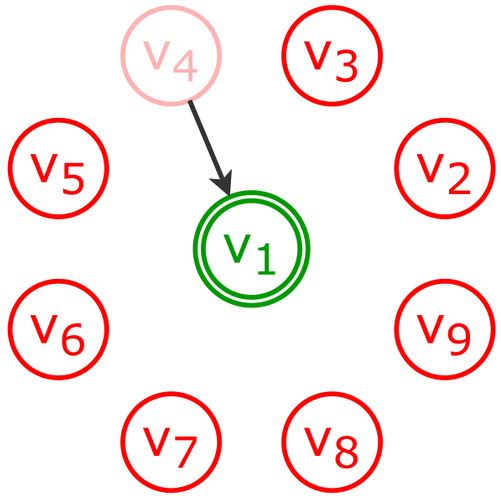

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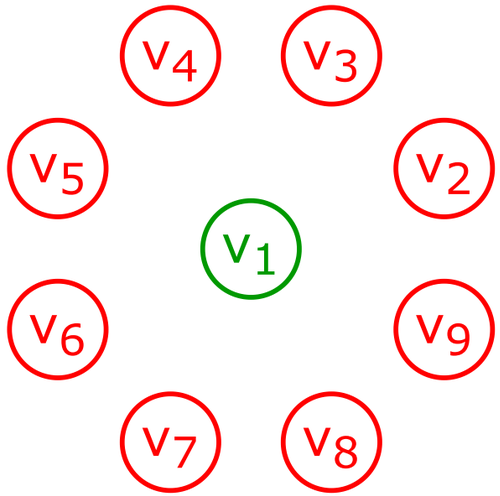
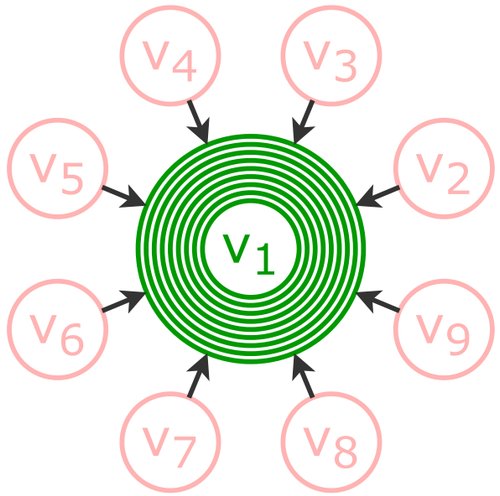
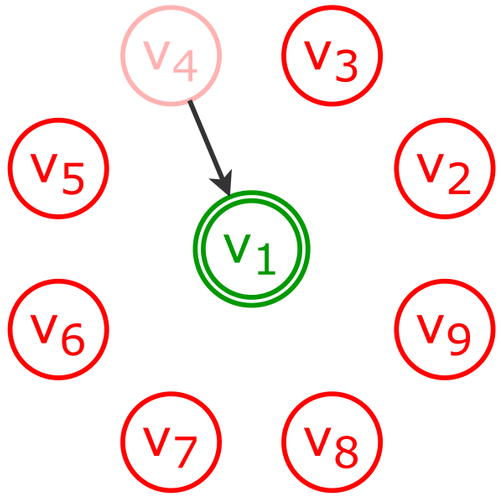


"Direct democracy" (93%)



"Epistocracy" (80%)



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Assume that there is an underlying directed graph G where the voters are vertices and delegation can only happen along edges of G.

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Proof. If there was such an algorithm, it could be used to solve this "gap problem":

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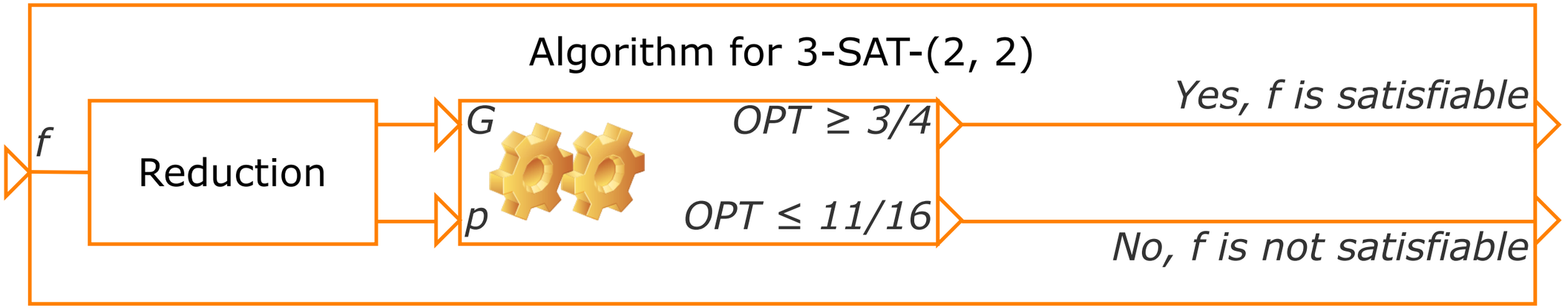
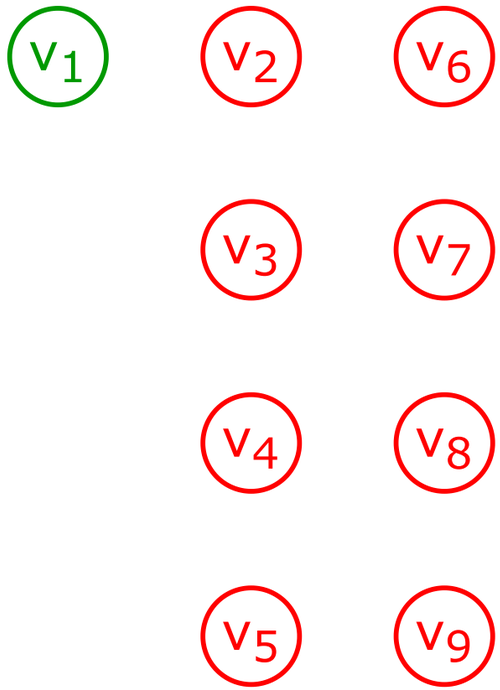


Where 3/4 and 11/16 come from



Suppose there is one voter with competence 1 and the rest have competence 1/2.

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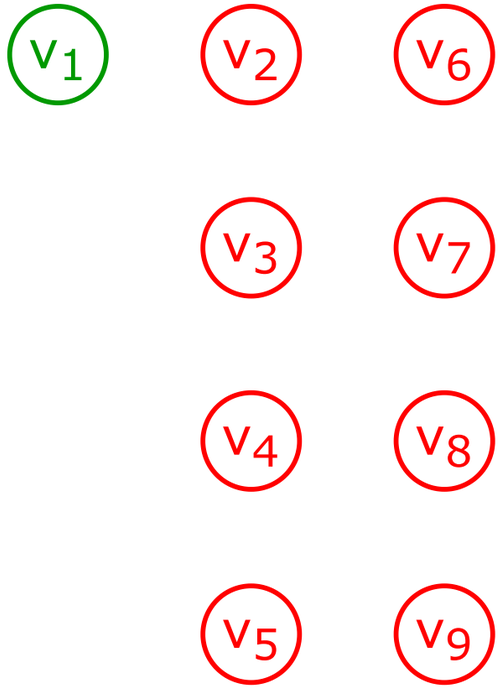

Suppose there is one voter with competence 1 and the rest have competence 1/2.

$$\Pr[\text{correct}] = \frac{\sum_{i=0}^{4} \binom{8}{i}}{2^8} = \frac{163}{256}$$

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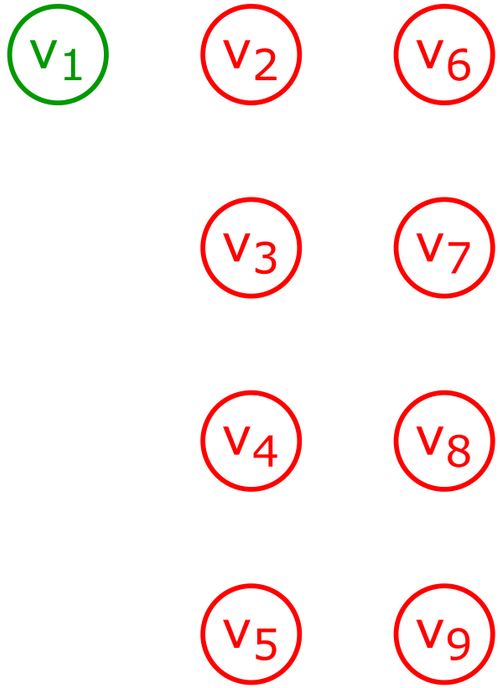

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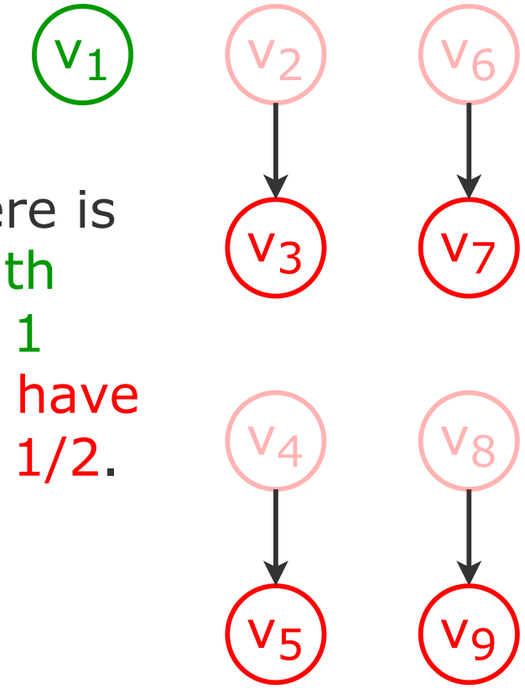

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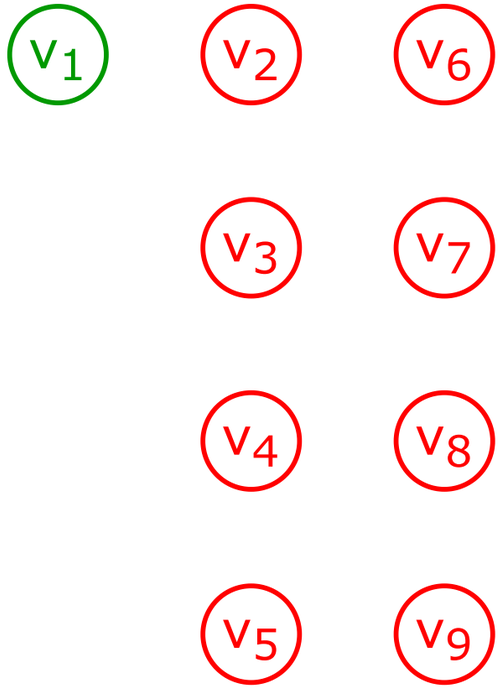

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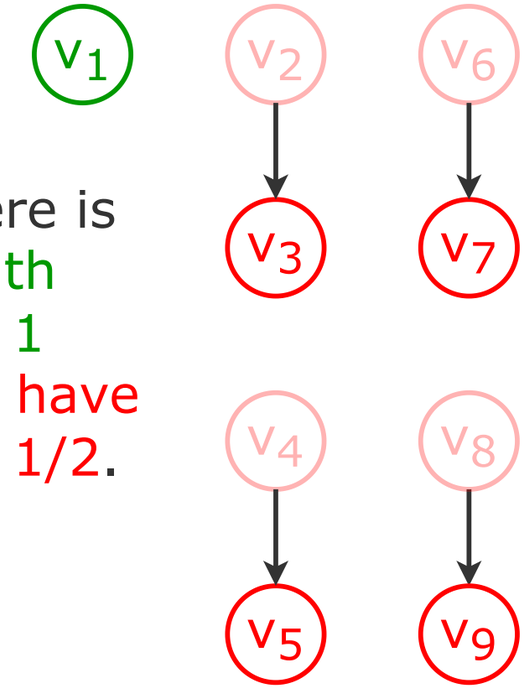

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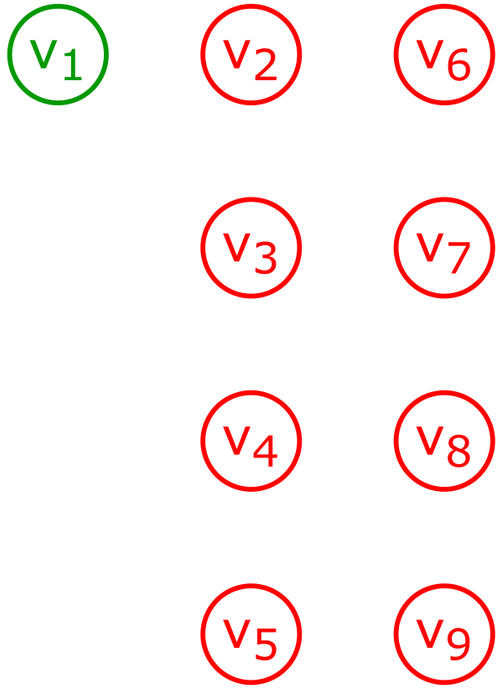

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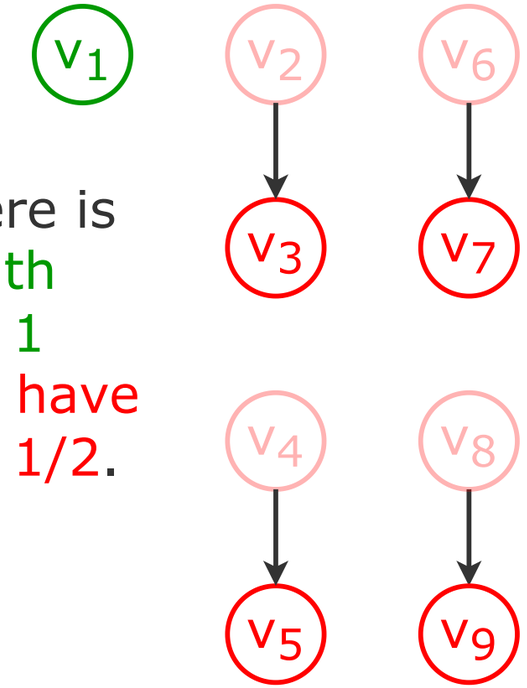

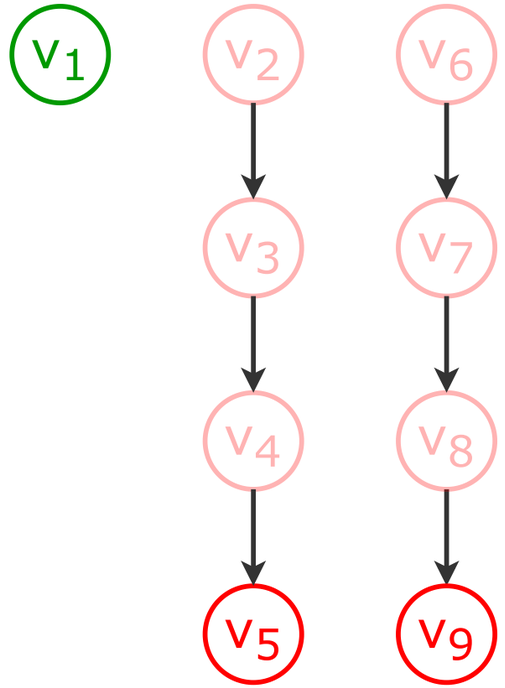

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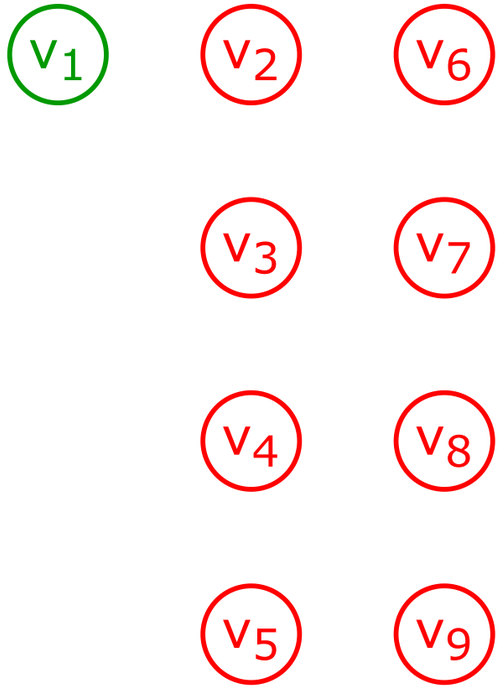

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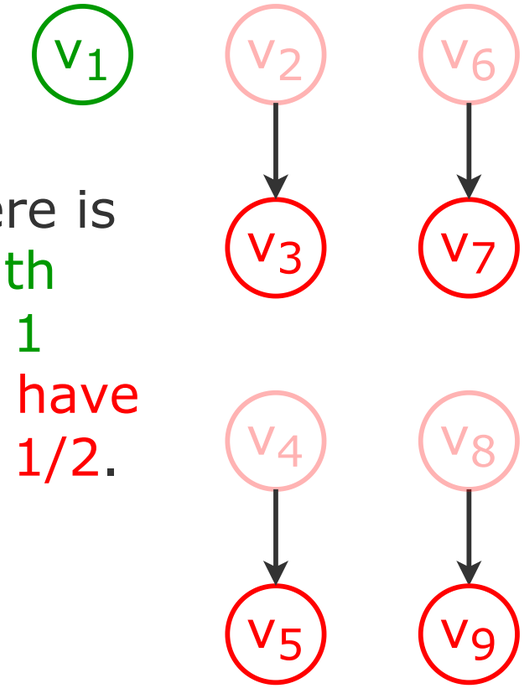

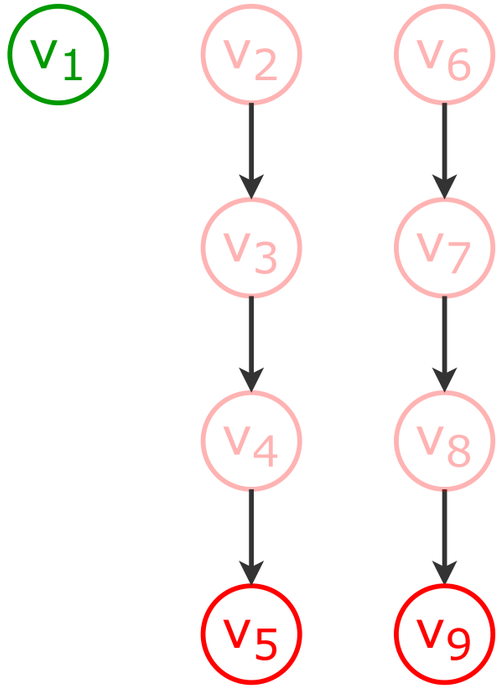

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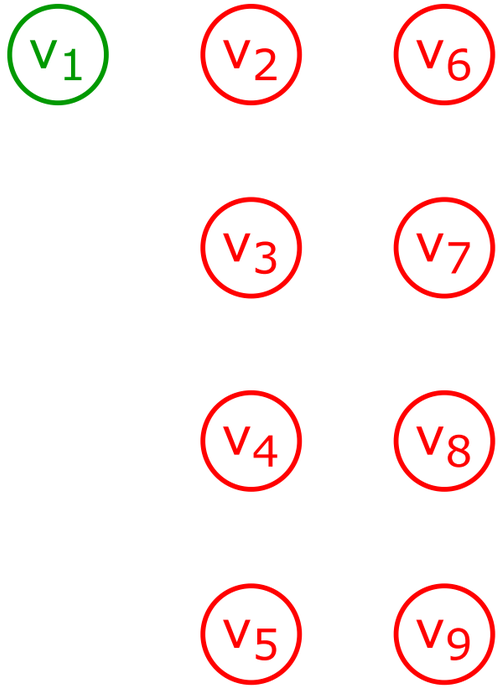
Where 3/4 and 11/16 come from



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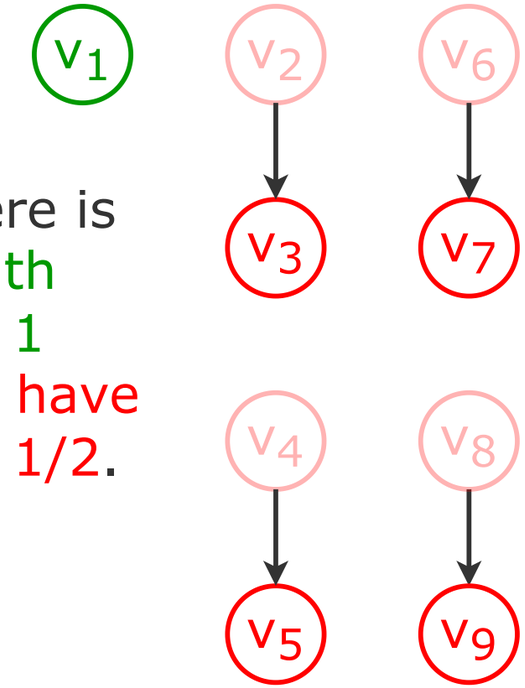

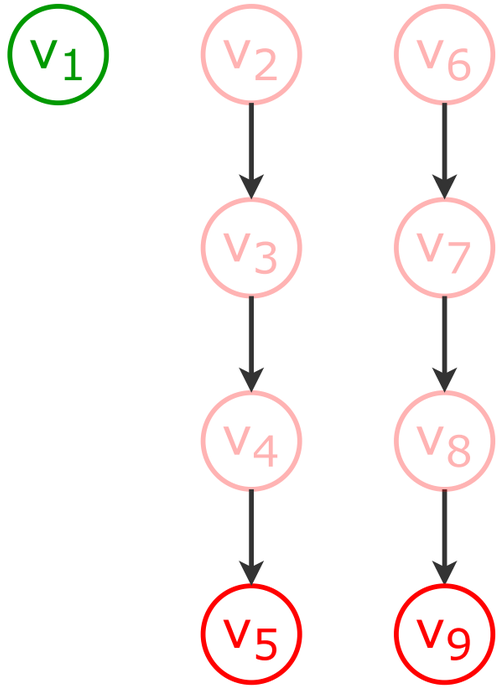

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Lemma

Let S be a multiset of positive integers summing to 2n.

- *If S = {n, n}, the probability a unif. random subset of S sums to at least n is 3/4.*
- Otherwise, *the probability is at most 11/16.*

Reduction from 3-SAT(2, 2)

$$(x_1 \vee x_1 \vee x_2) \wedge (\overline{x_1} \vee \overline{x_1} \vee \overline{x_2}) \wedge (x_2 \vee x_3 \vee x_3) \wedge (\overline{x_2} \vee \overline{x_3} \vee \overline{x_3})$$

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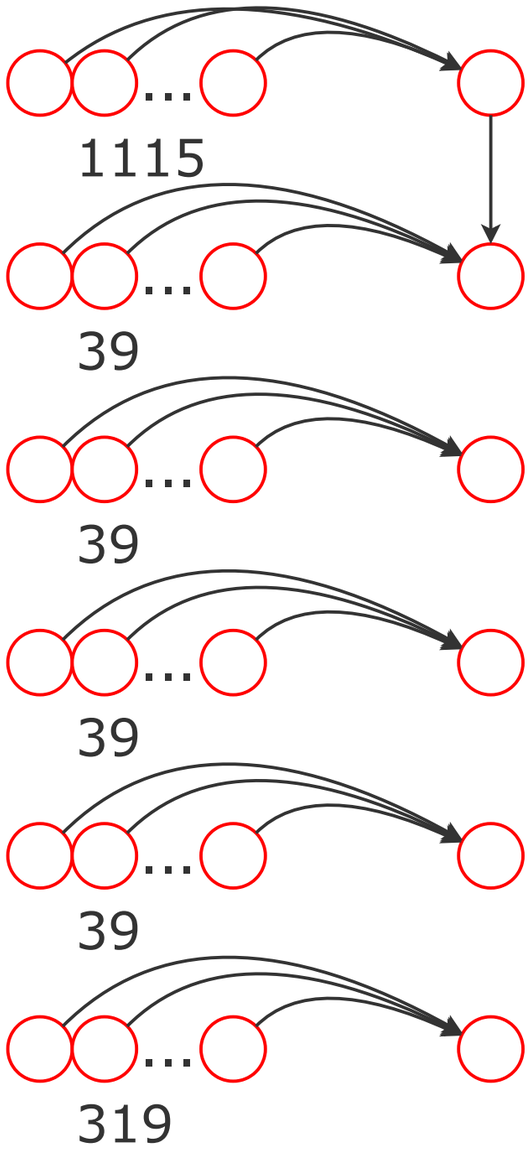

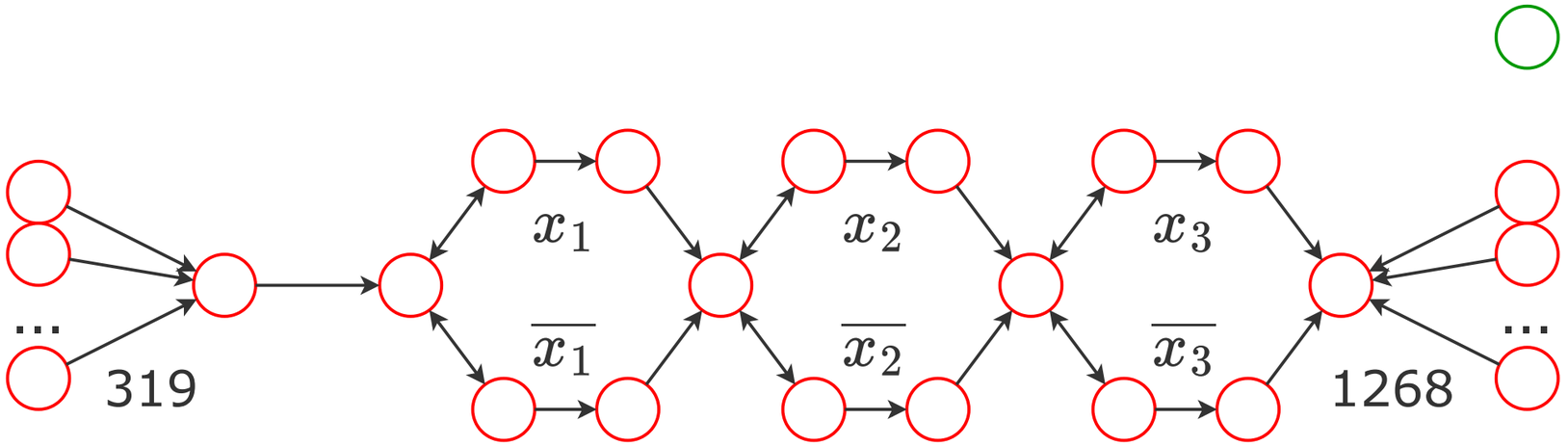

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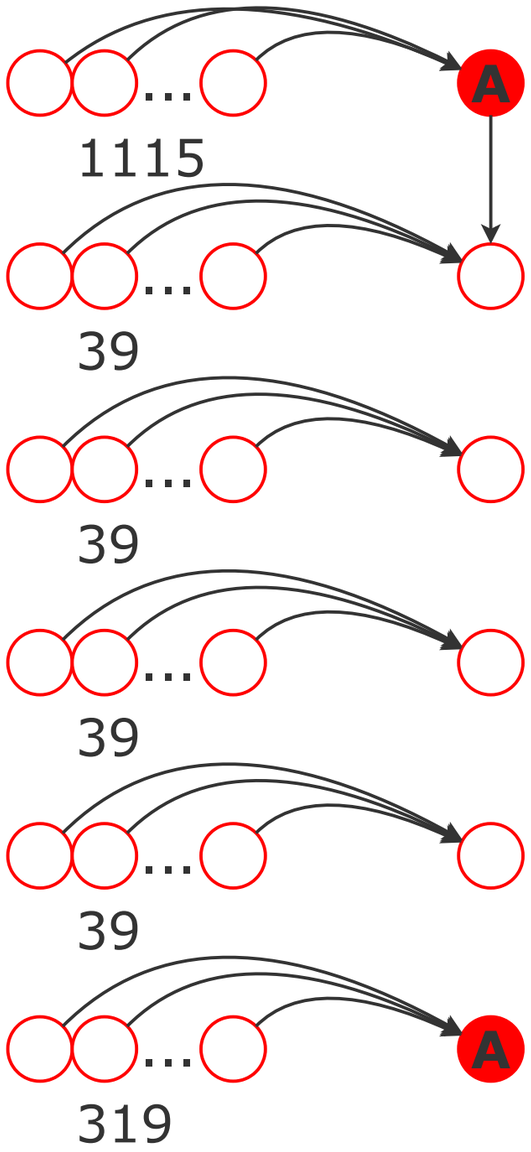

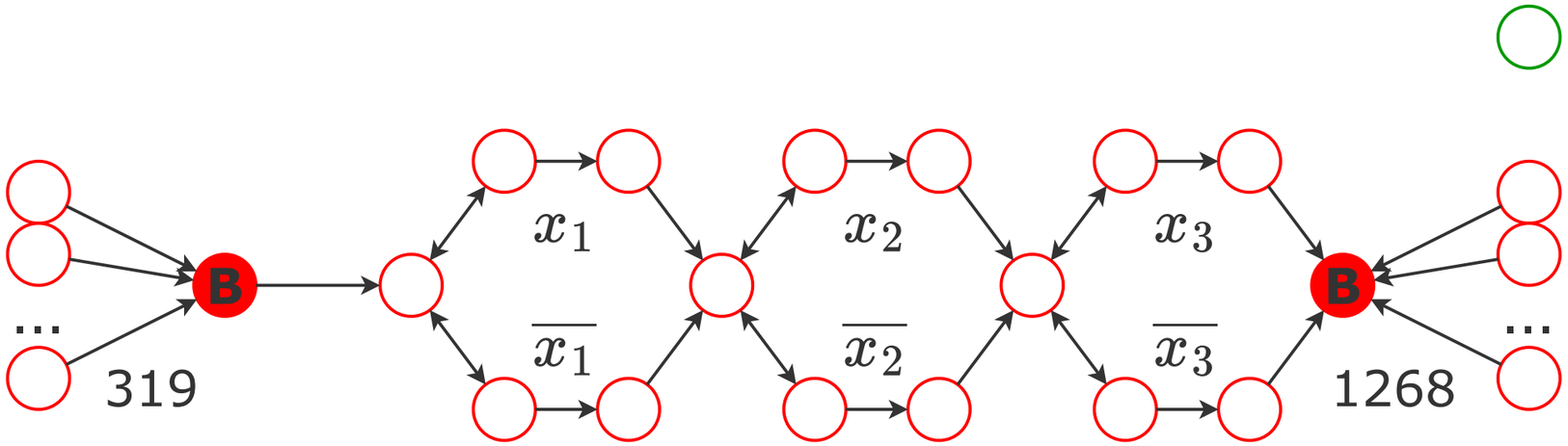

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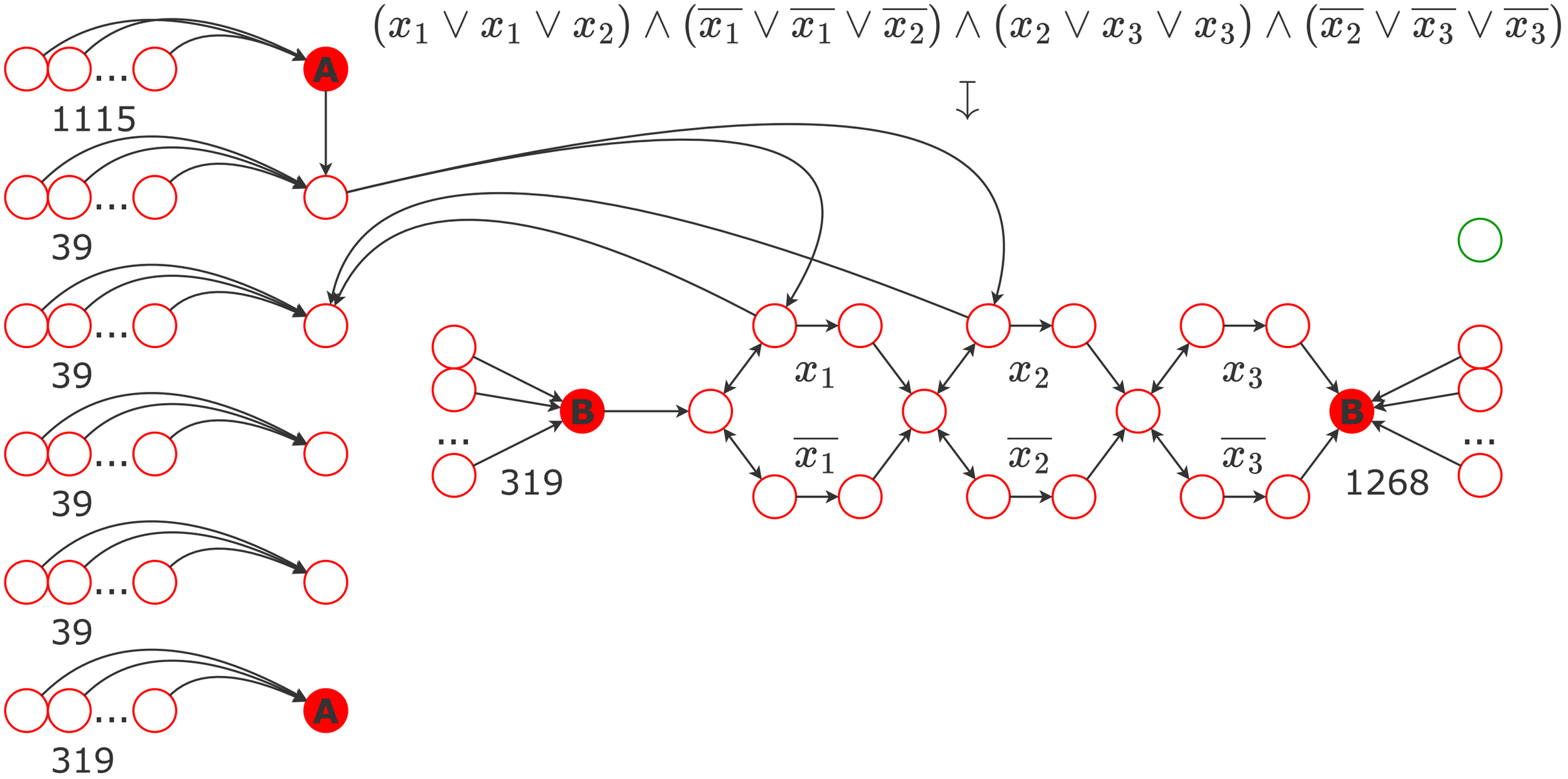

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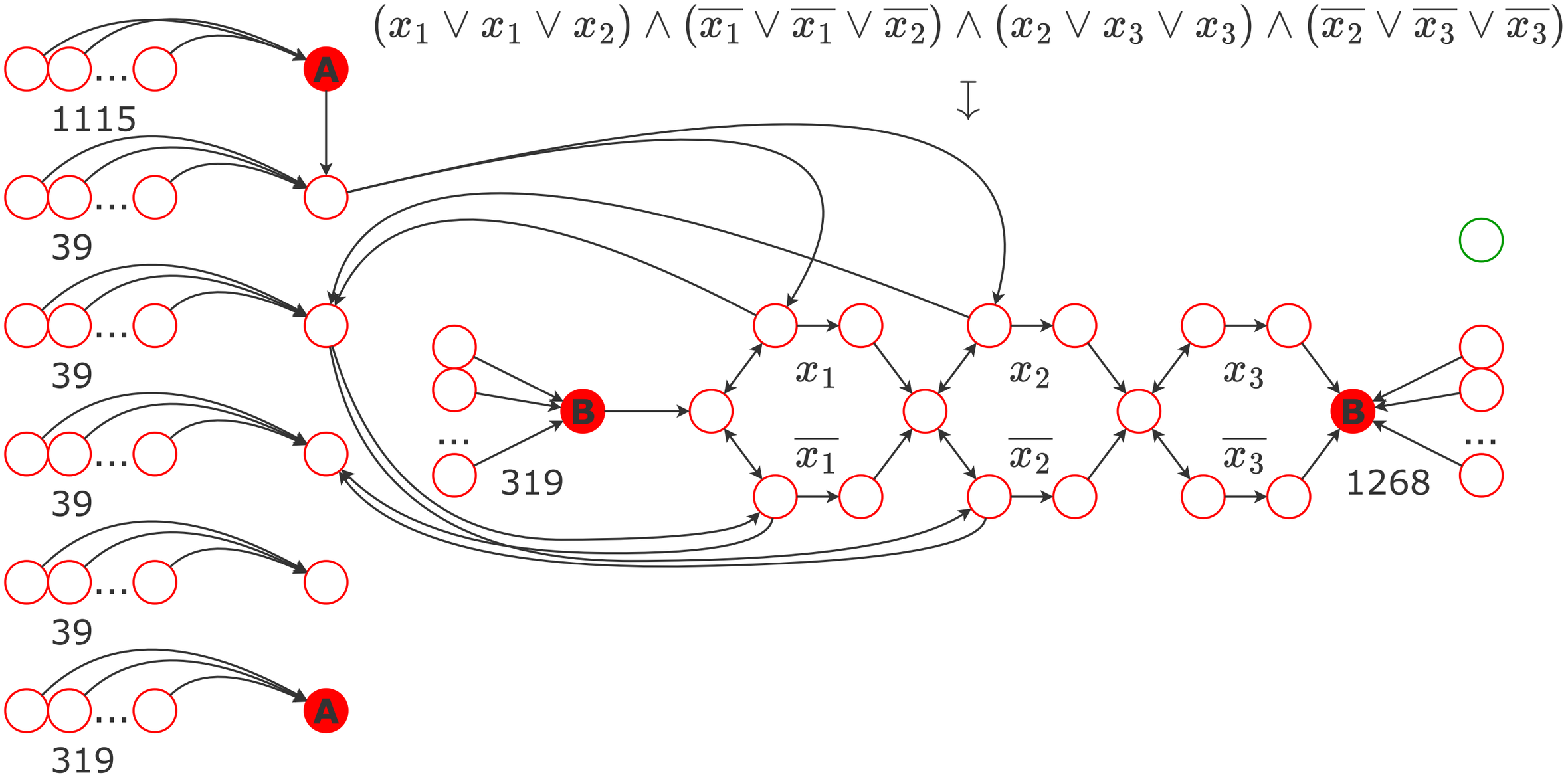

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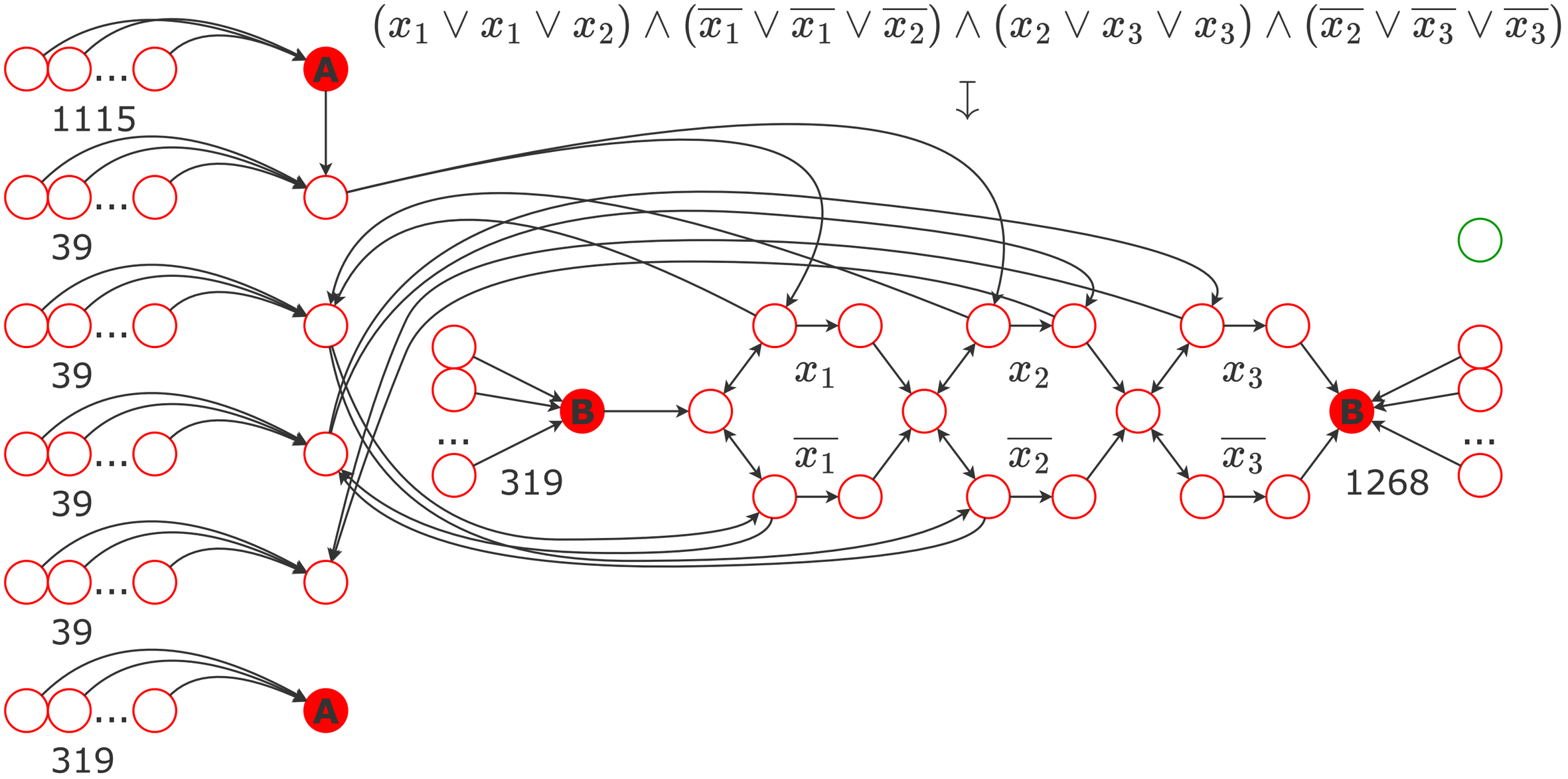

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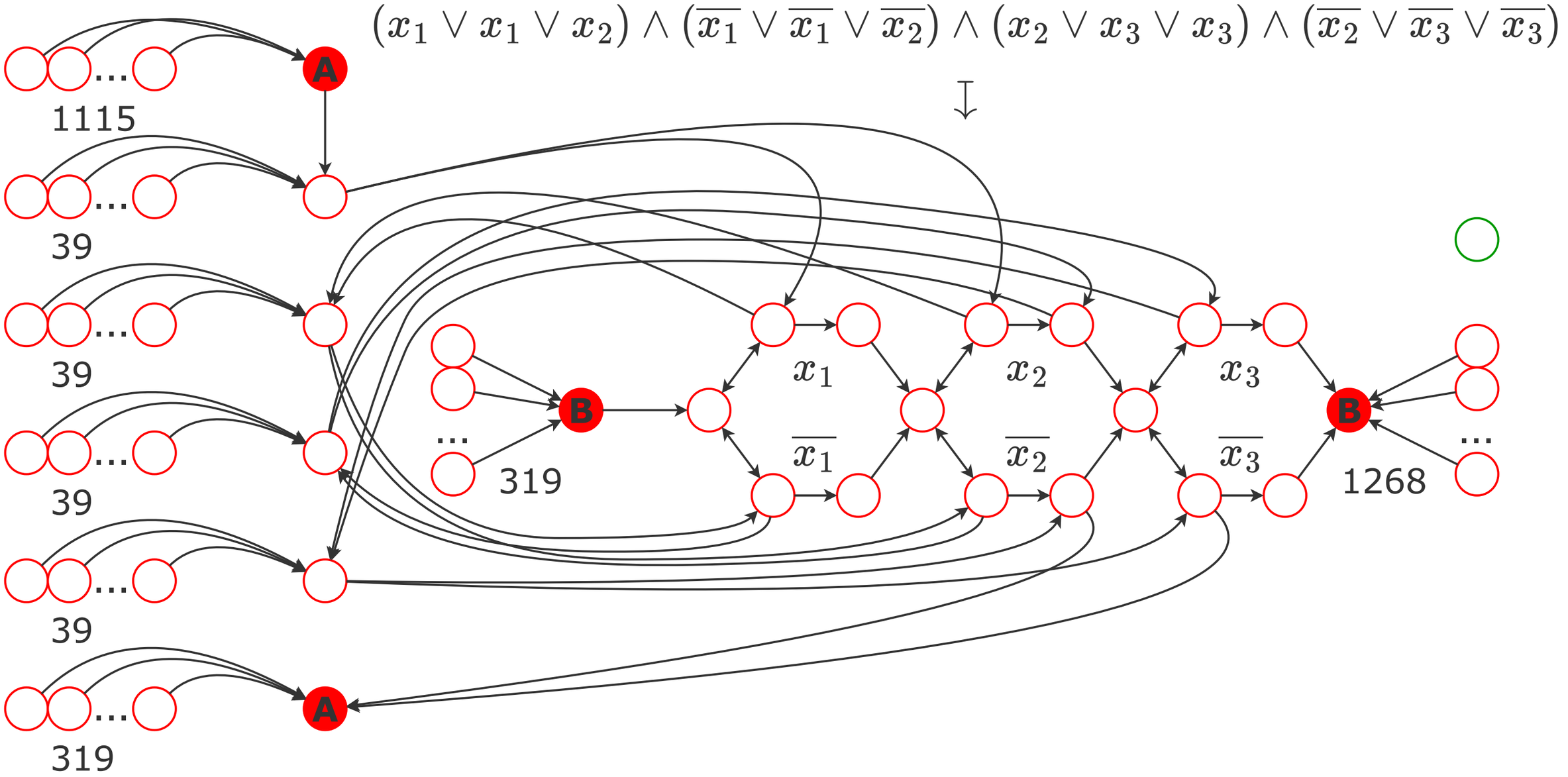

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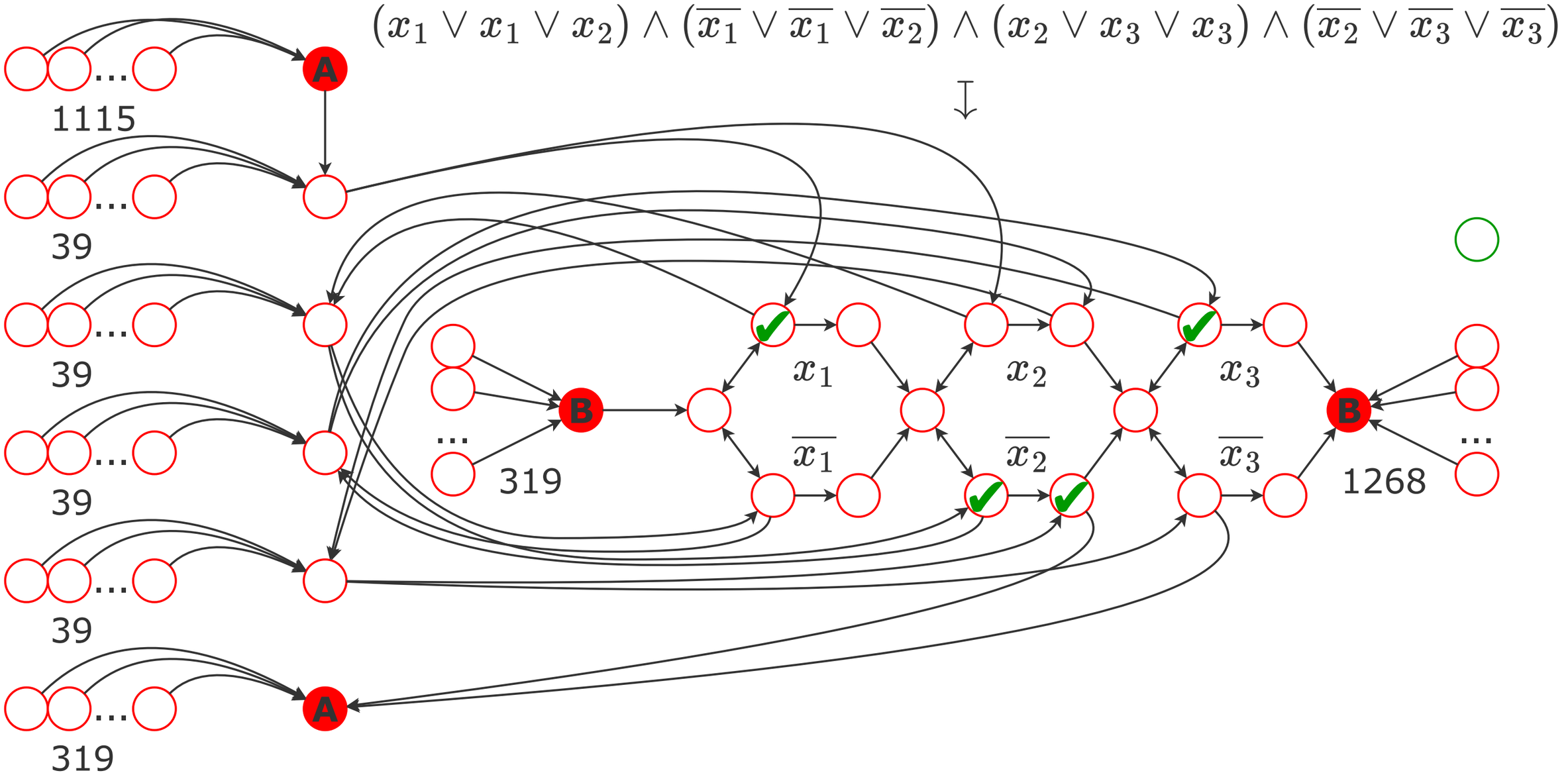

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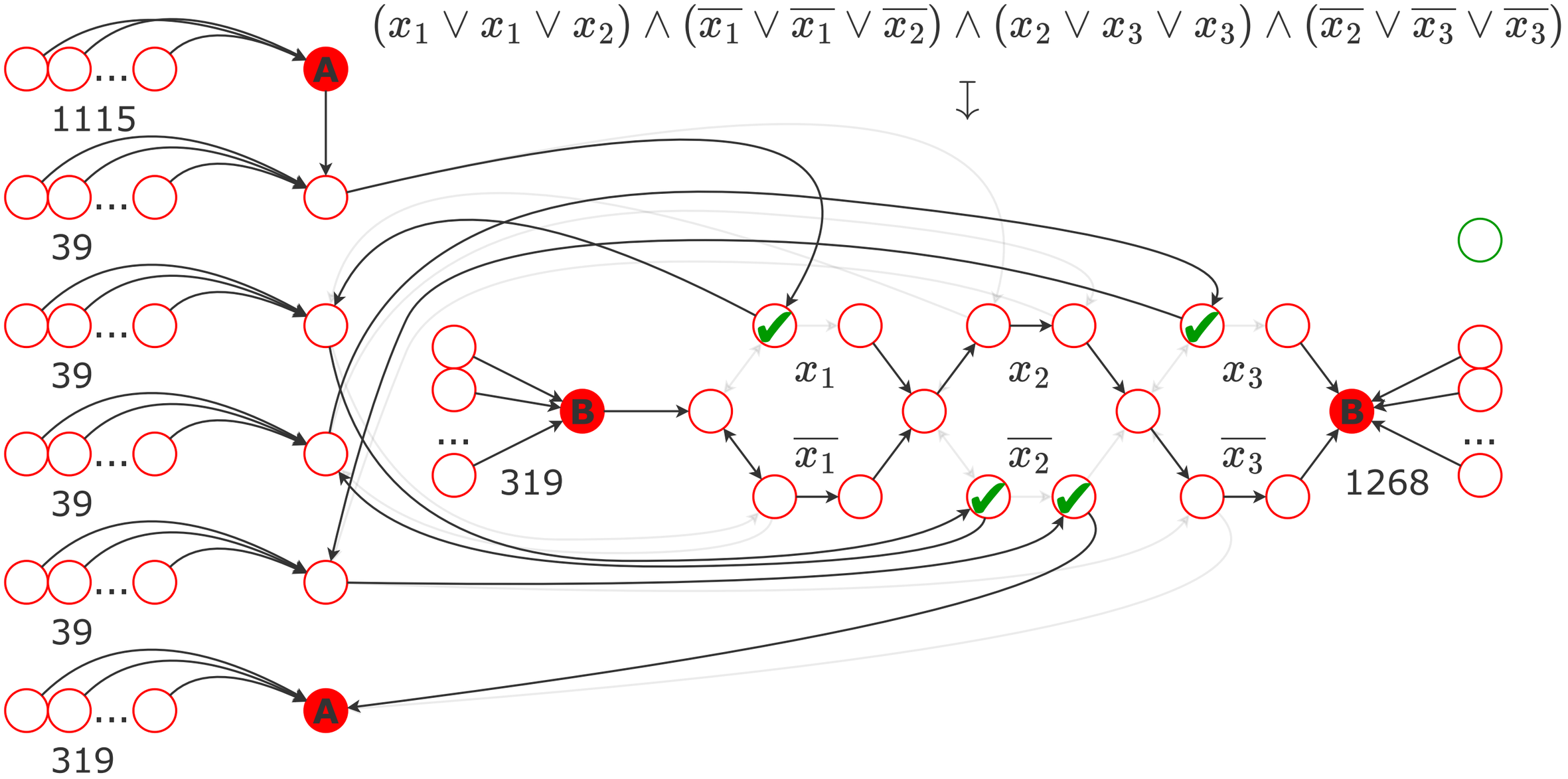

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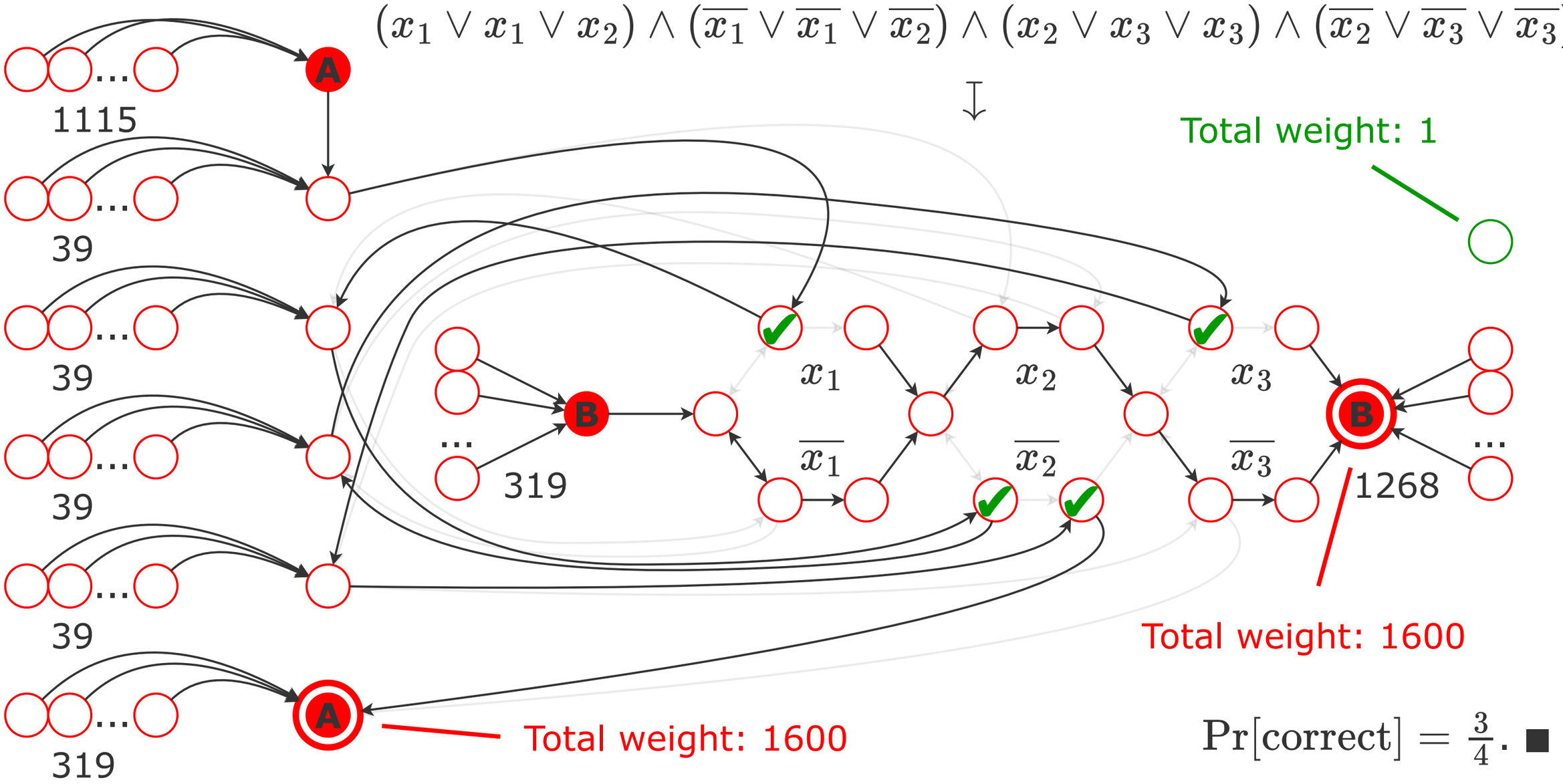

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New assumption: For a constant $\alpha \in [0, 1)$, a voter can only delegate to "approved" voters - those who are more competent by an additive alpha.

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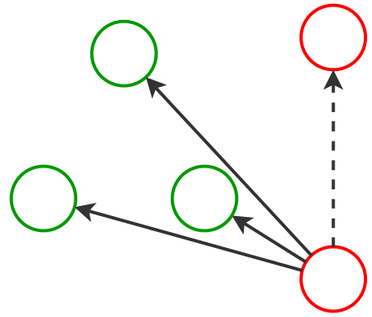

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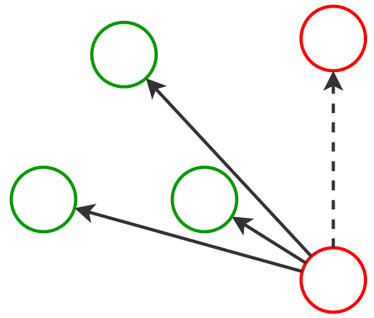

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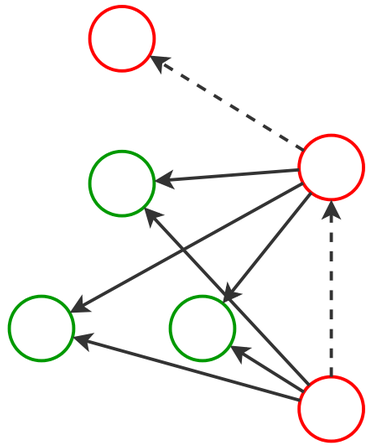

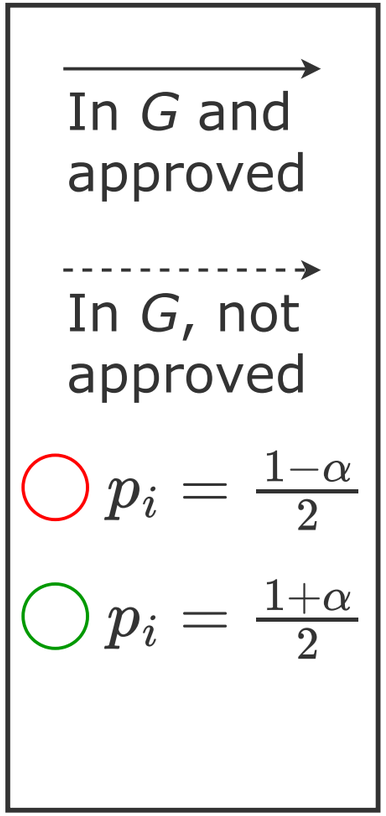

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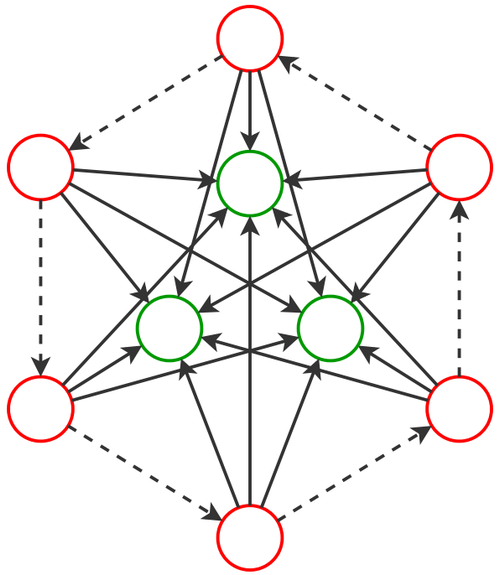

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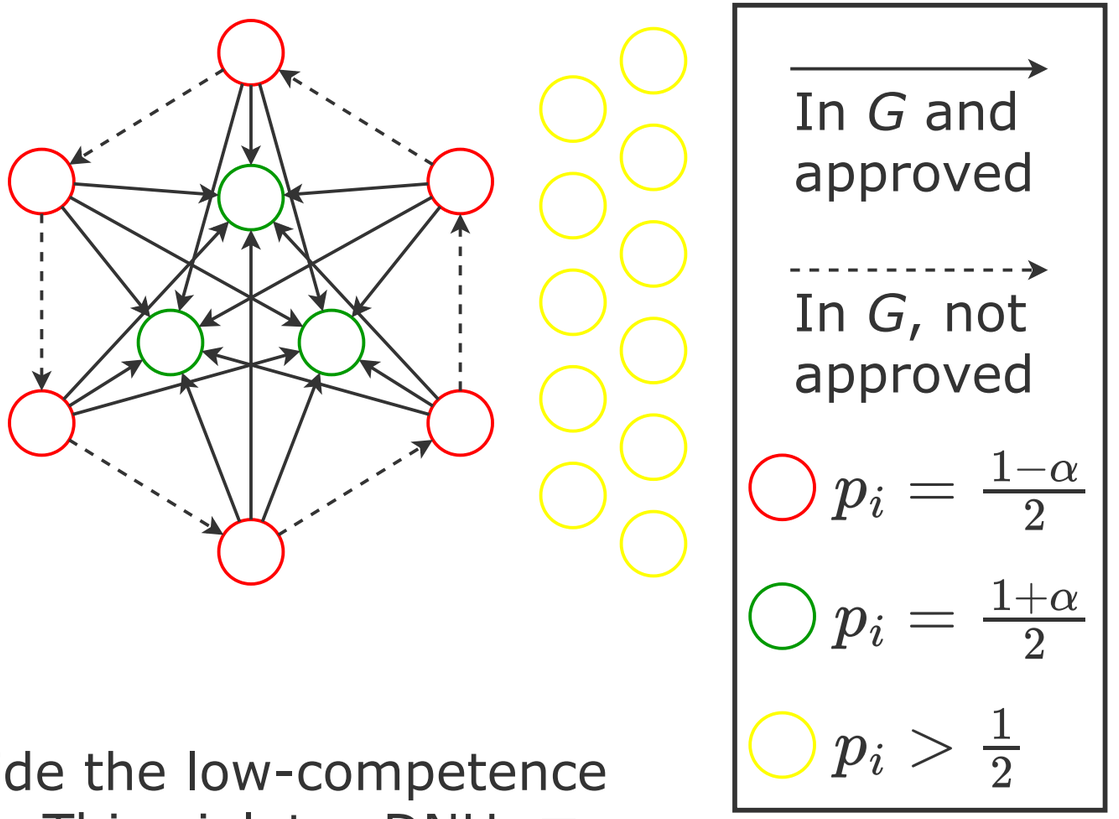

Isolated voters are competant enough to override the low-competence voters, but not if all central voters are incorrect. This violates DNH. ■