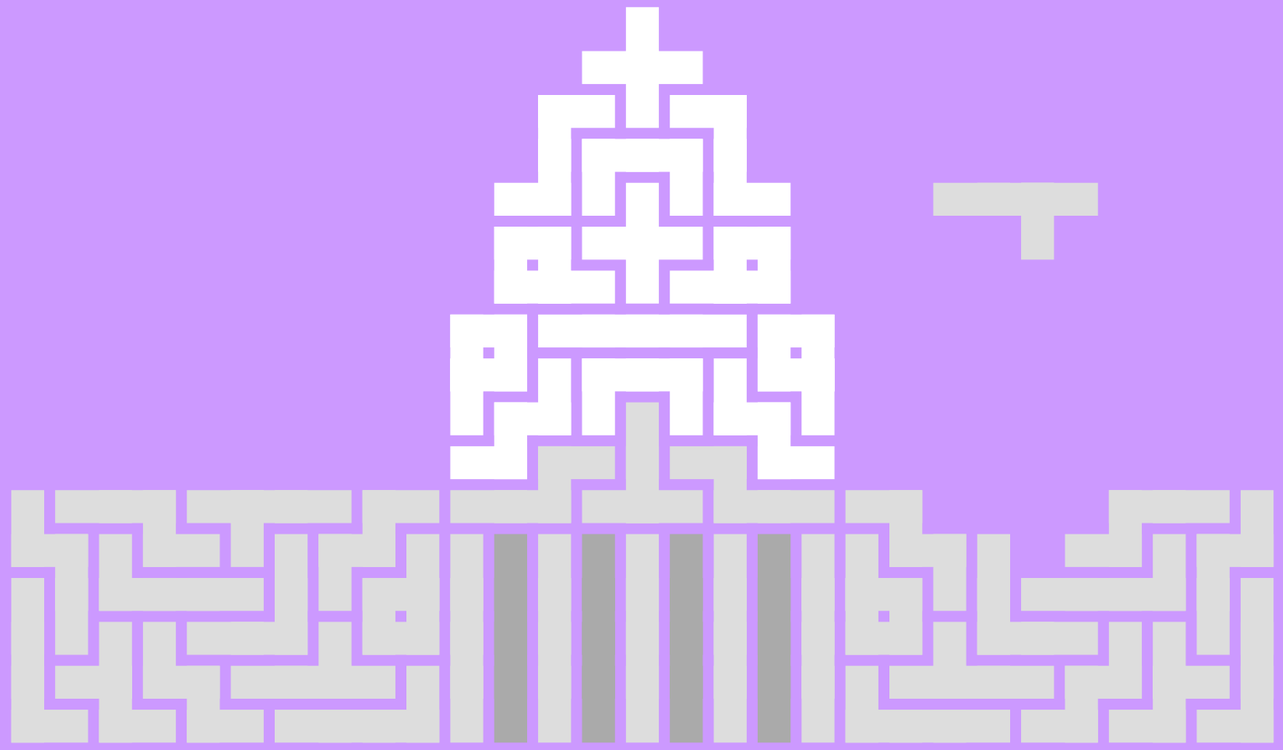


Algorithms For Democratic Decision-Making

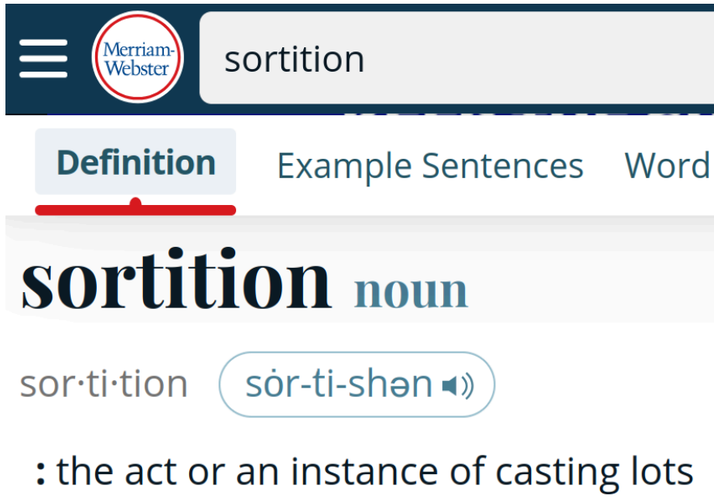
Jamie Tucker-Foltz • Yale University • Spring 2026

Lecture 17: **Citizens Assemblies**

Announcements

Next week I will be at a conference so we will meet over Zoom only at the usual link. Check Canvas for an announcement.

Sortition


Merriam-Webster
sortition
Definition
Example Sentences
Word
sortition noun
sor·ti·tion
sȯr-ti-shən
: the act or an instance of casting lots

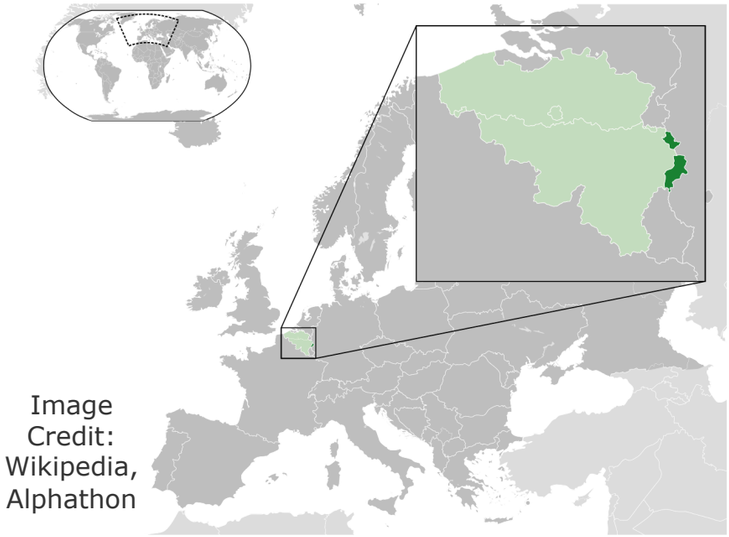
Sortition

The New York Times

Democracy Is in Trouble. This Region Is Turning to Its People.

A small corner of Belgium is recruiting ordinary citizens to help create policies. Participants say it's renewed their faith in government.

Listen · 6:30 min Share full article 74



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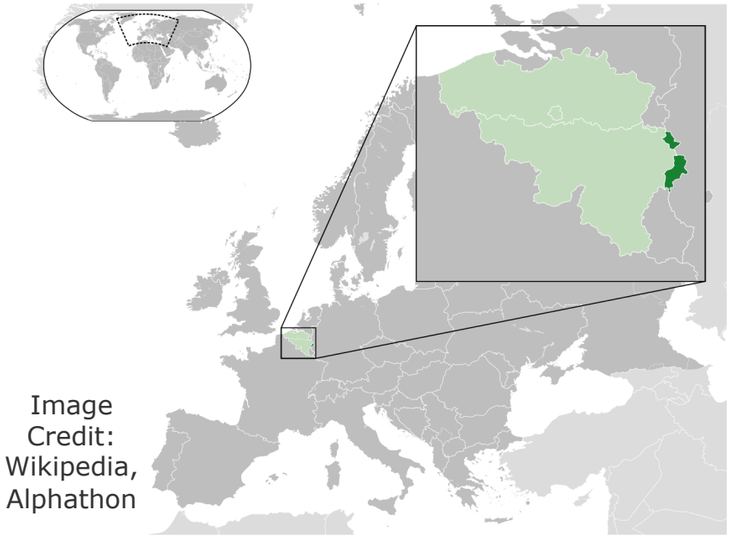

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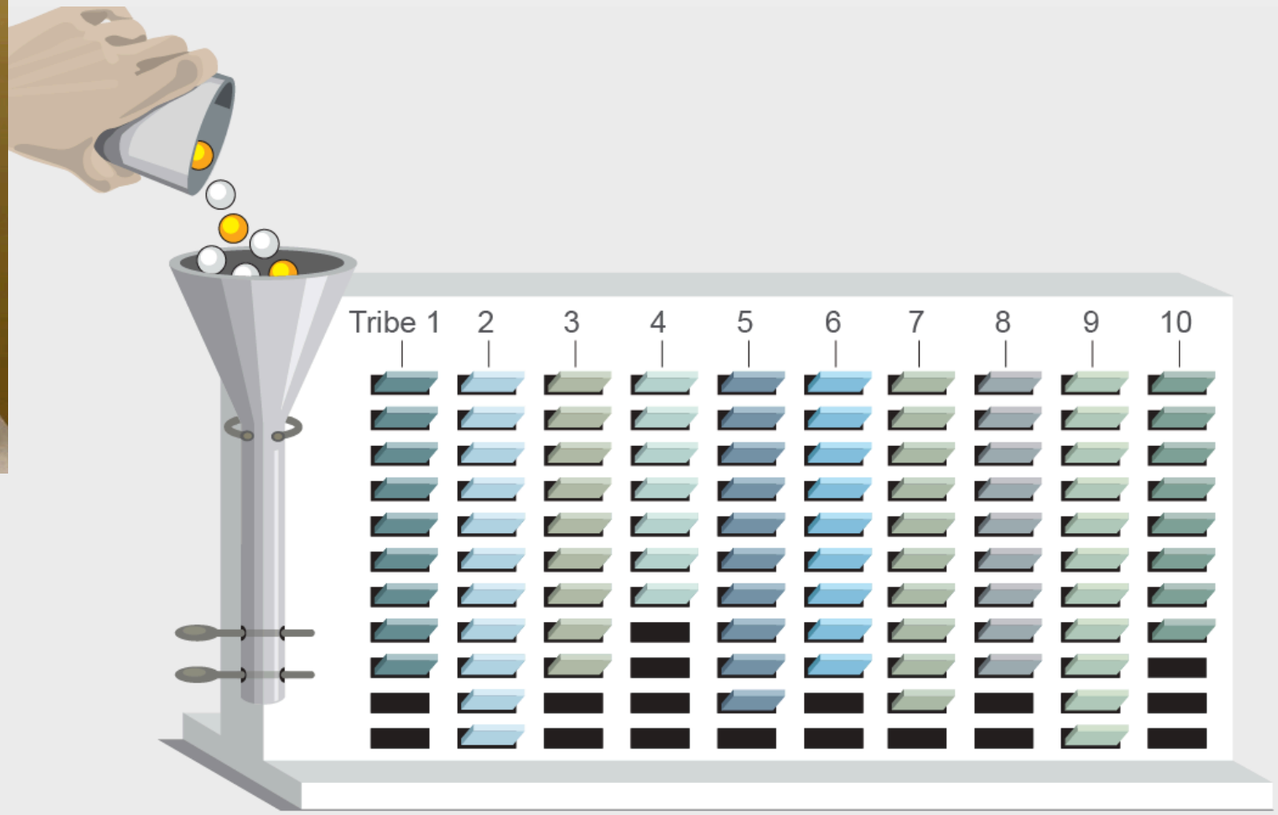
"About 1,500 letters are sent once a year to randomly selected residents in Ostbelgien. Of those who indicate interest, about 30 are chosen to join the citizens' assembly...
Each participant is paid a stipend of about 115 euros ($133) per day...
Though the assemblies' recommendations are not binding, lawmakers are required to consider them, and many have been adopted."

Democracy in Ancient Greece

Democracy in Ancient Greece

Image Credit: Jen Christiansen

Democracy in Ancient Greece



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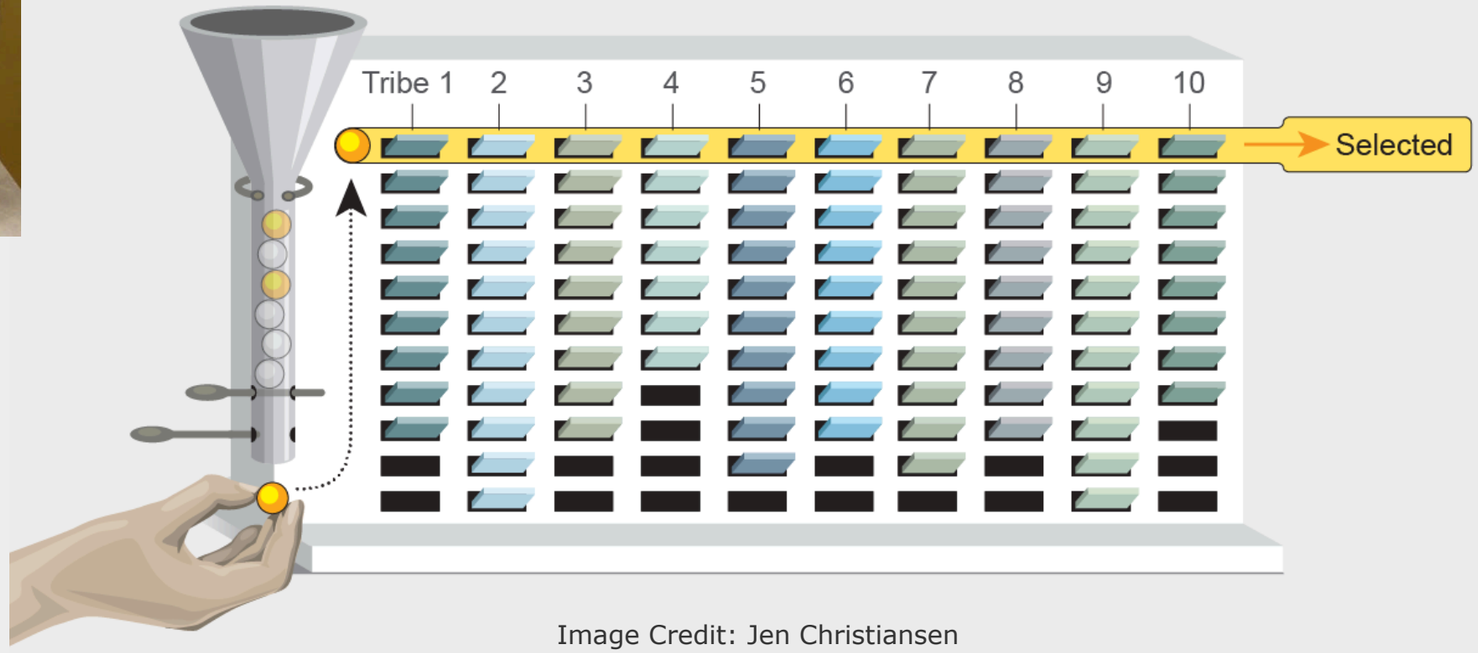

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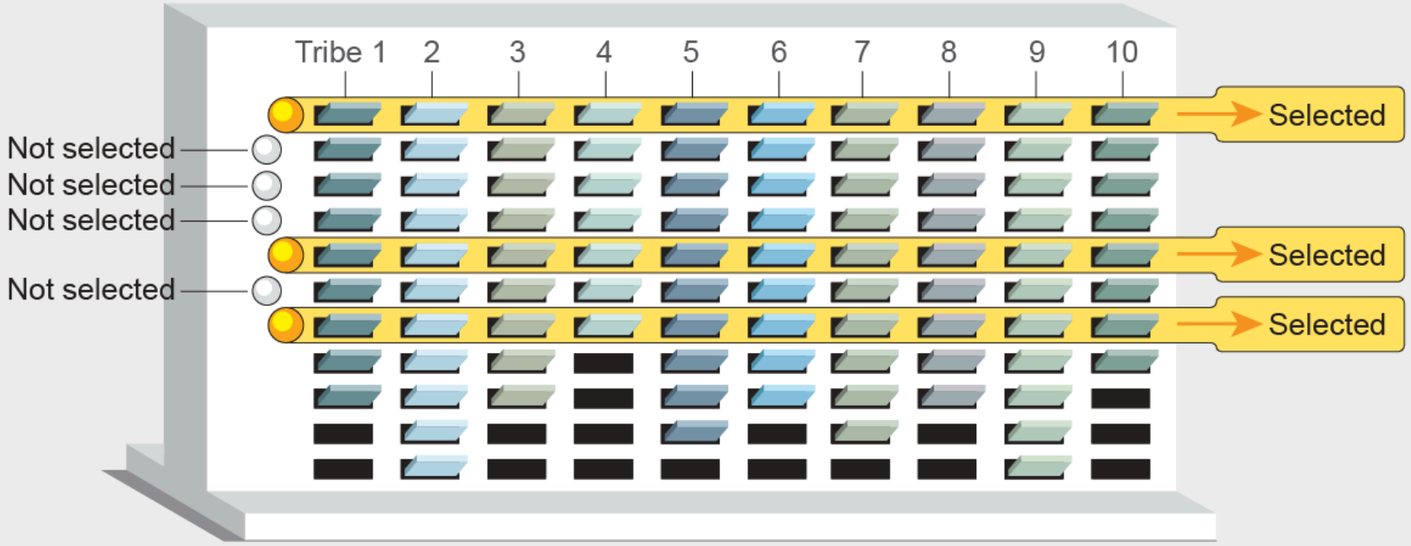

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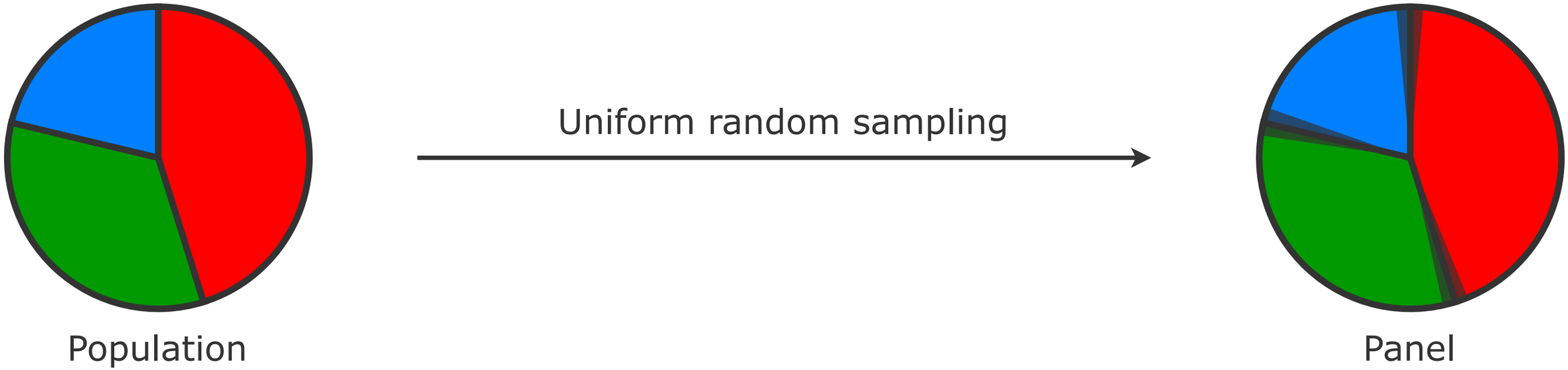

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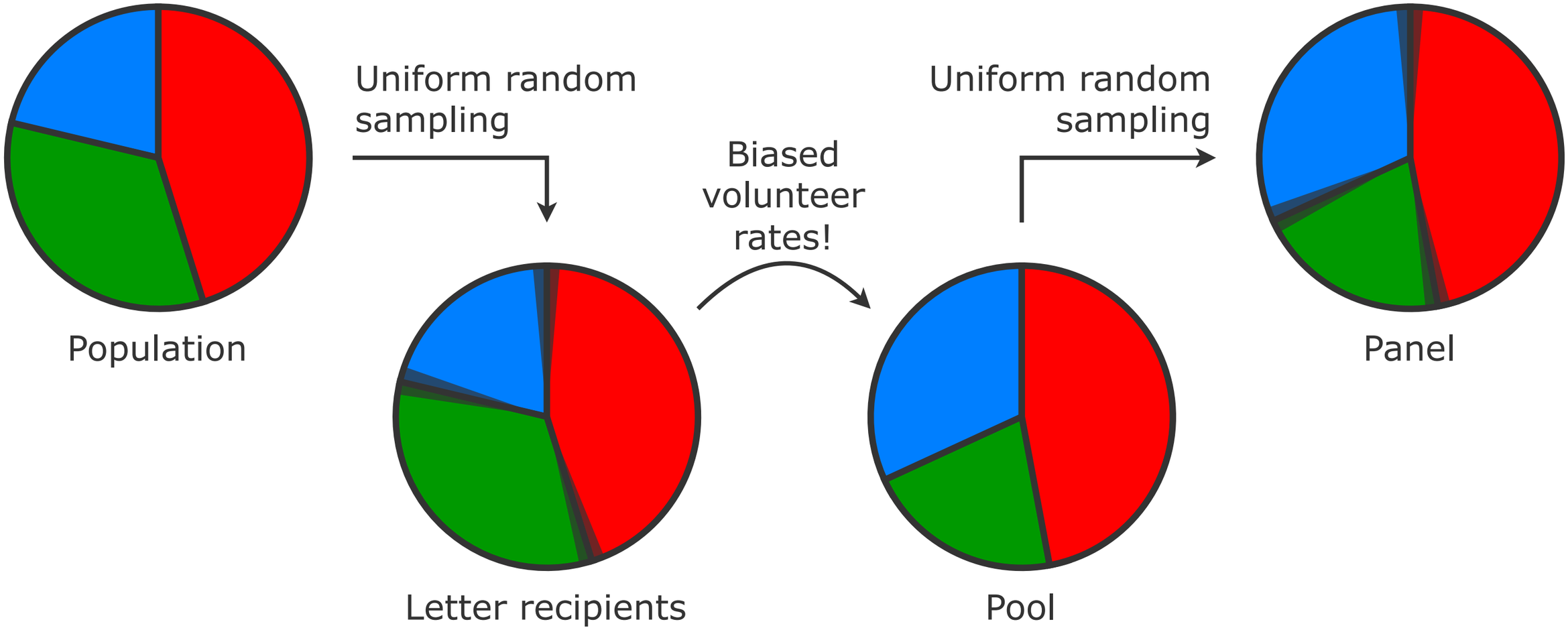

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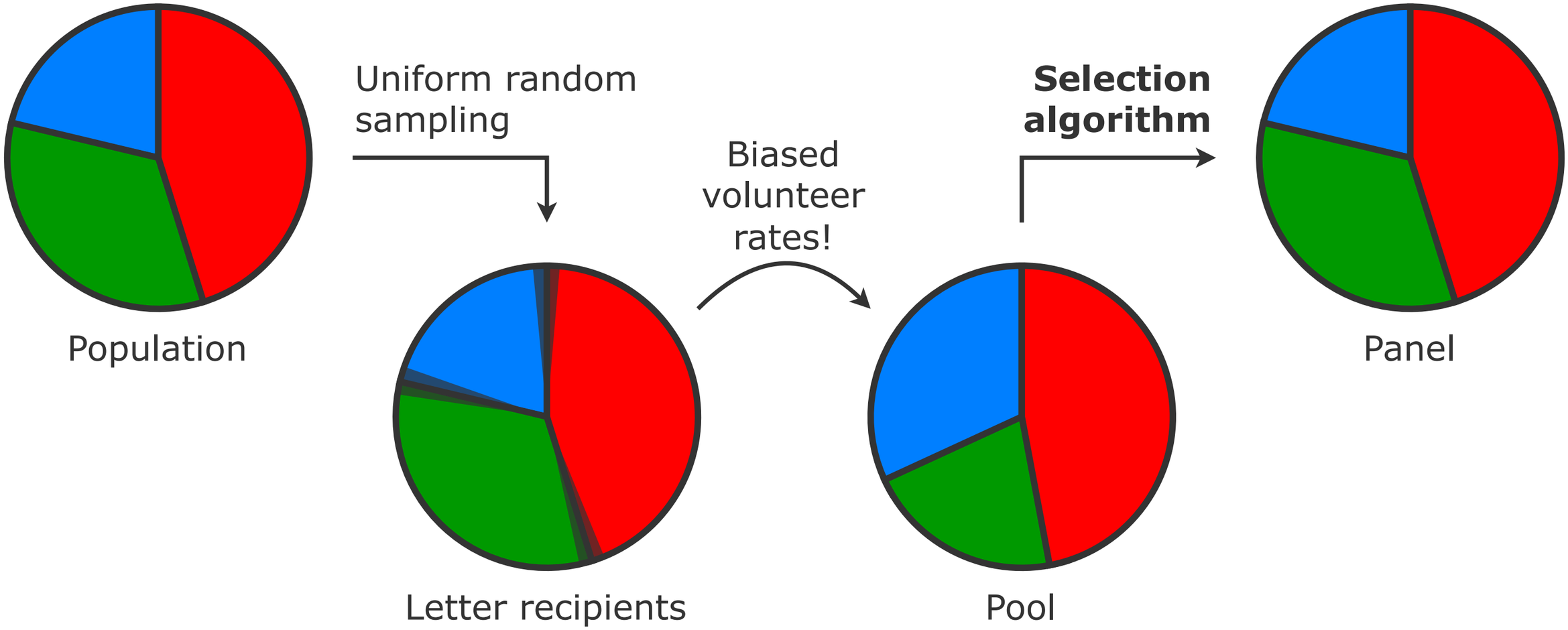
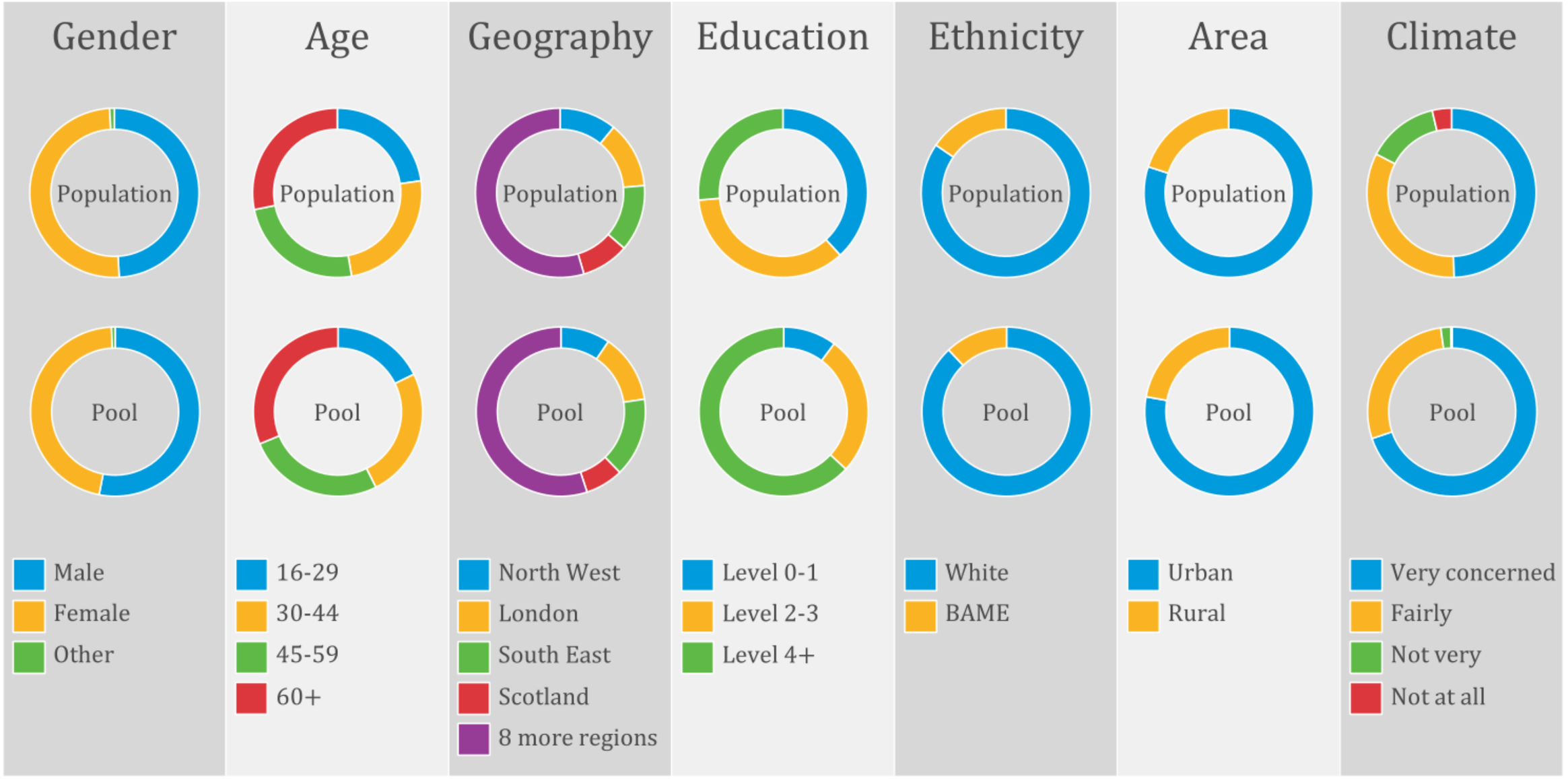


Data from the UK Climate Assembly



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- Panel size $k < n$

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Proposition

Deciding whether there is any quota-compliant panel is NP-complete.

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Deciding whether there is any quota-compliant panel is NP-complete.

Note: In practice, this problem is actually easy to solve!

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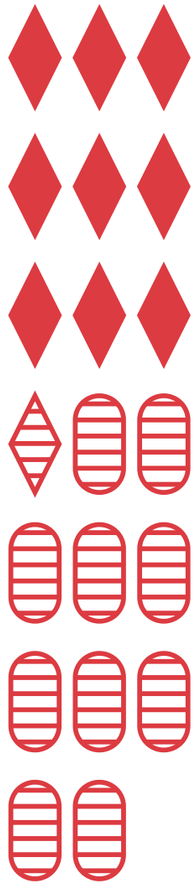
f:	◇	▯	≡	●
ℓ_f:	4	4	4	4

$n = 20,\ k = 10$

A P



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Priority:	$\frac{4}{10}$			

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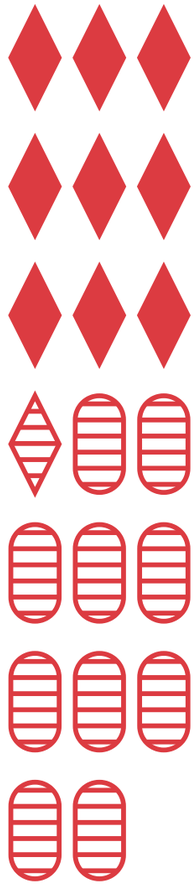

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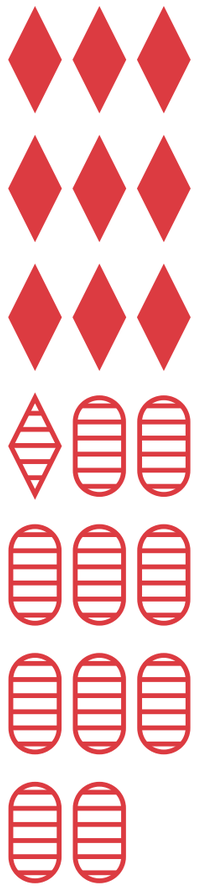

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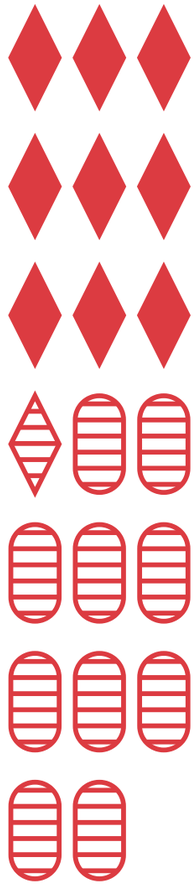

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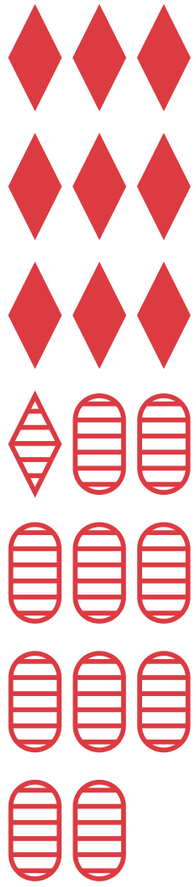

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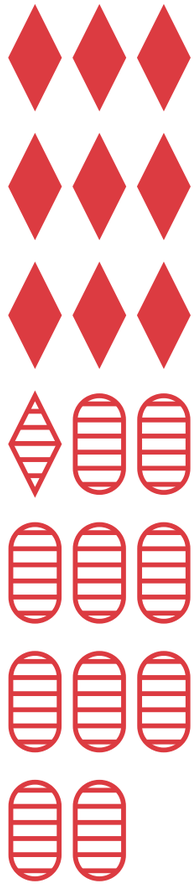

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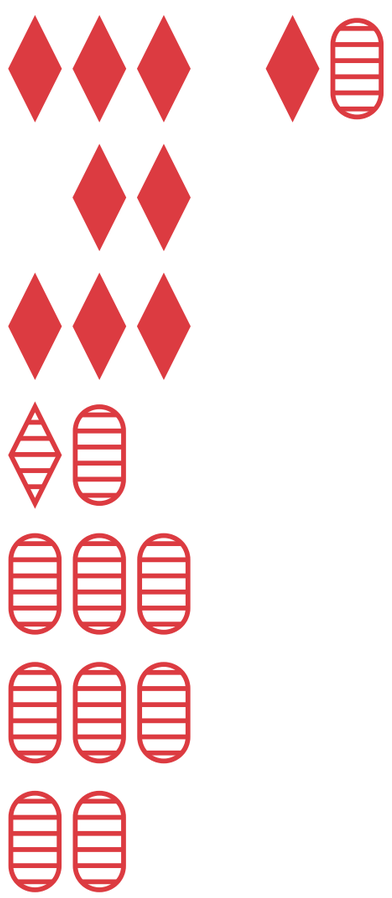

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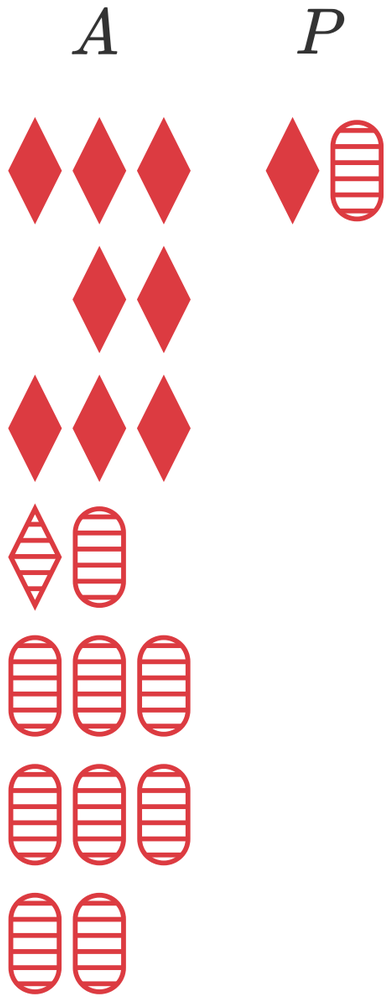

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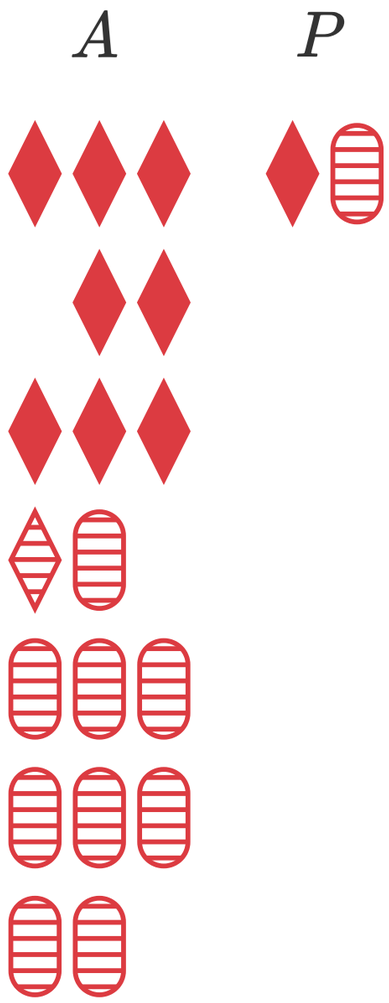
f:	◊	▯	≡	●
ℓ_f:	4	4	4	4
Priority:	$\frac{3}{9}$	$\frac{3}{9}$	$\frac{3}{10}$	$\frac{3}{8}$

$n = 20,\ k = 10$



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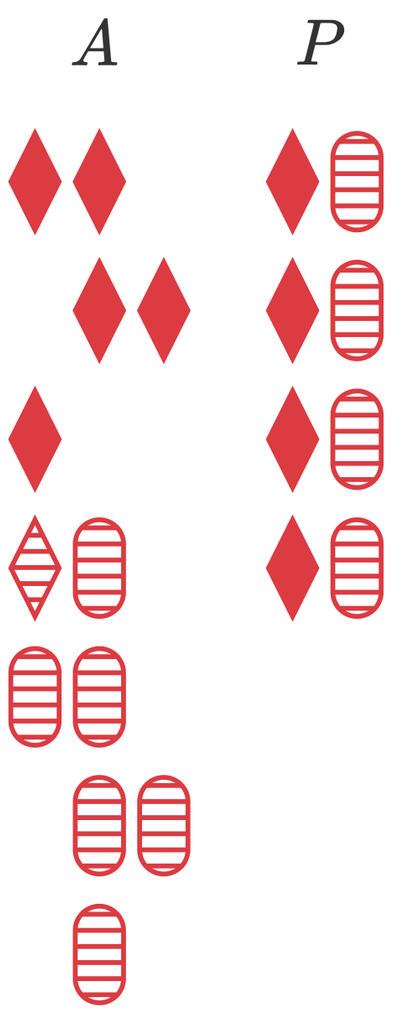

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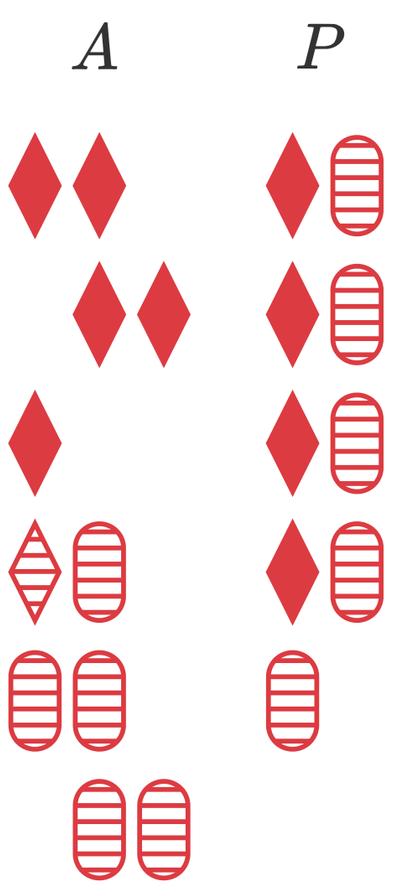

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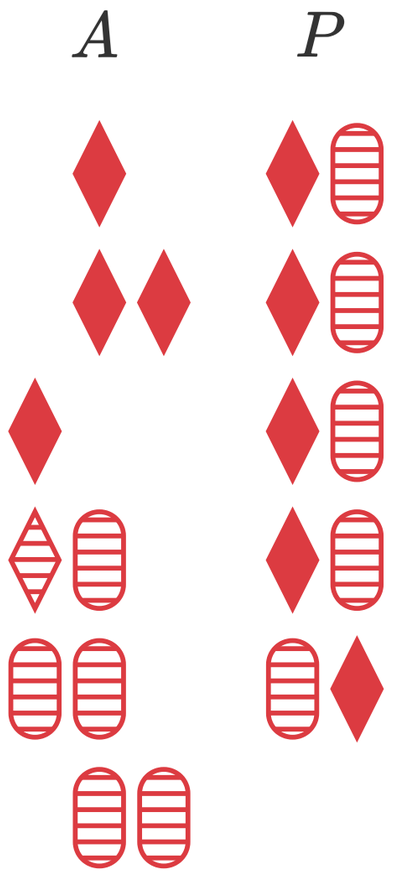

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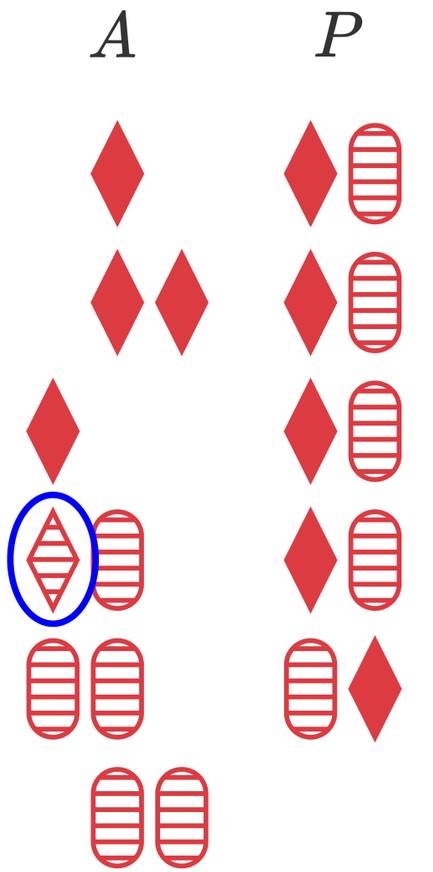

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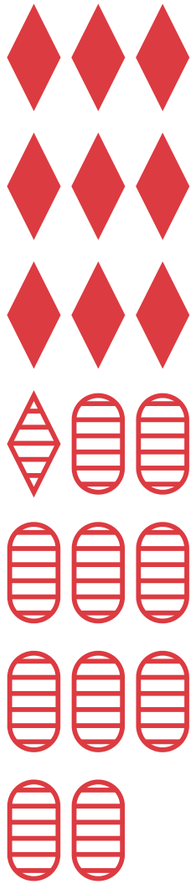

▸ What is the probability the striped diamond is selected under leximin?

Respond at:

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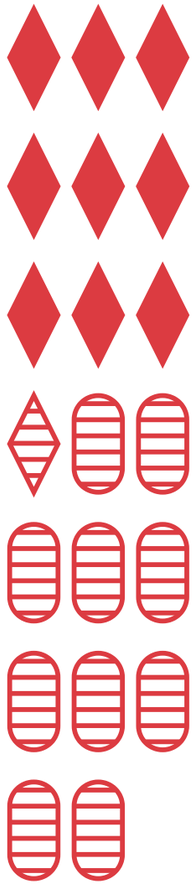

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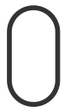
f:				
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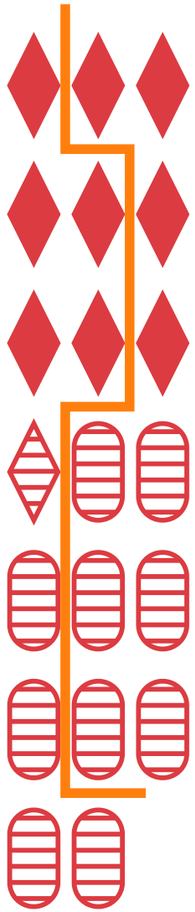

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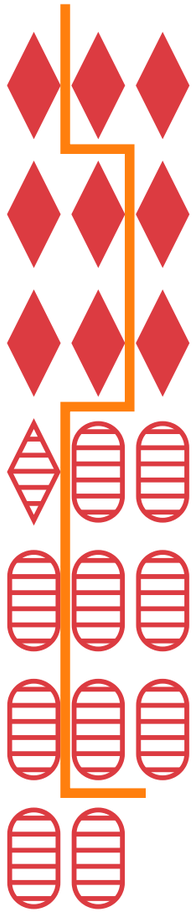

Panelot

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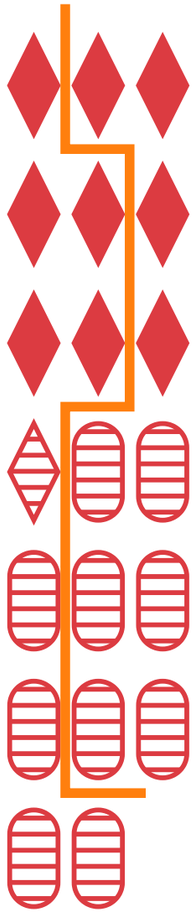

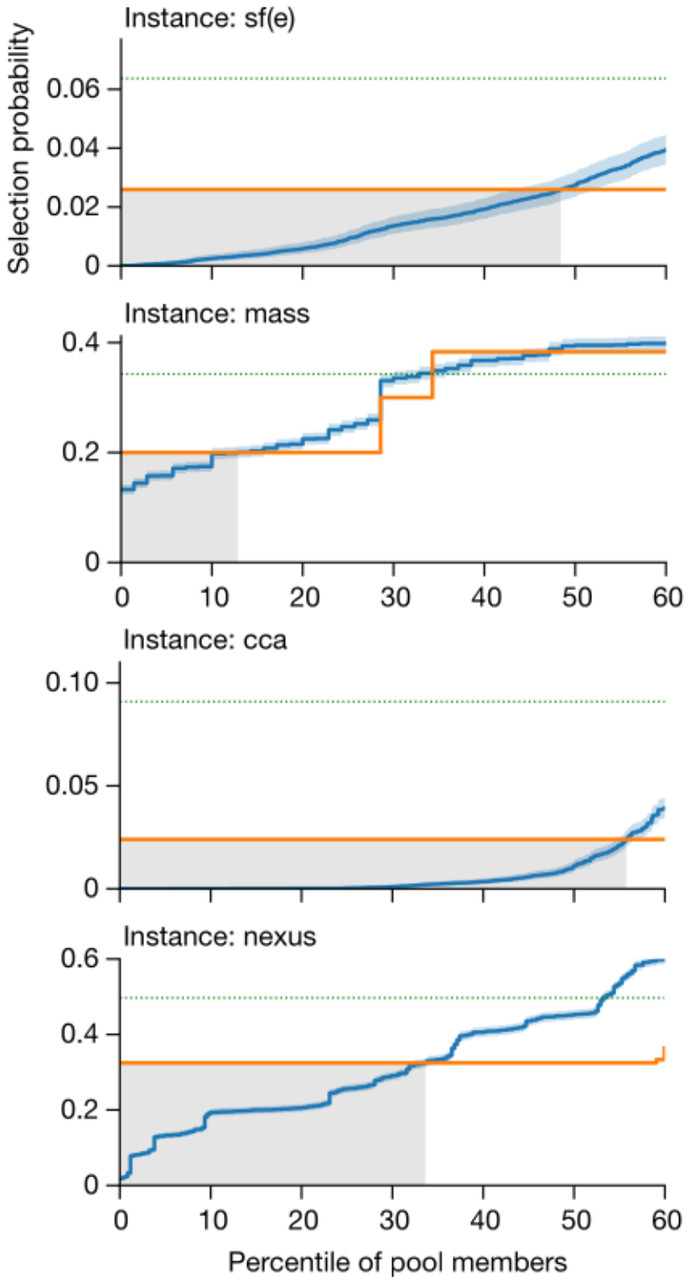

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A ◊ can pretend to be a ⬭, increasing its probability from 1/90 to 9/11. ■

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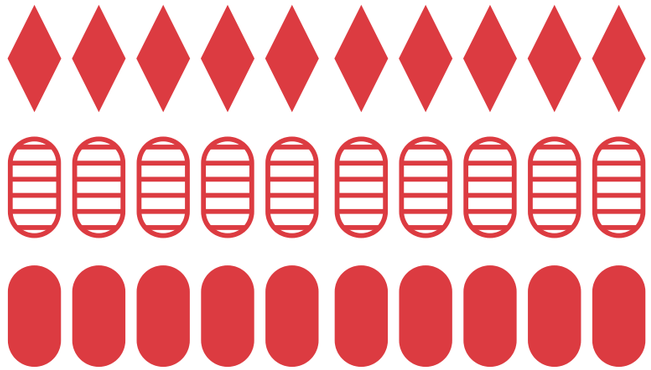

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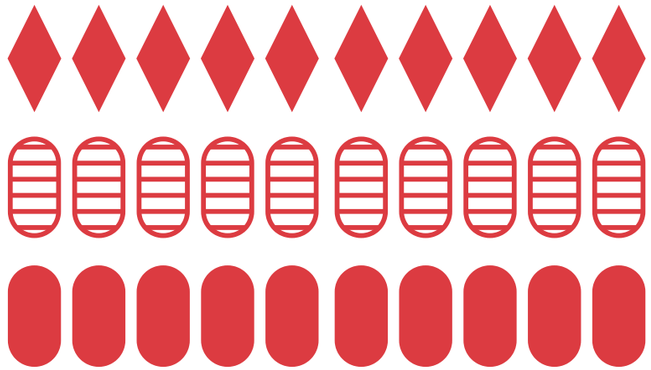

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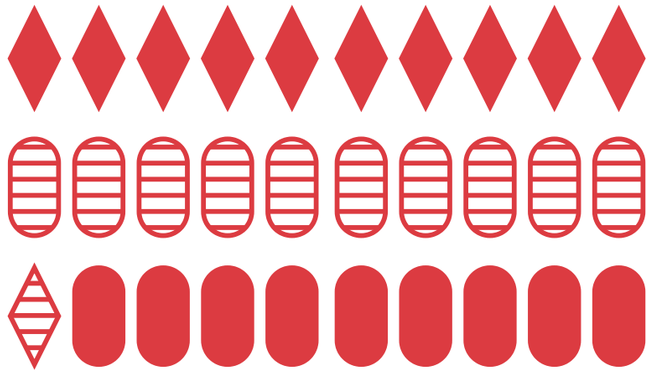

Always ~~Never~~ selected!

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Then they could have misreported for a gain of k/n if their true type was  . ■

Algorithm 4: Goldilocks (Baharav, Flanigan, 2024)

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