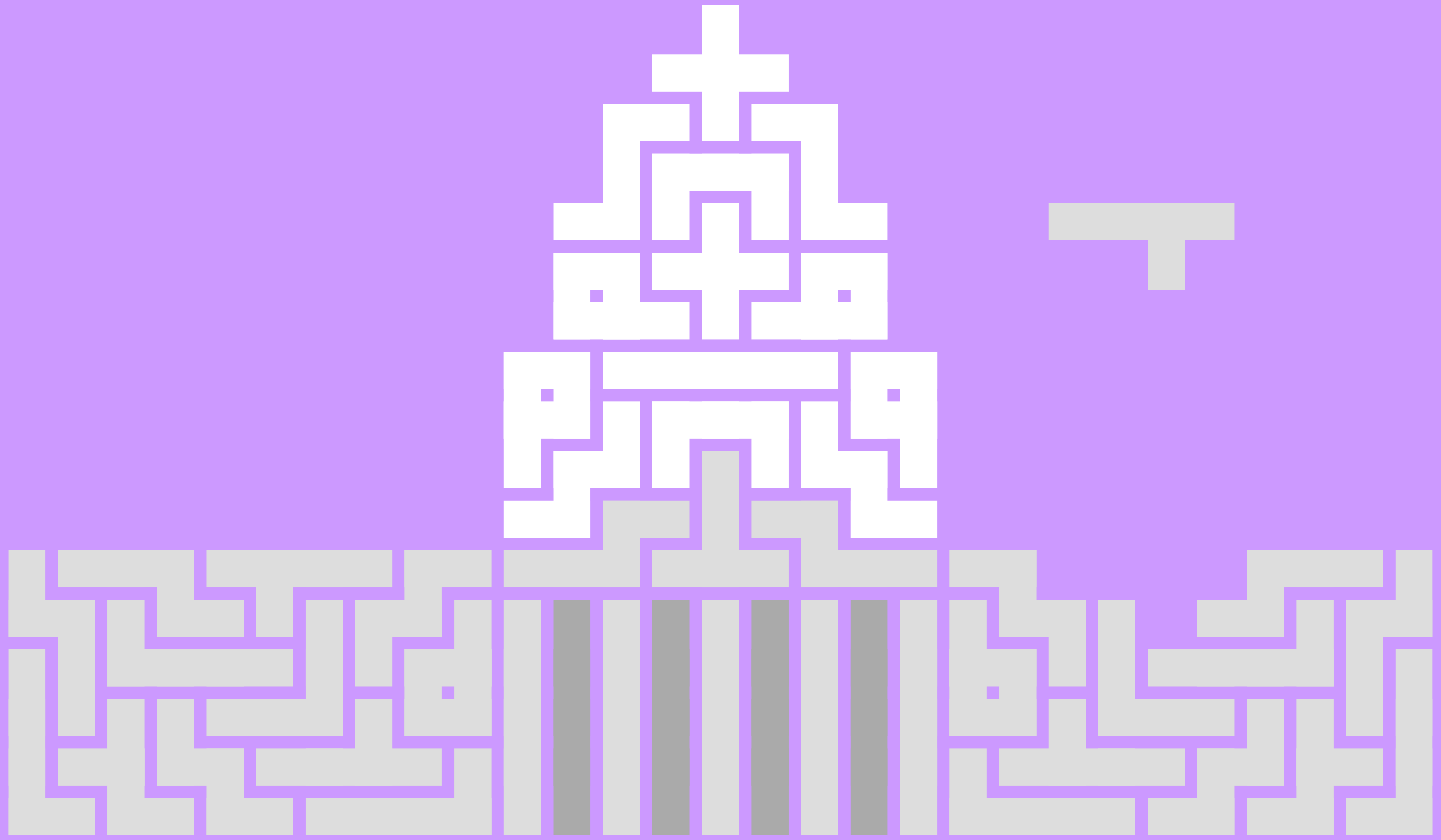

Algorithms For Democratic Decision-Making

Jamie Tucker-Foltz • Yale University • Spring 2026

Lecture 21: **Redistricting as a Fair Division Problem**

Announcements

Project presentations start next Wednesday! Fill out poll linked from Canvas announcement by **tomorrow night** if you have preferences over the date/order.

Redistricting

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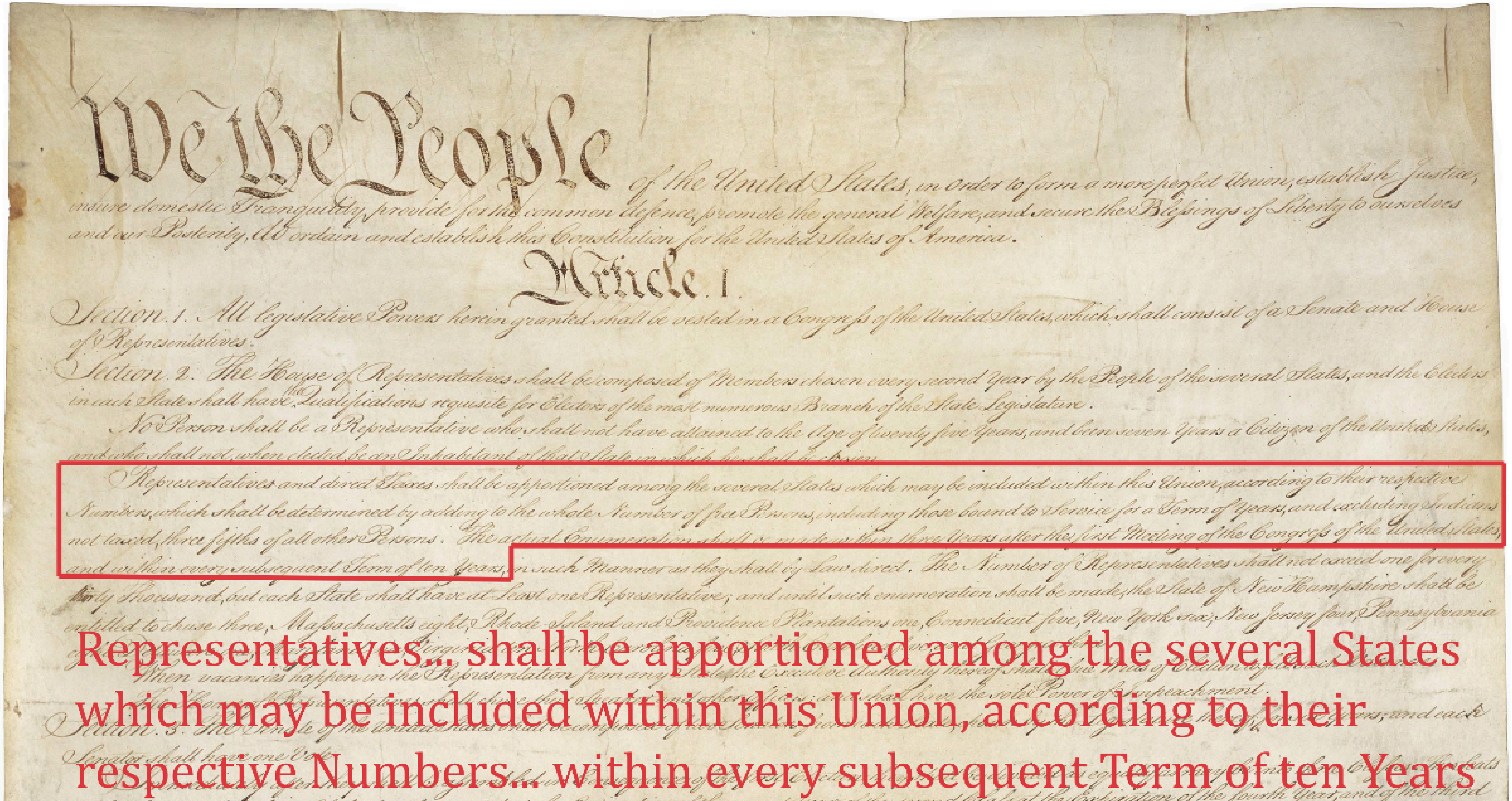
We the People of the United States, in Order to form a more perfect Union, establish Justice, insure domestic Tranquility, provide for the common defence, promote the general Welfare, and secure the Blessings of Liberty to ourselves and our Posterity, do ordain and establish this Constitution for the United States of America.

Article. I

Section. 1. All legislative Powers herein granted shall be vested in a Congress of the United States, which shall consist of a Senate and House of Representatives.

Section. 2. The House of Representatives shall be composed of Members chosen every second Year by the People of the several States, and the Electors in each State shall have the Qualifications requisite for Electors of the most numerous Branch of the State Legislature.

No Person shall be a Representative who shall not have attained to the Age of twenty five Years, and been seven Years a Citizen of the United States, and who shall not, when elected, be an Inhabitant of that State in which he shall be chosen.

Representatives and direct Taxes shall be apportioned among the several States which may be included within this Union, according to their respective Numbers, which shall be determined by adding to the whole Number of free Persons, including those bound to Service for a Term of Years, and excluding Indians not taxed, three fifths of all other Persons. The actual Enumeration shall be made within three Years after the first Meeting of the Congress of the United States, and within every subsequent Term of ten Years, in such Manner as they shall by Law direct. The Number of Representatives shall not exceed one for every thirty Thousand, but each State shall have at Least one Representative; and until such enumeration shall be made, the State of New Hampshire shall be entitled to chuse three, Massachusetts eight, Rhode Island and Providence Plantations one, Connecticut five, New York six, New Jersey four, Pennsylvania

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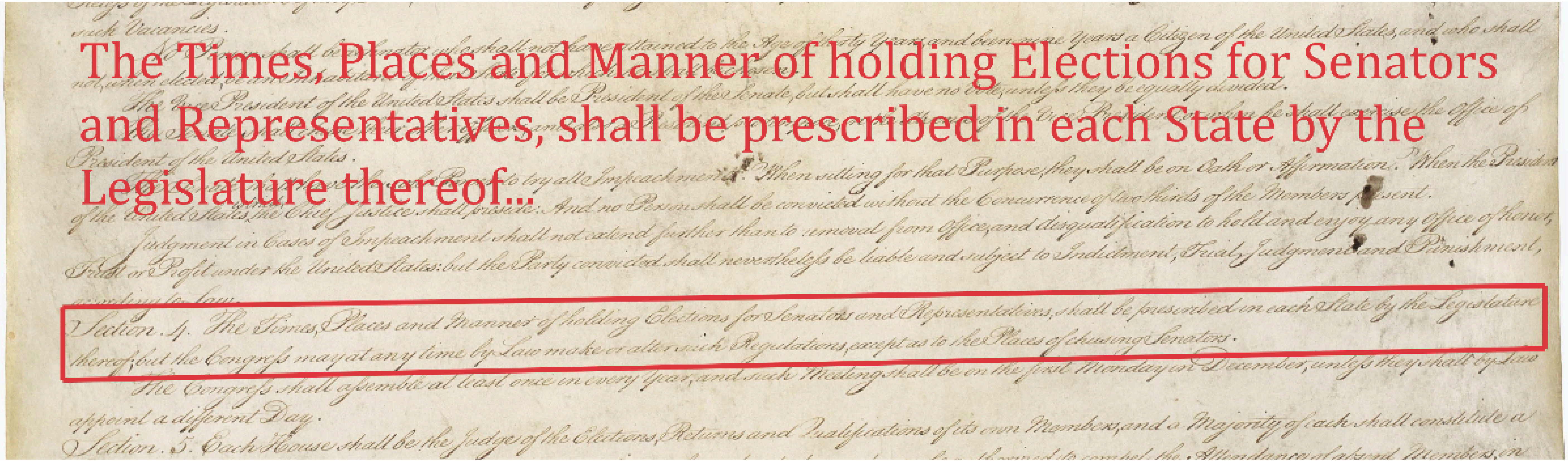
Redistricting



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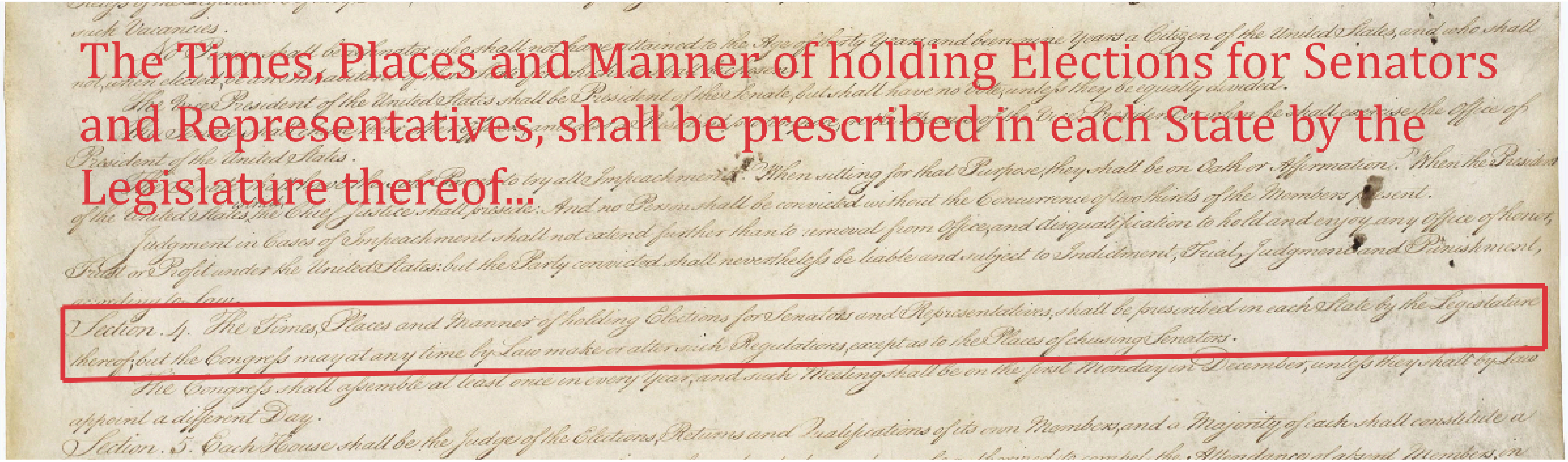

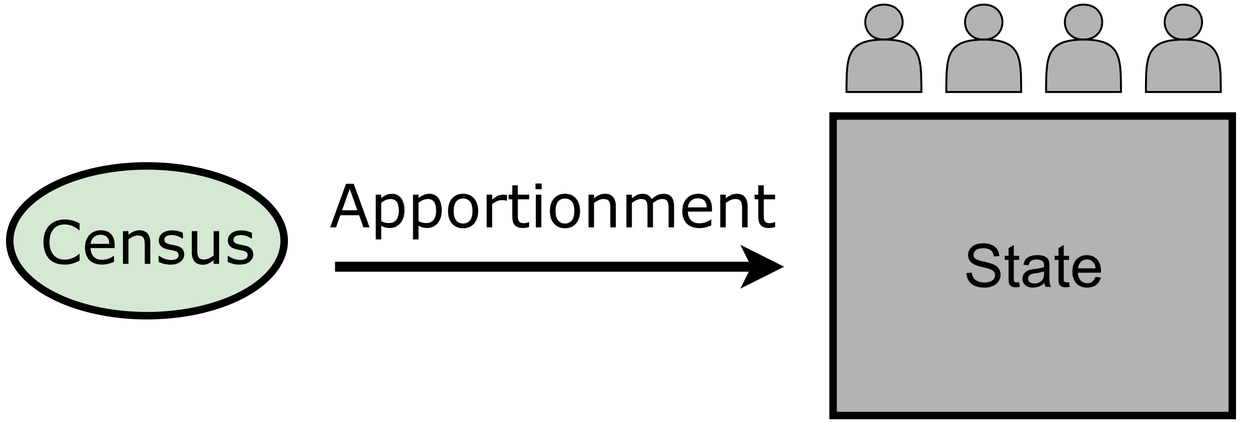

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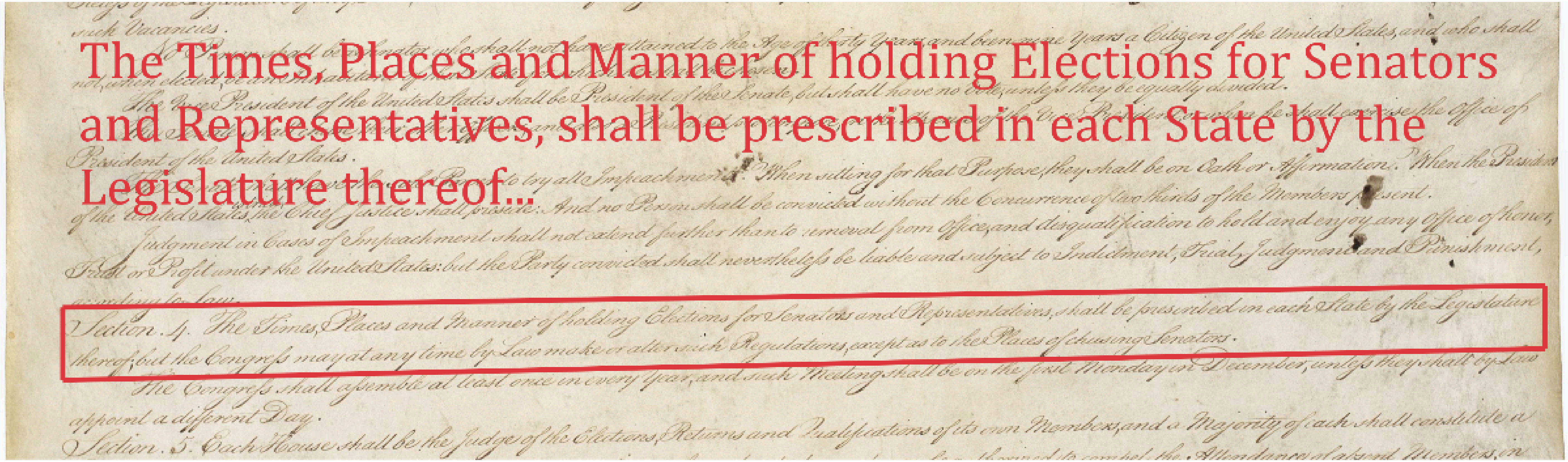

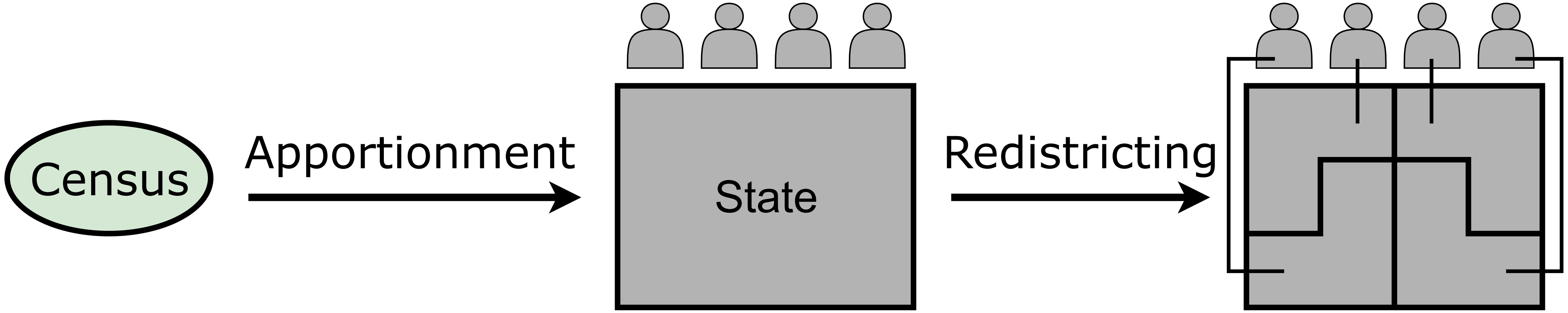

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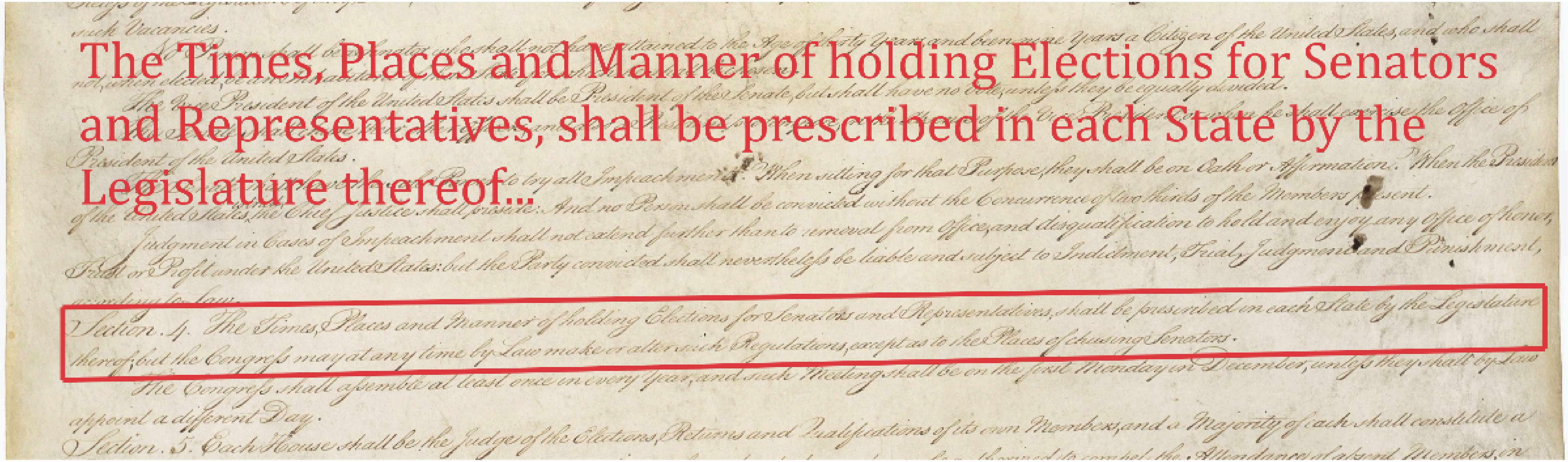
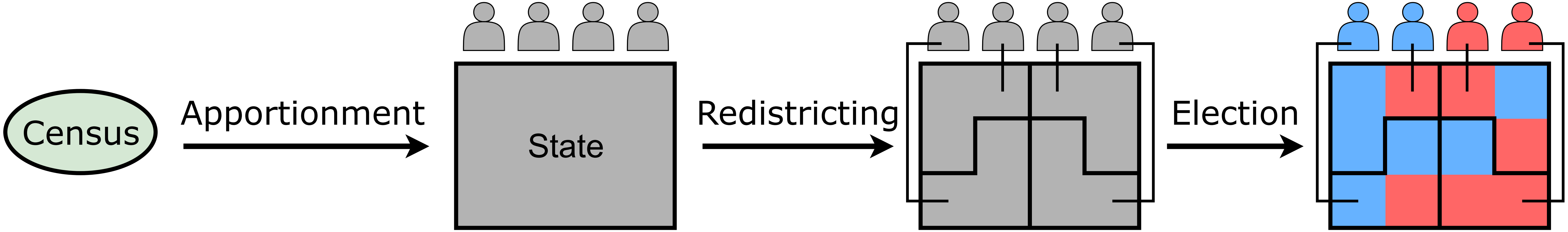
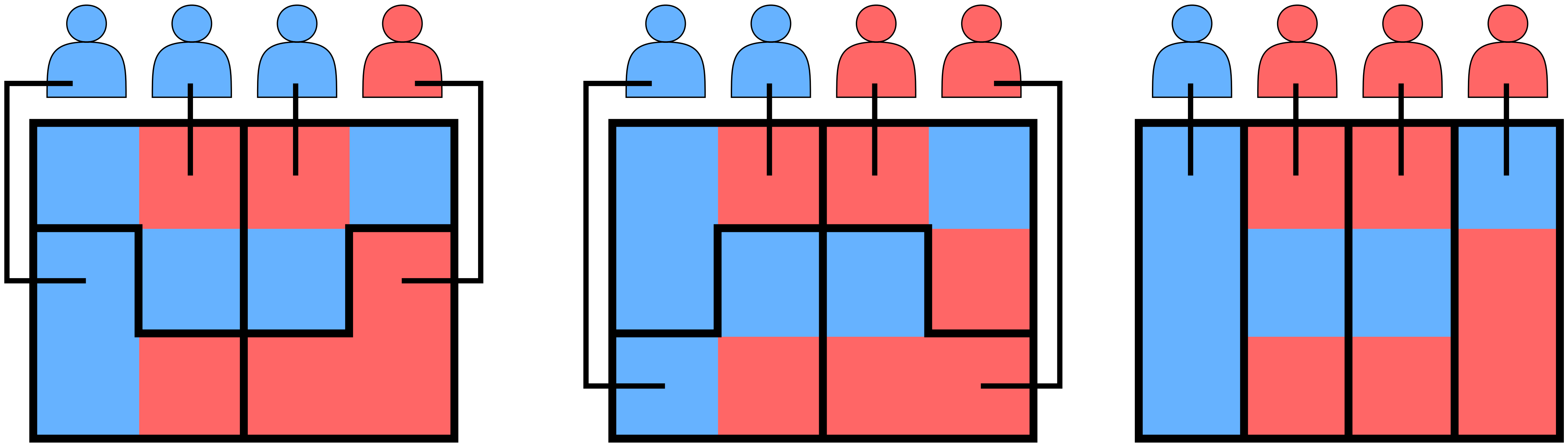




Gerrymandering



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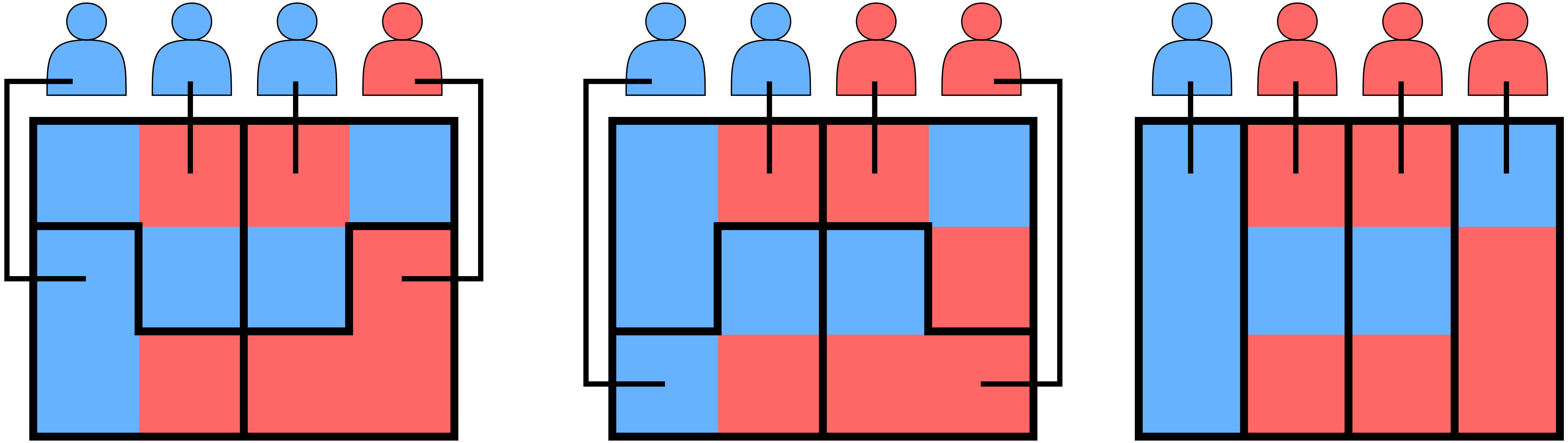
Gerrymandering: Drawing district lines to advantage one party or group over another.



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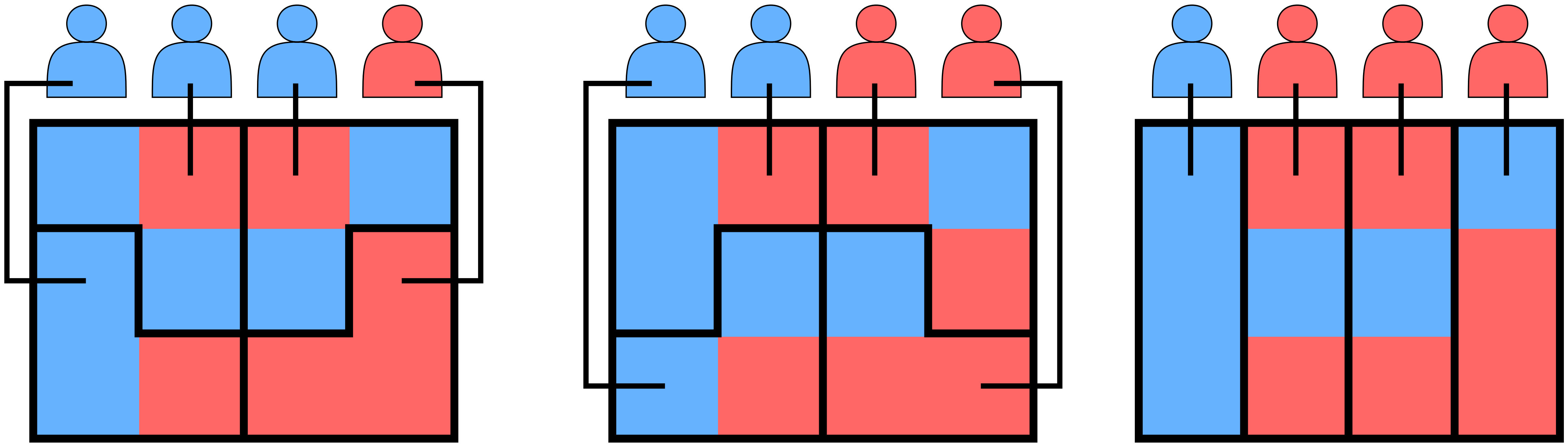

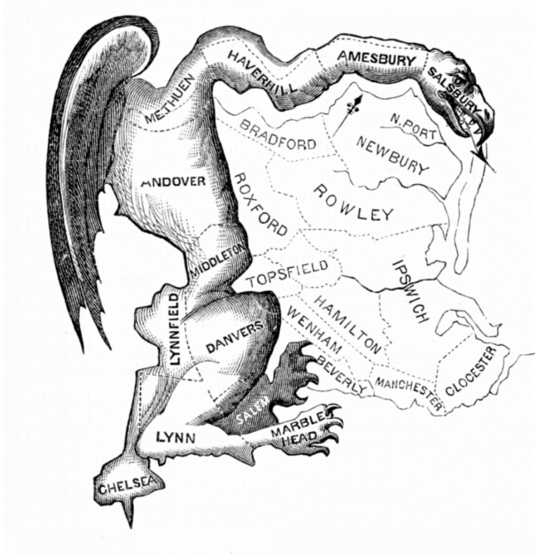

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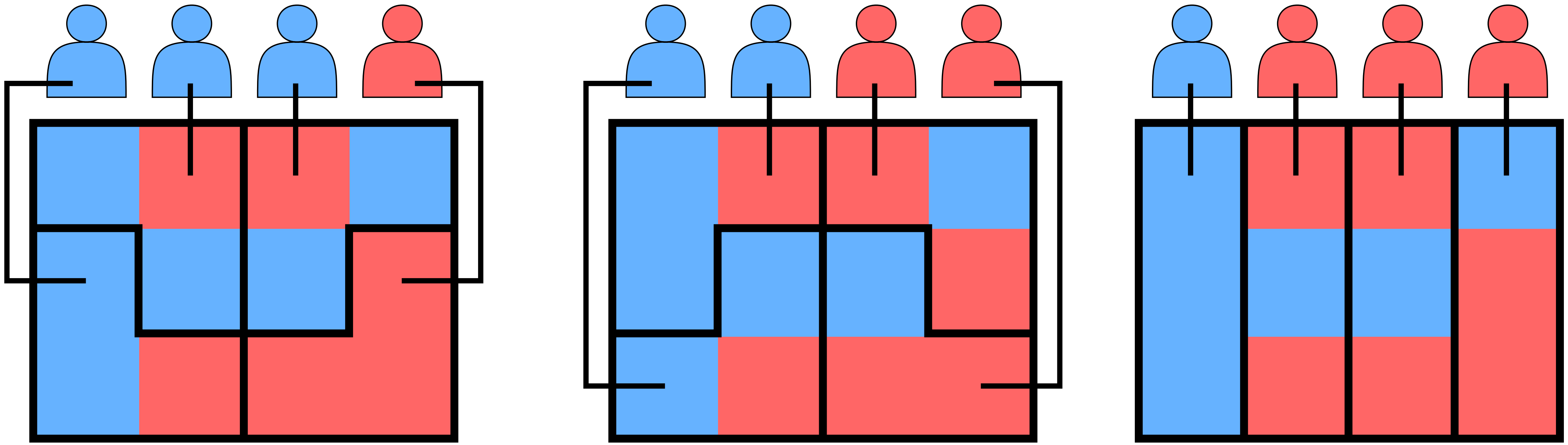

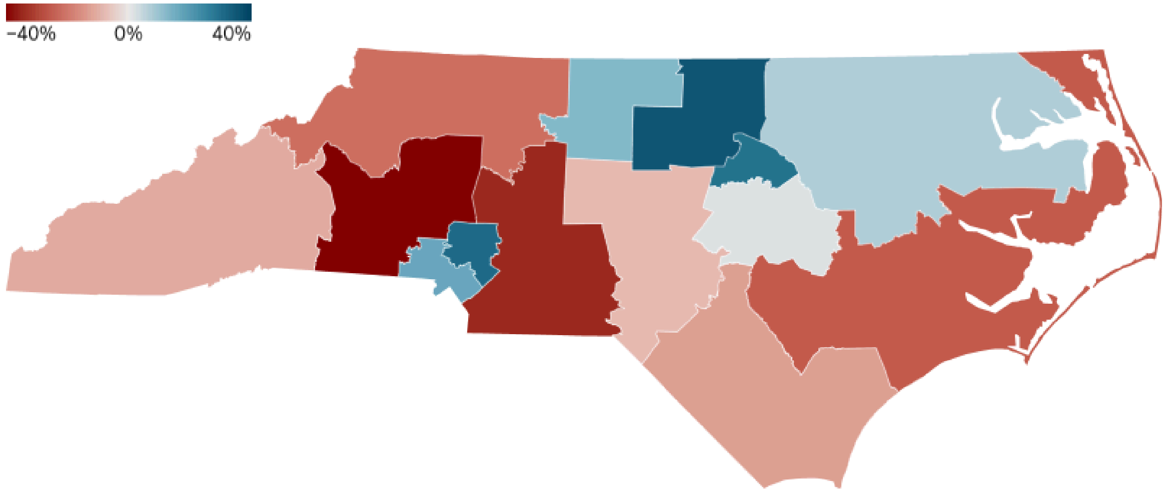

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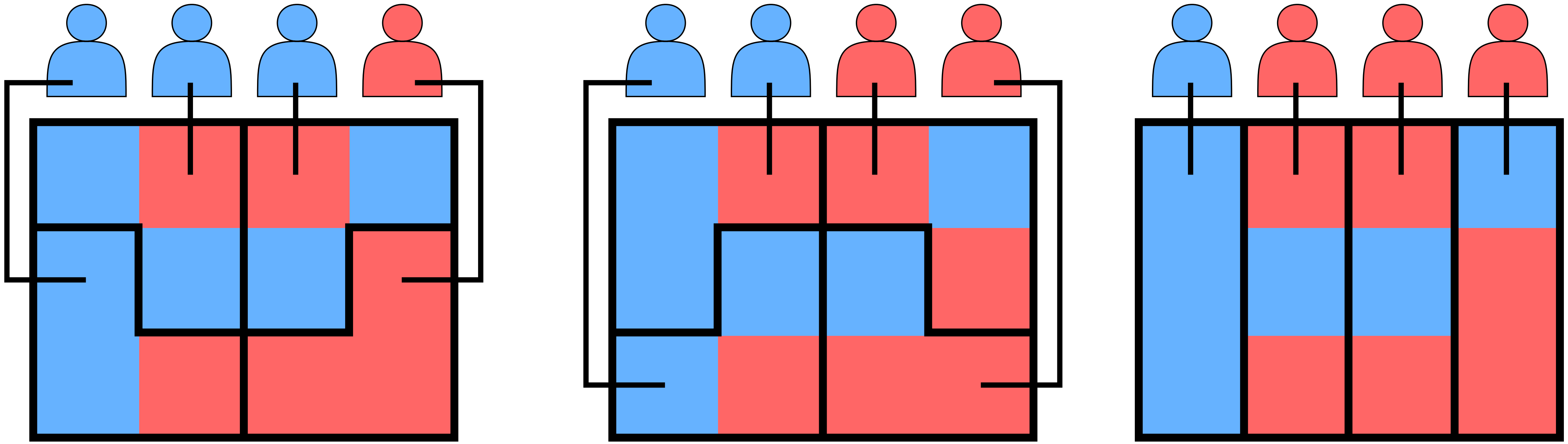

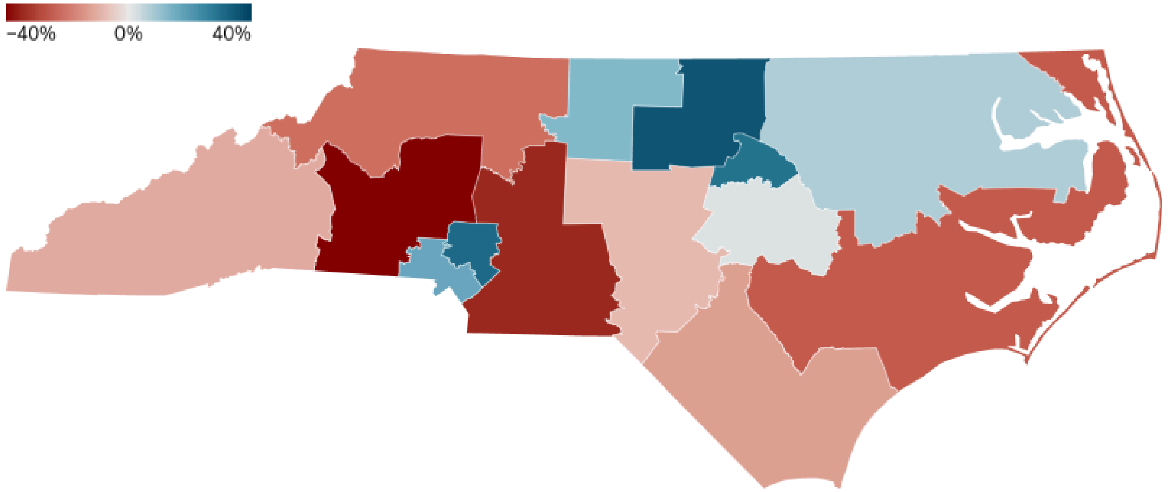

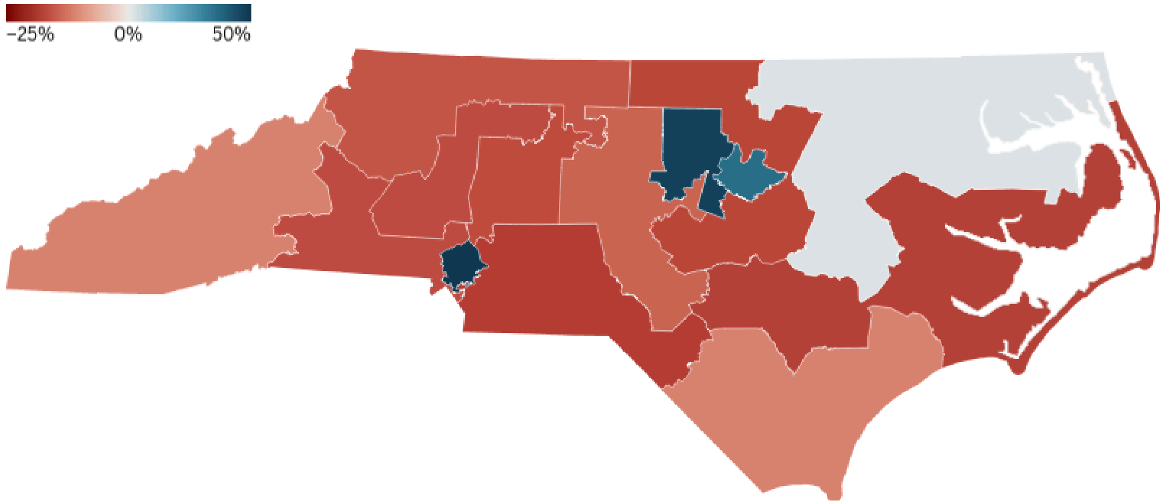

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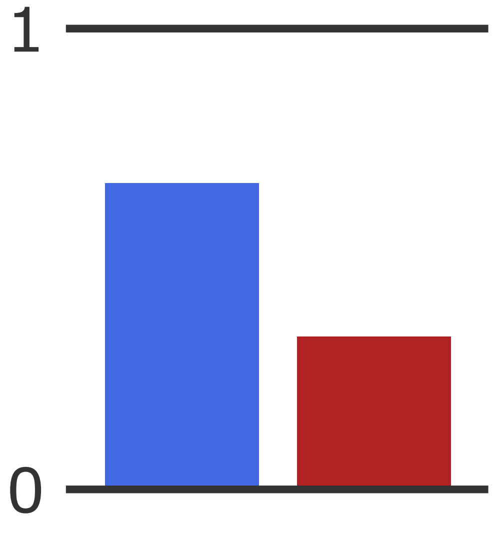

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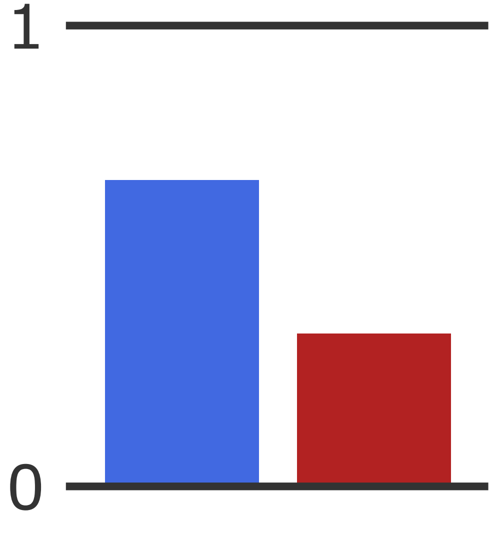

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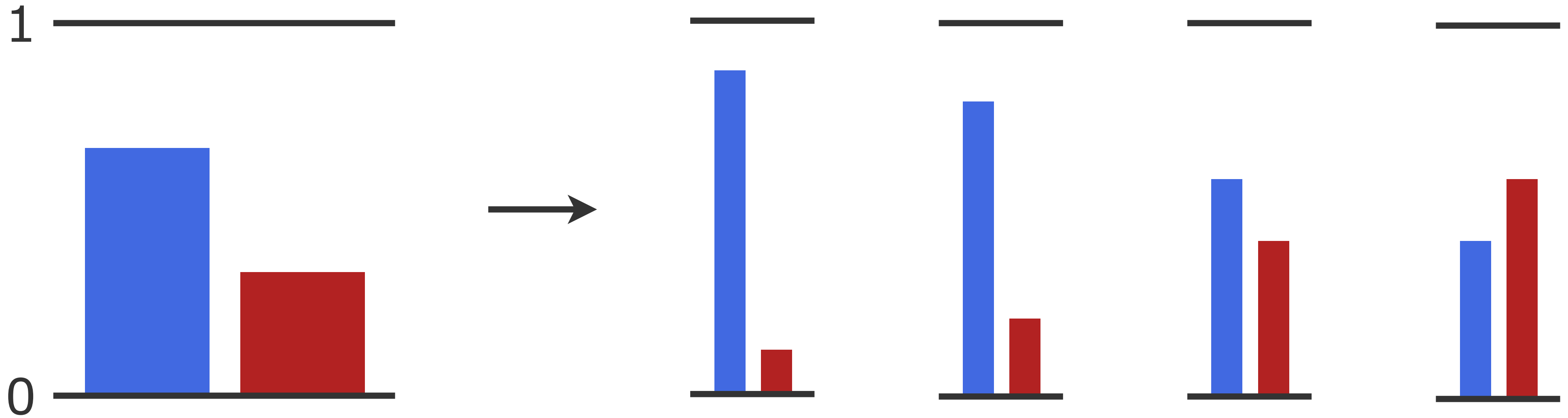
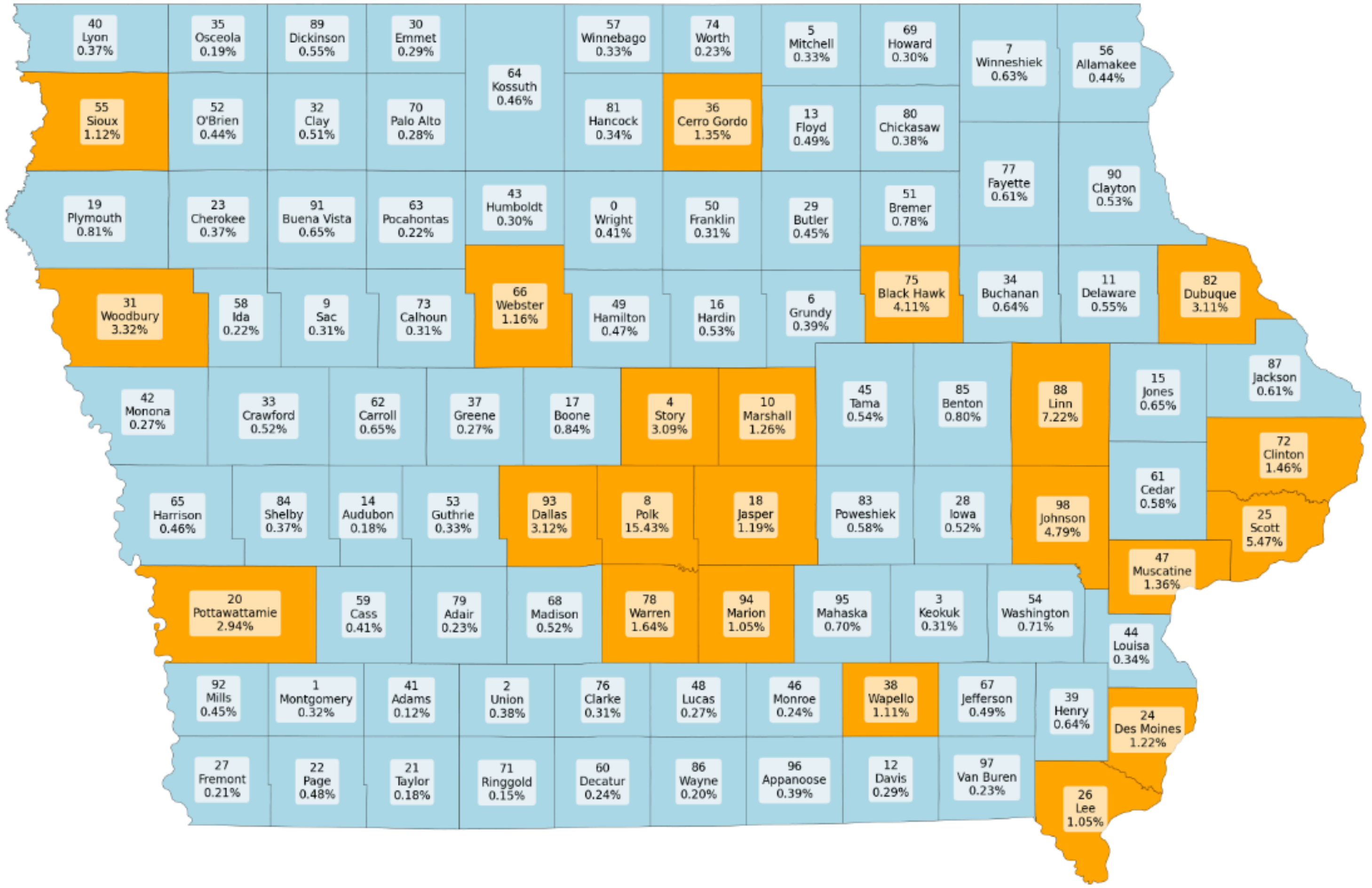
A redistricting map is a partition of the voting mass $P = D_1, D_2, \ldots, D_k$ where each D_ℓ is a pair of populations $D_\ell = (p_\ell, \frac{1}{k} - p_\ell)$, with $v^1(D_\ell) := p_\ell$, $v^2(D_\ell) := 1 - p_\ell$.
A feasible partition is one where $\sum_{\ell=1}^{k} v^1(D_\ell) = p$.



Model #1: Graph partitioning

For legal purposes, this is what redistricting actually is!

(But it's harder to work with analytically)



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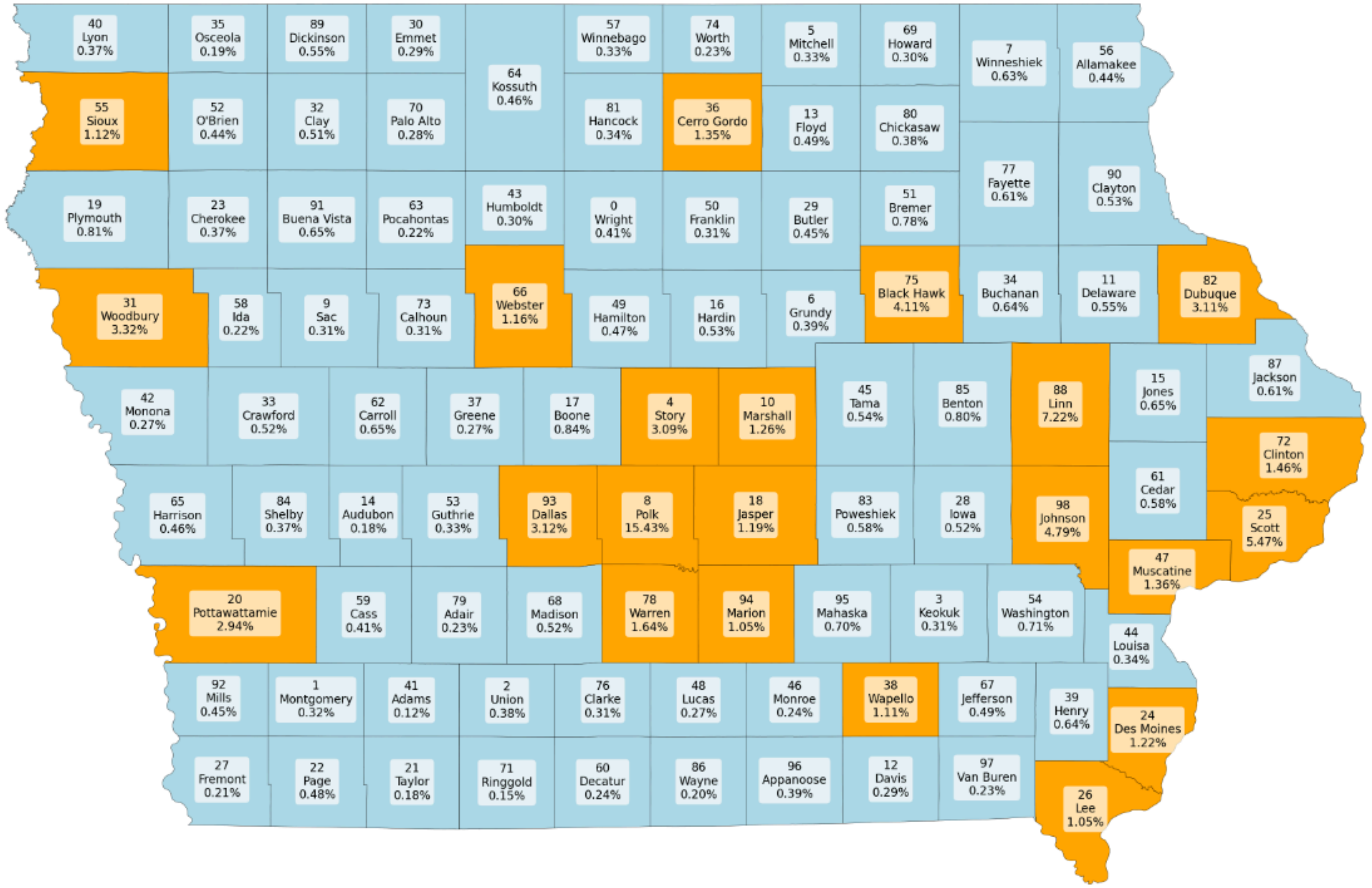
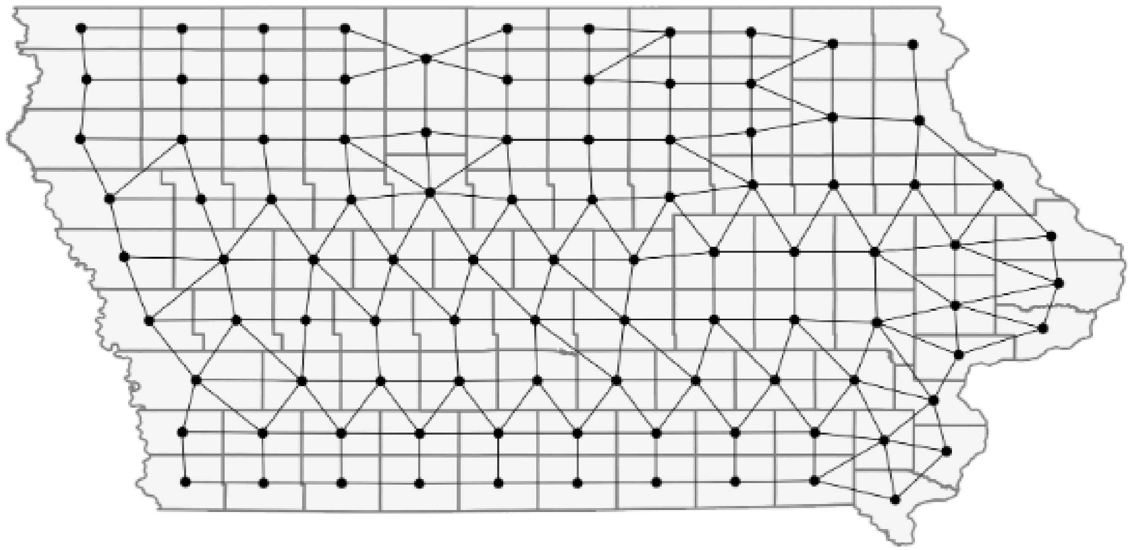
Underlying vertex-weighted graph: G





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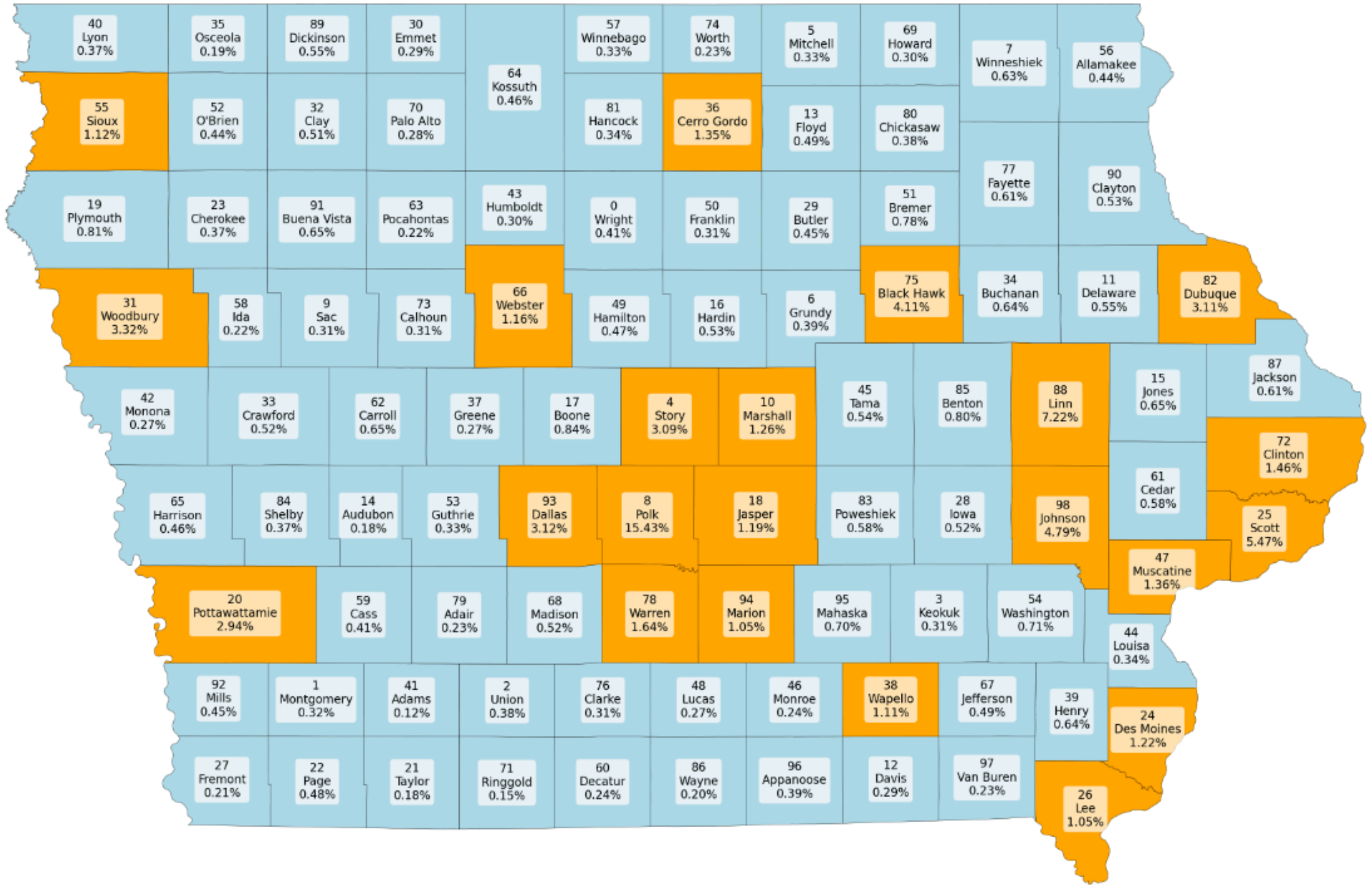

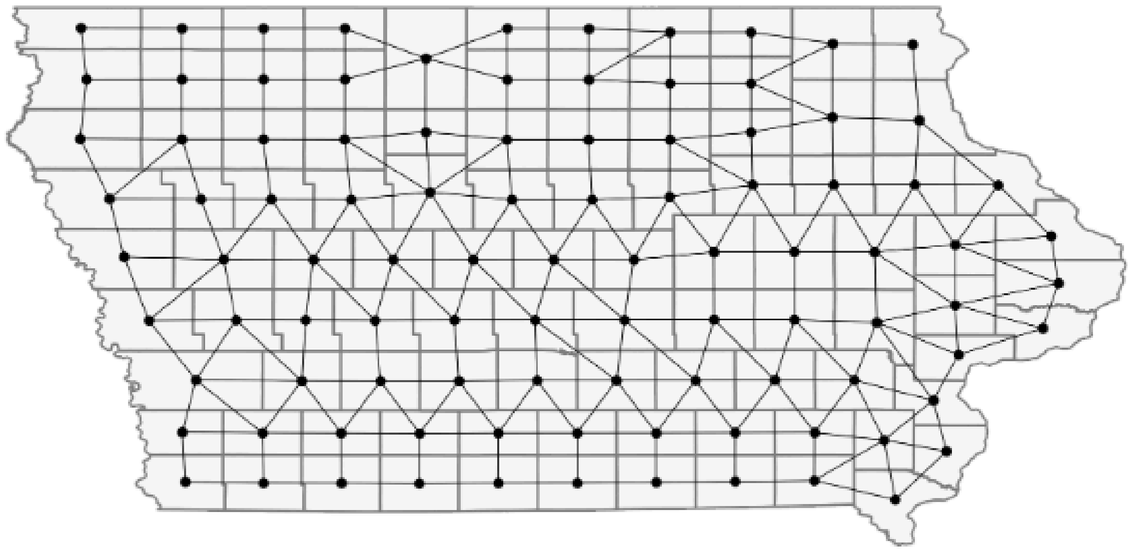

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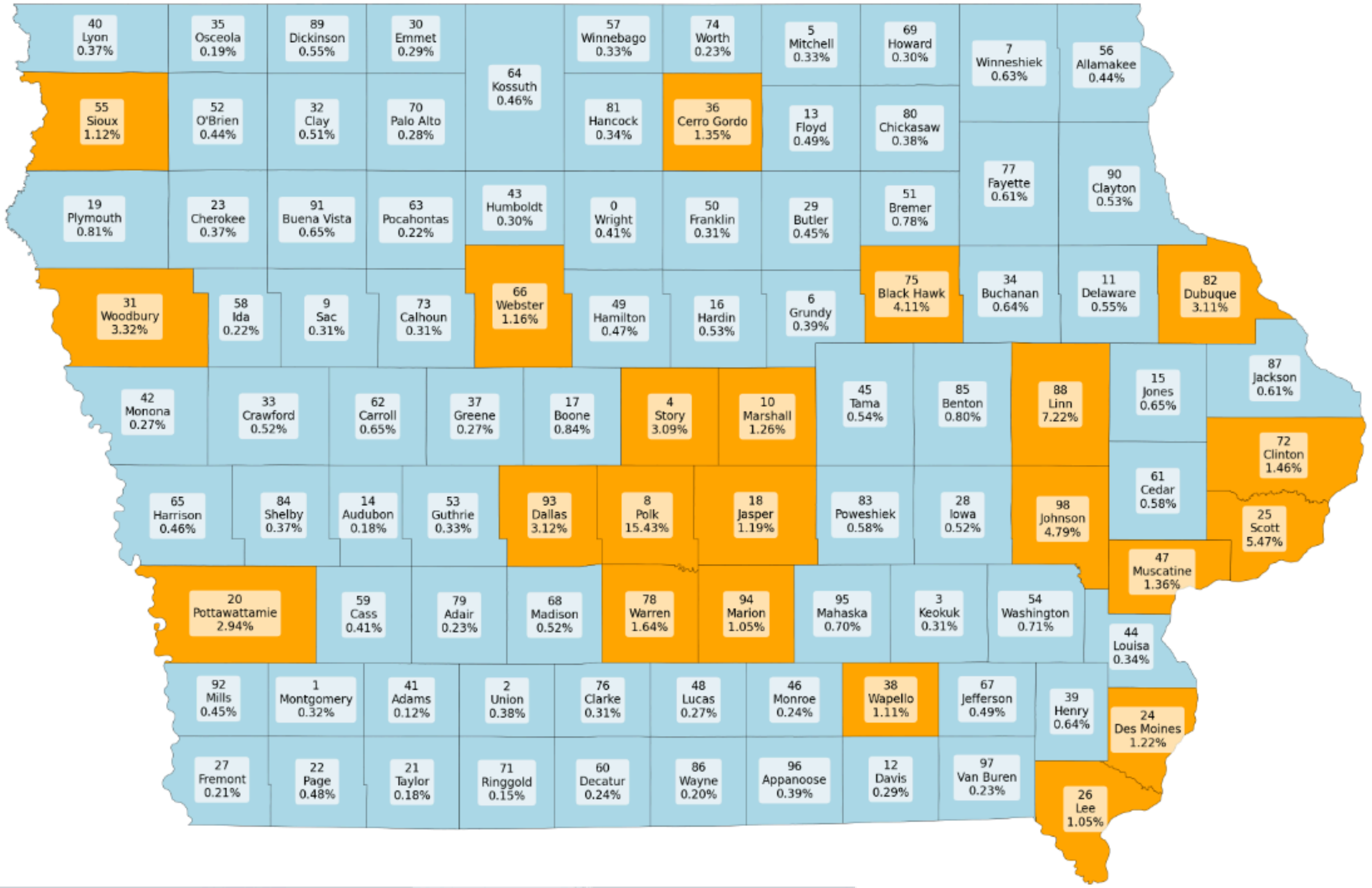

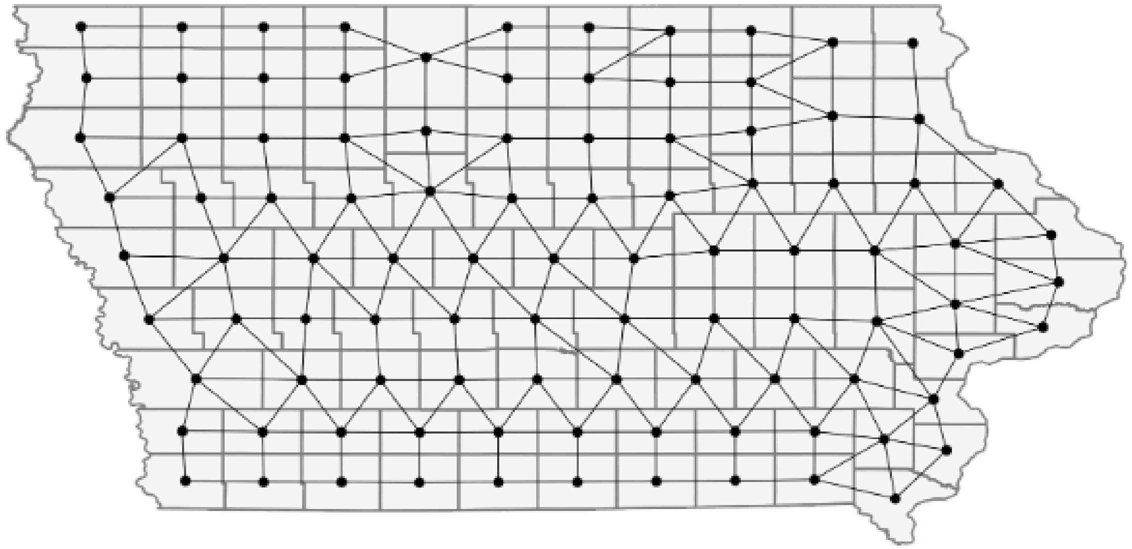

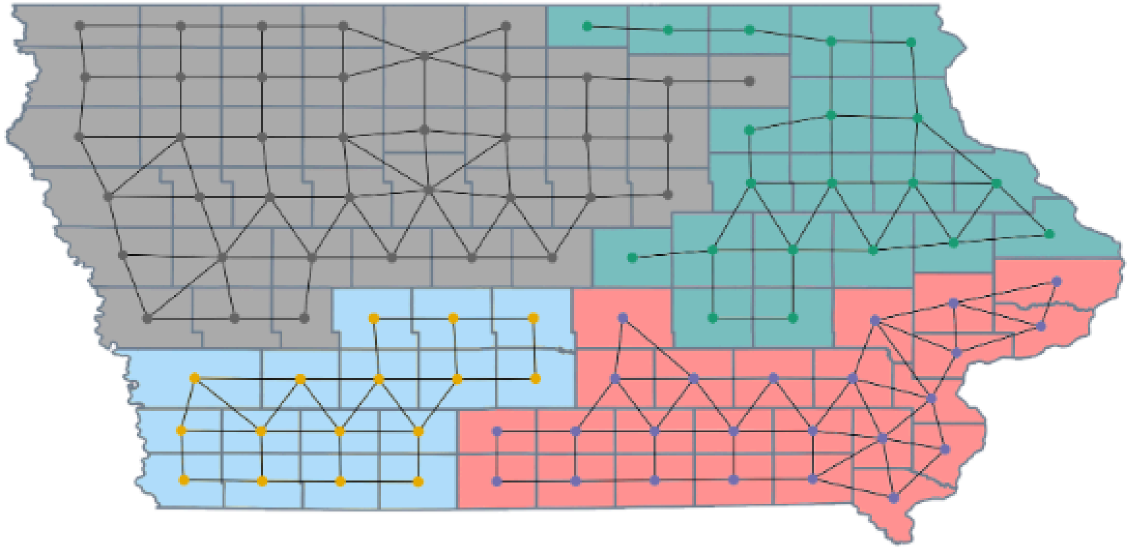

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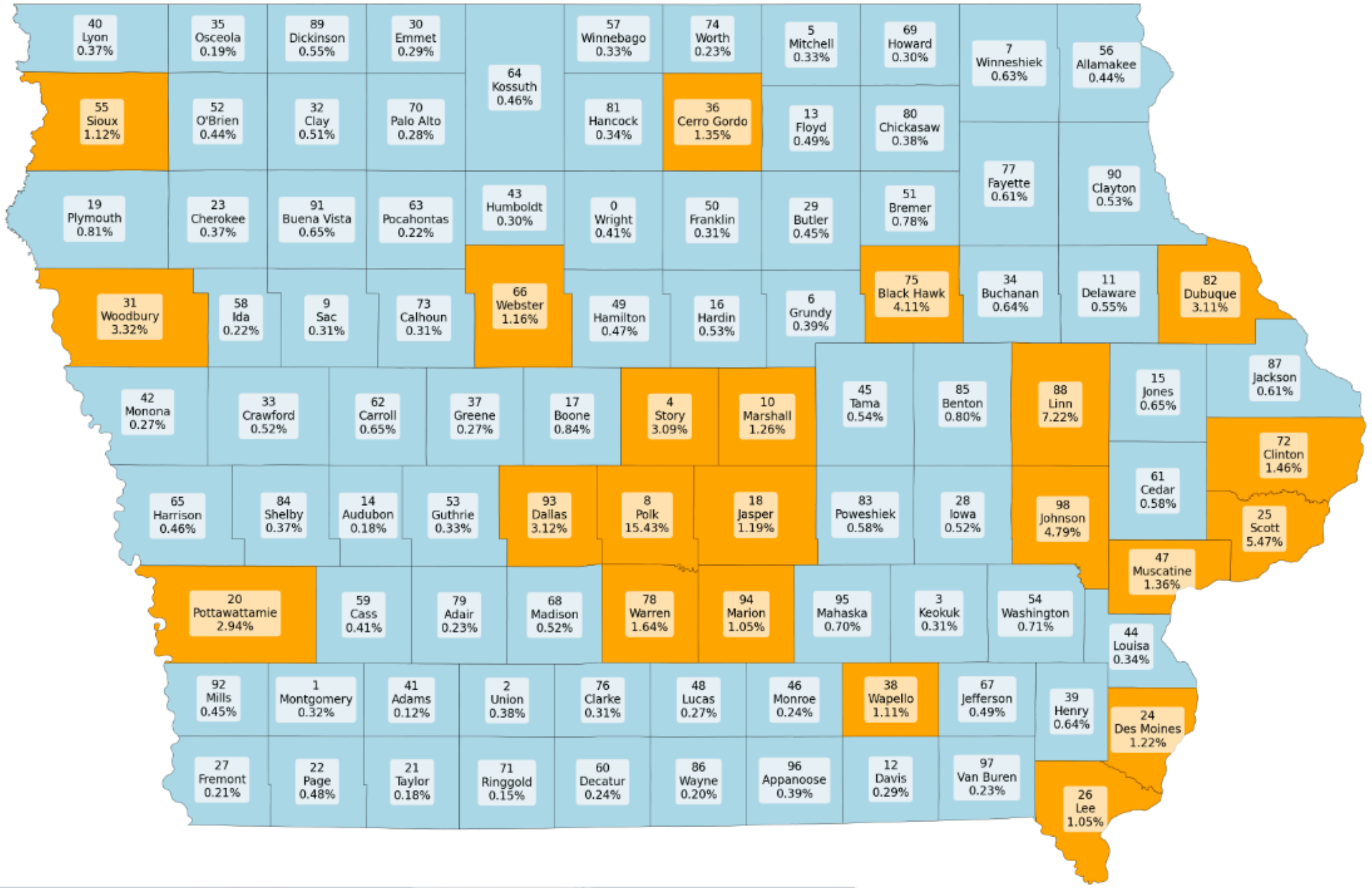
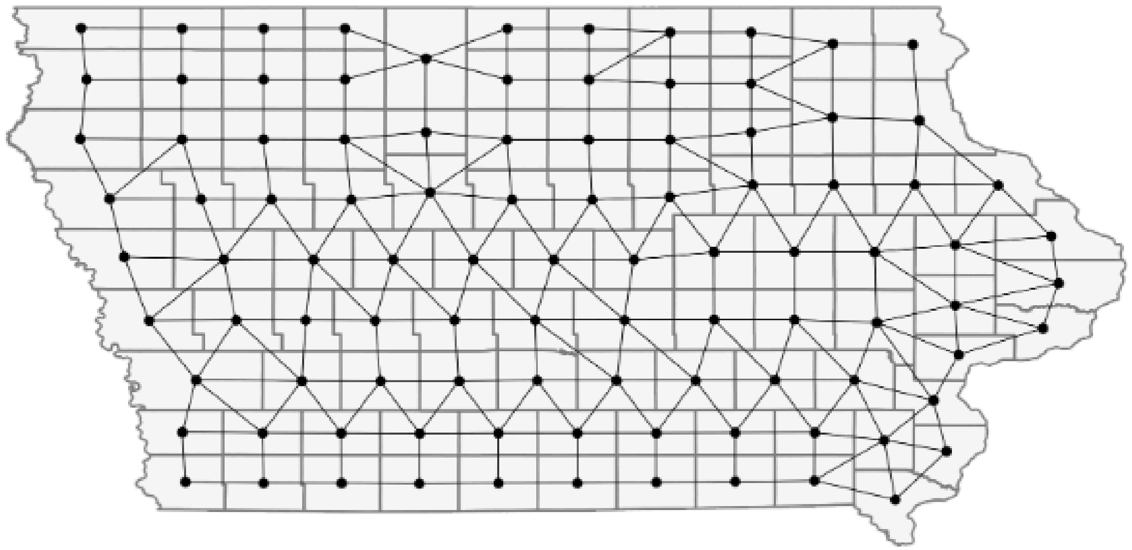
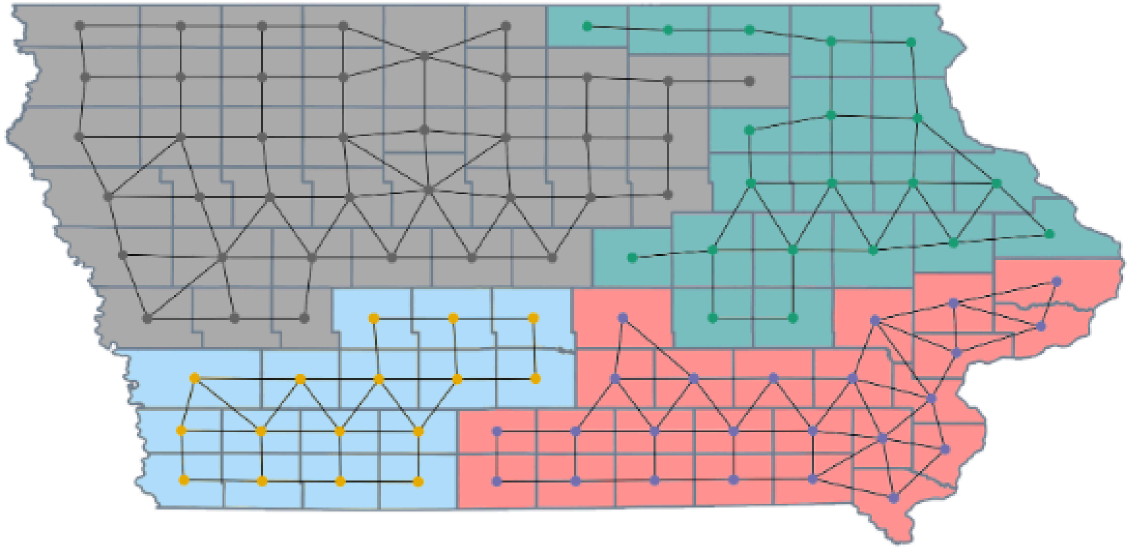
For $i \in N$, and $\ell \in [k]$, $v^i(D_\ell)$ obtained by summing over vertices, using historical data







Fairness axioms

For a partition $P = (D_1, D_2, \ldots, D_k)$, denote the utility of each party i by

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▸ In the geometry-free model, which axiom requires that the minority party win (weakly) more seats?

- **Proportionality**
- **Incomparable**
- **Geometric target**
- **Equivalent**

Respond at:

pollev.com/jtuckerfoltz255 or

bit.ly/jtfpoll or

text jtuckerfoltz255 to 37607

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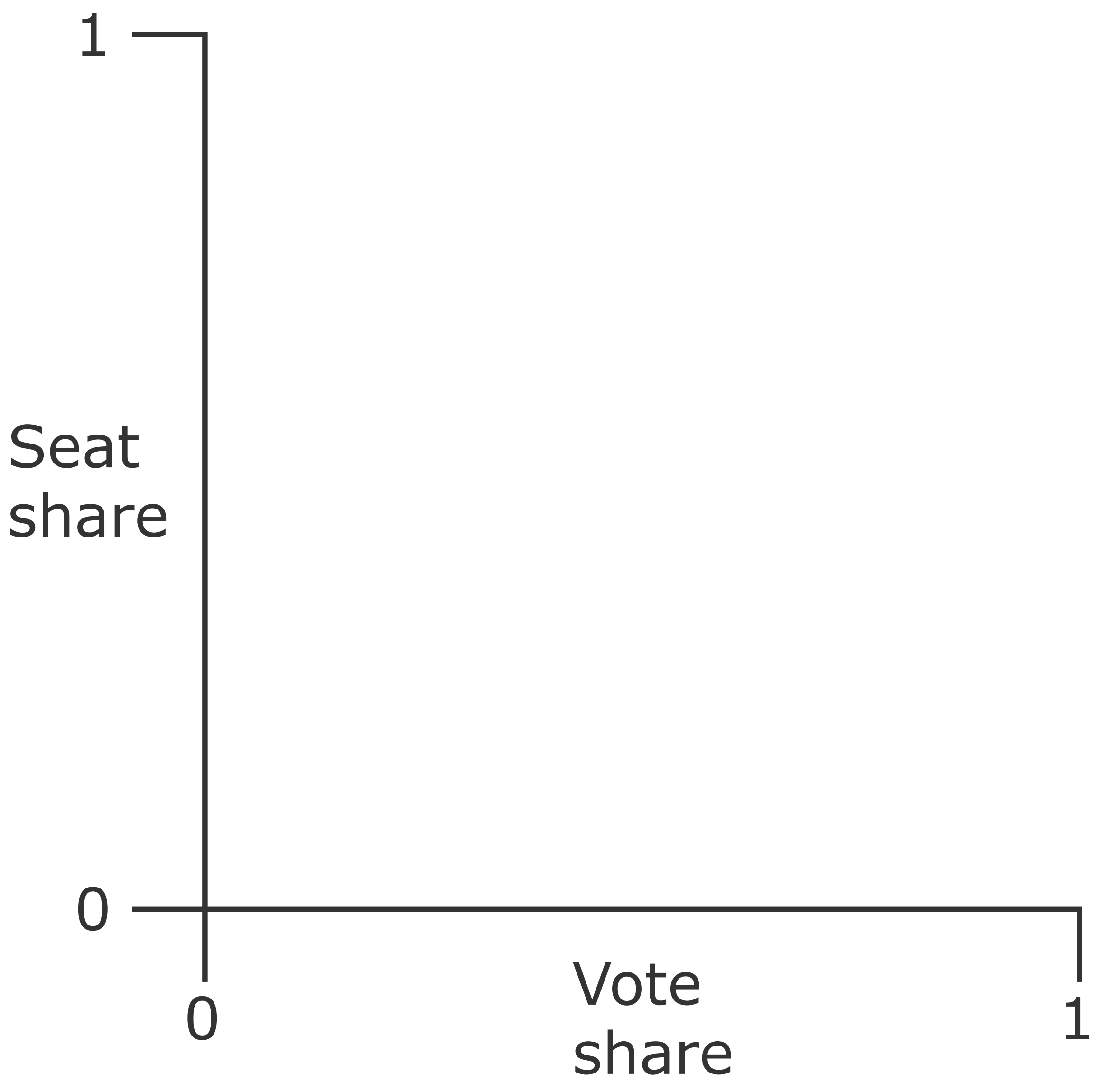

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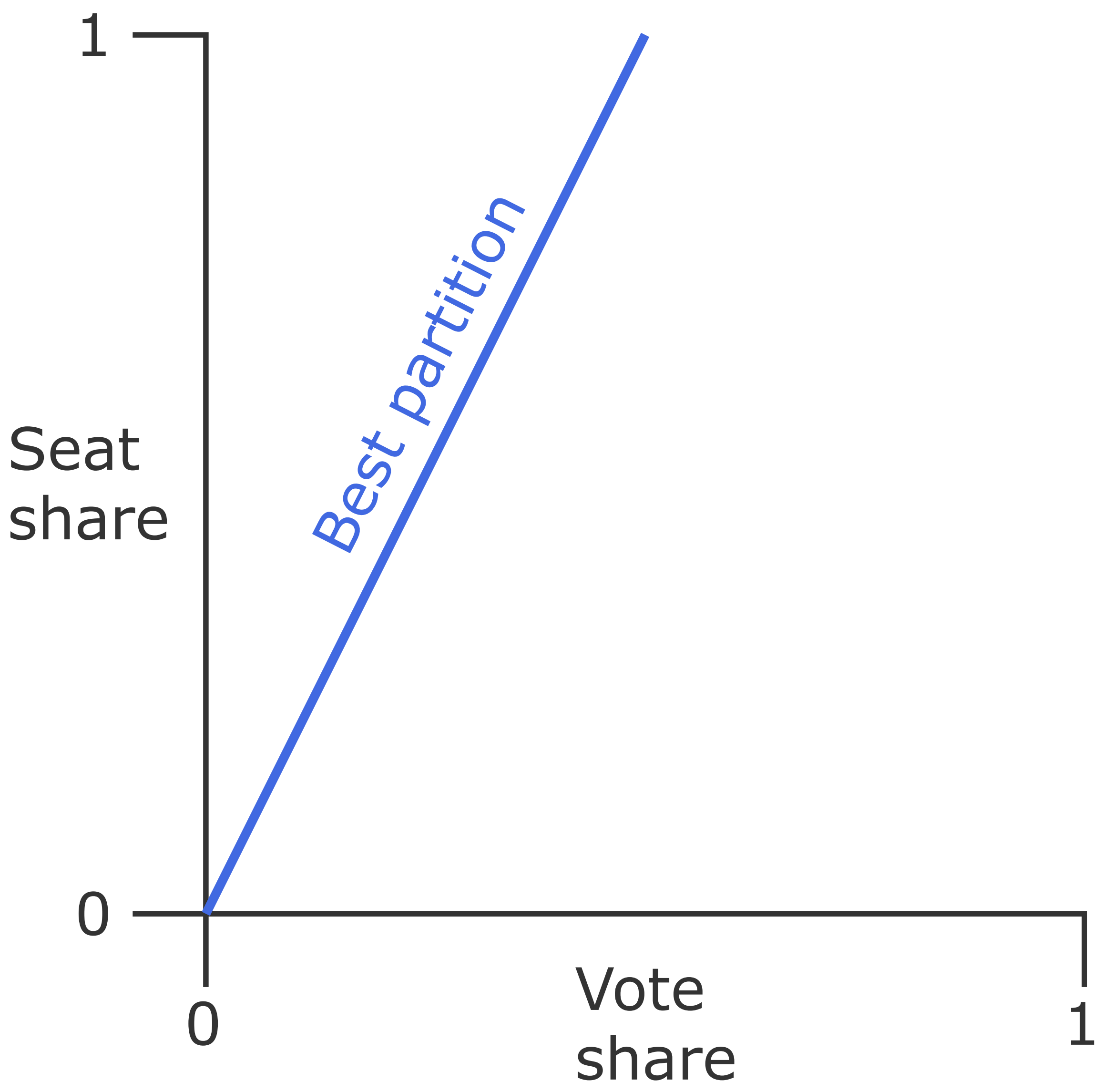

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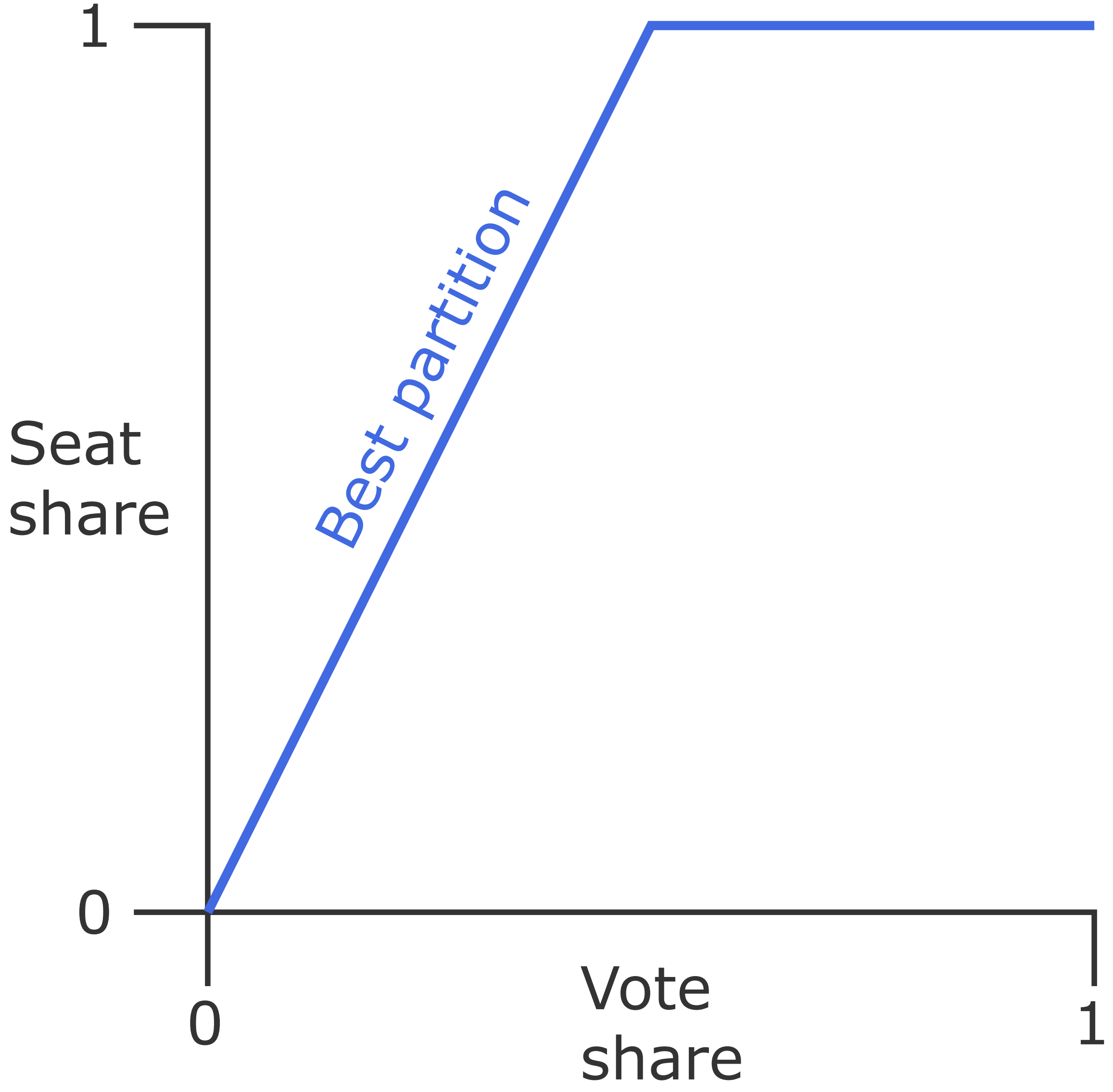

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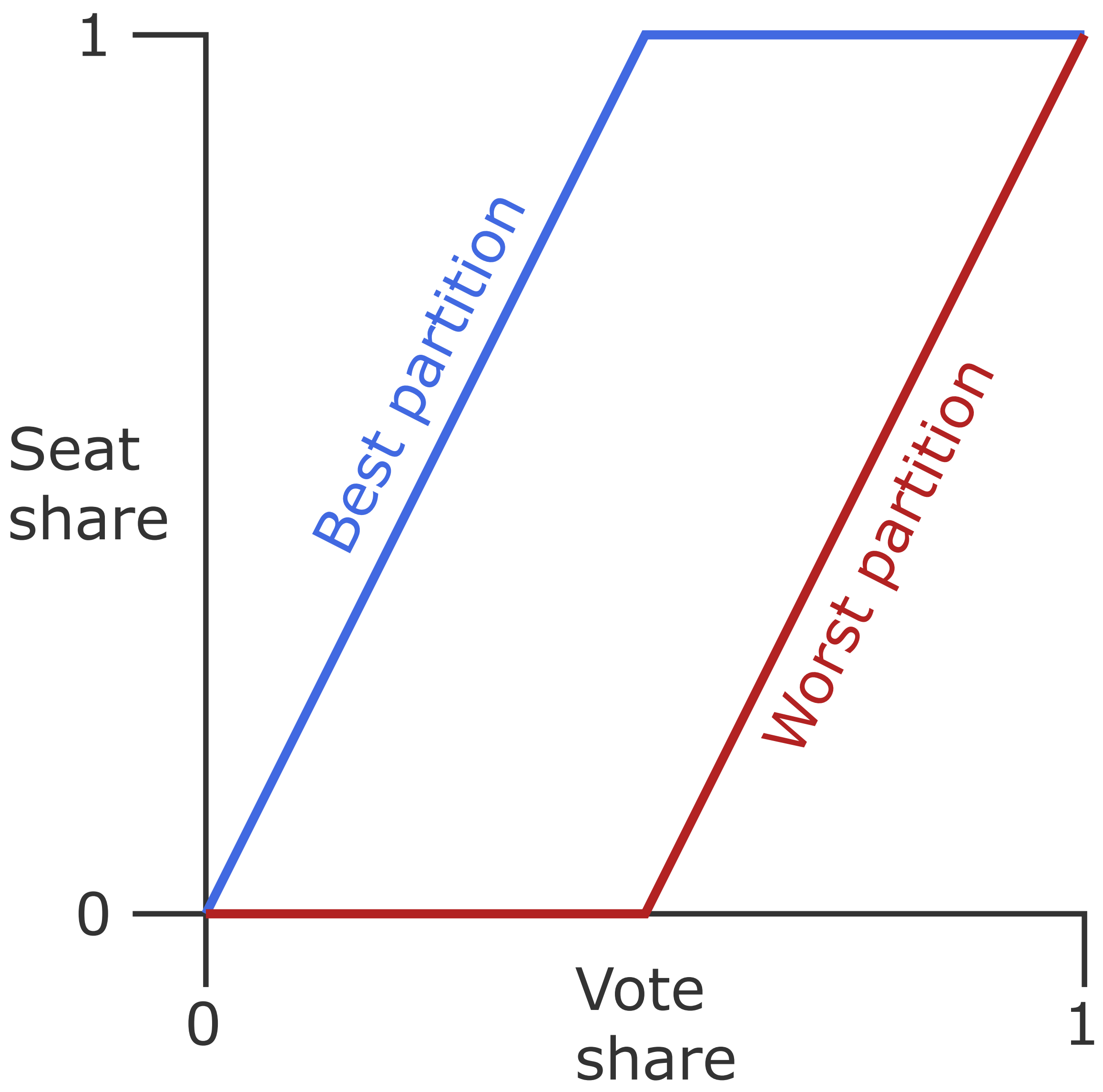

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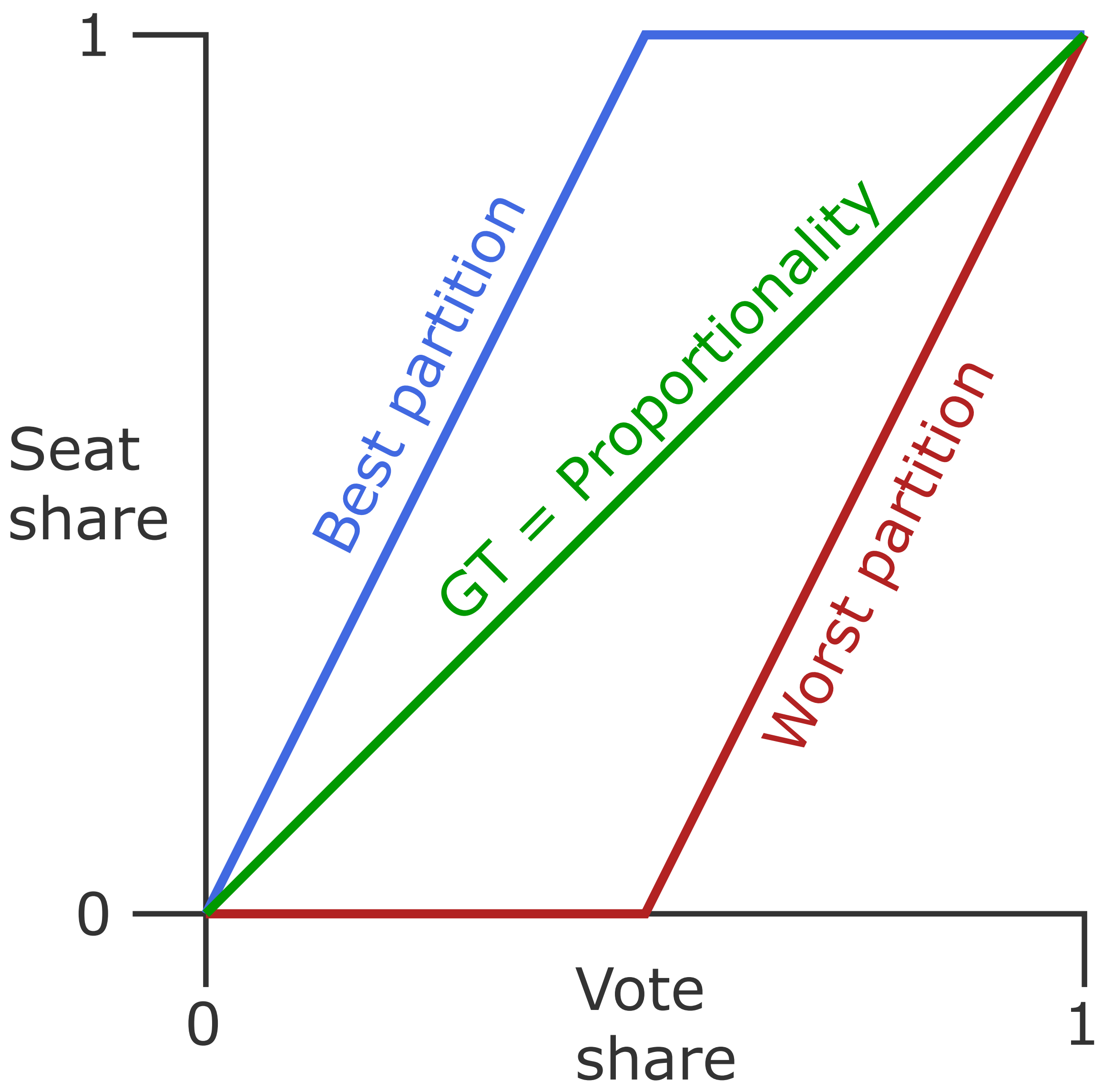
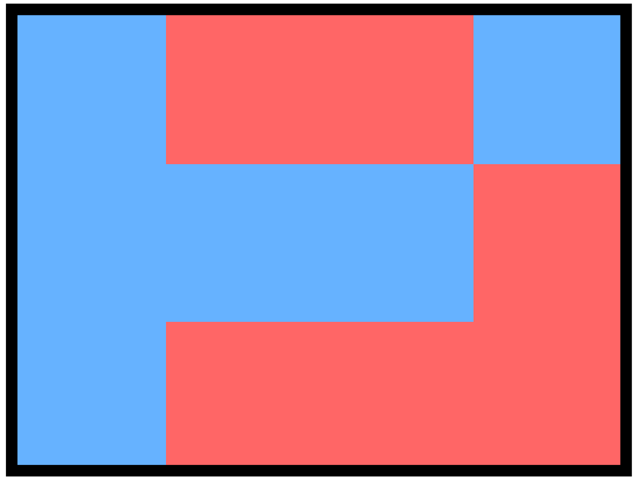


Protocol #1: I-Cut-You-Freeze



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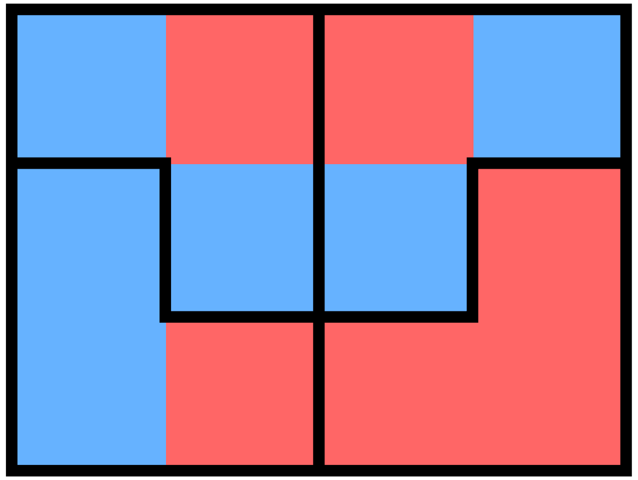
- Party 1 partitions the map into k districts



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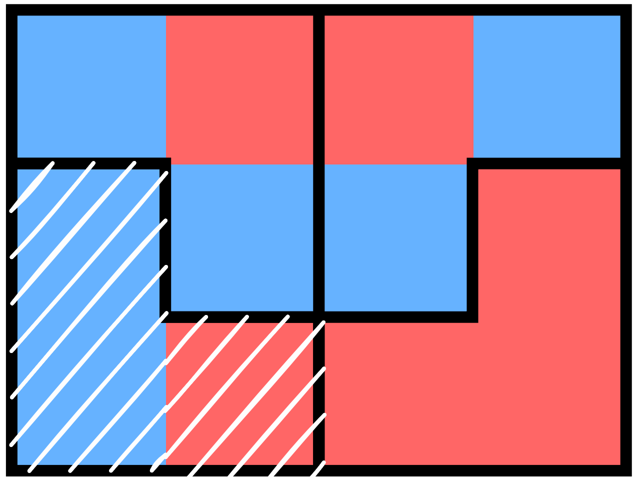

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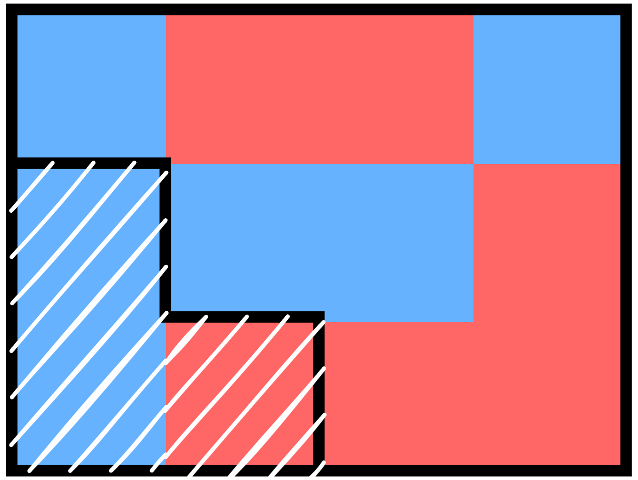

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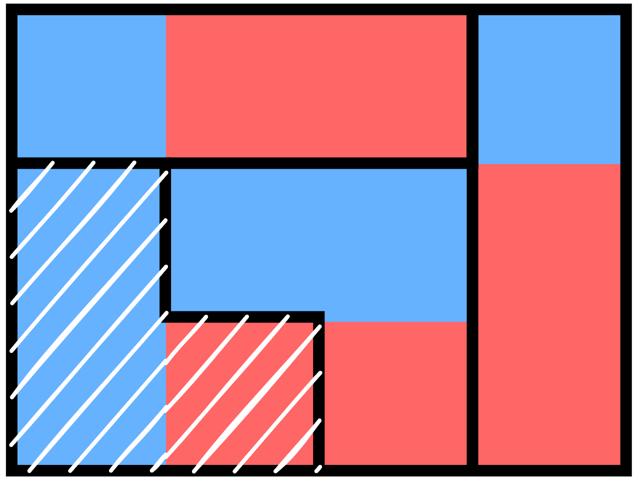

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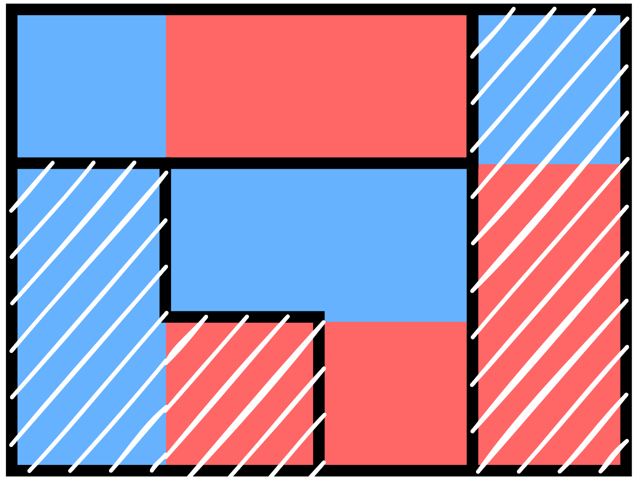

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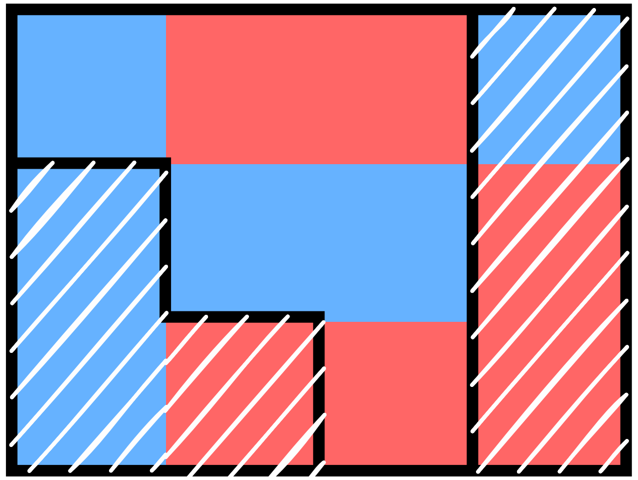
- ...
- Continue until all districts are frozen



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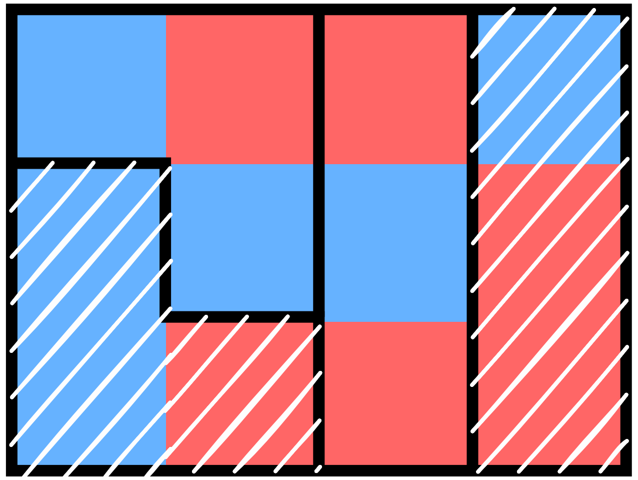

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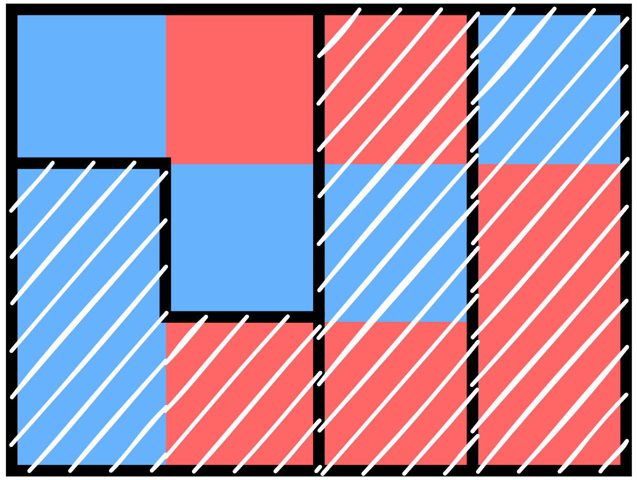

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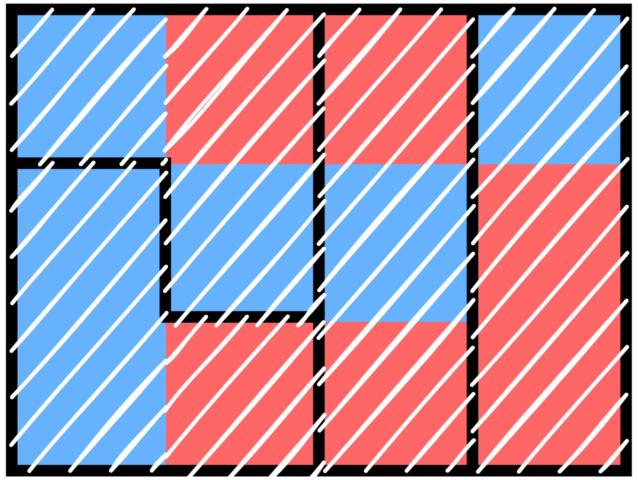

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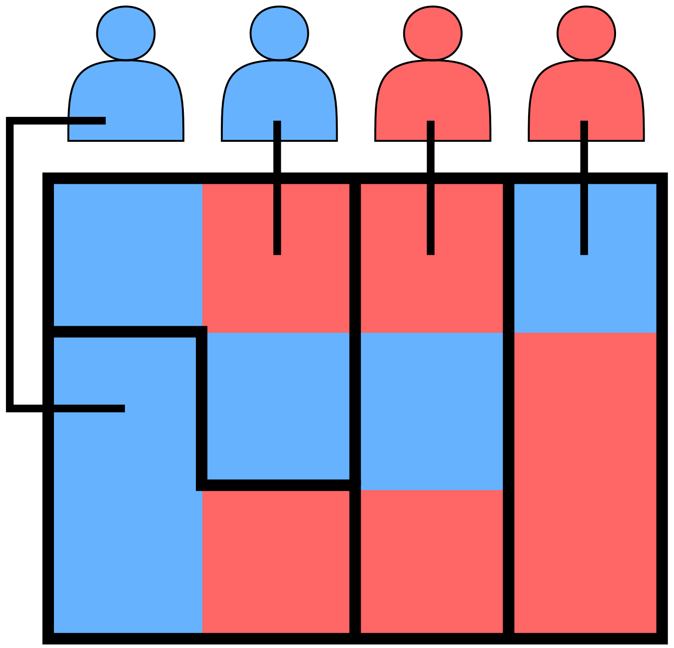

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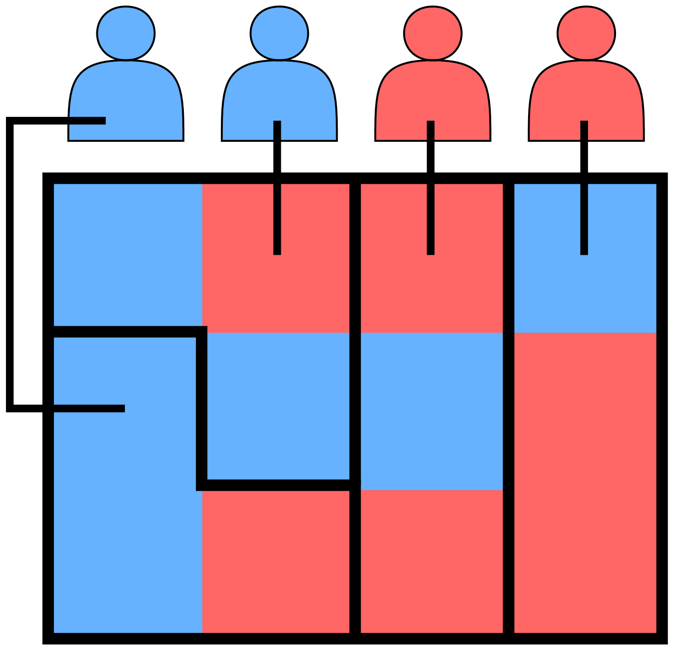
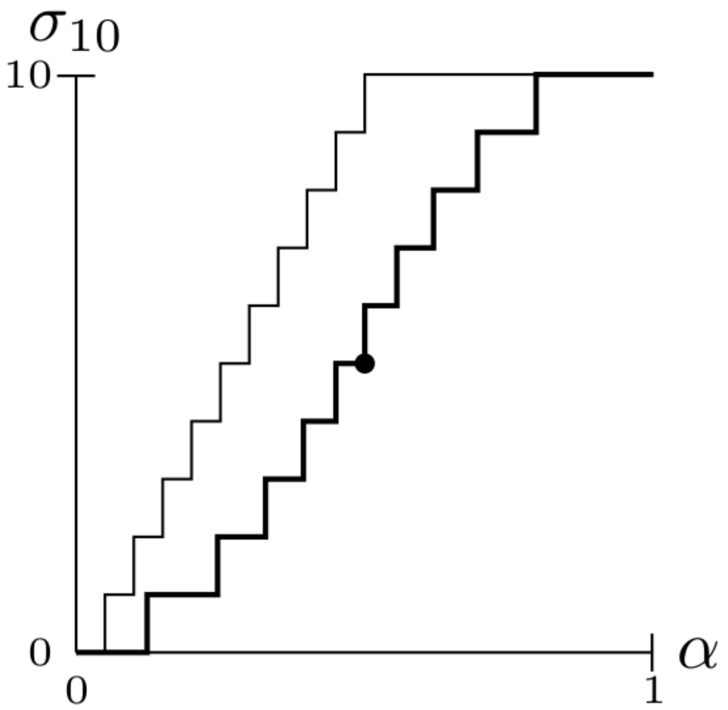
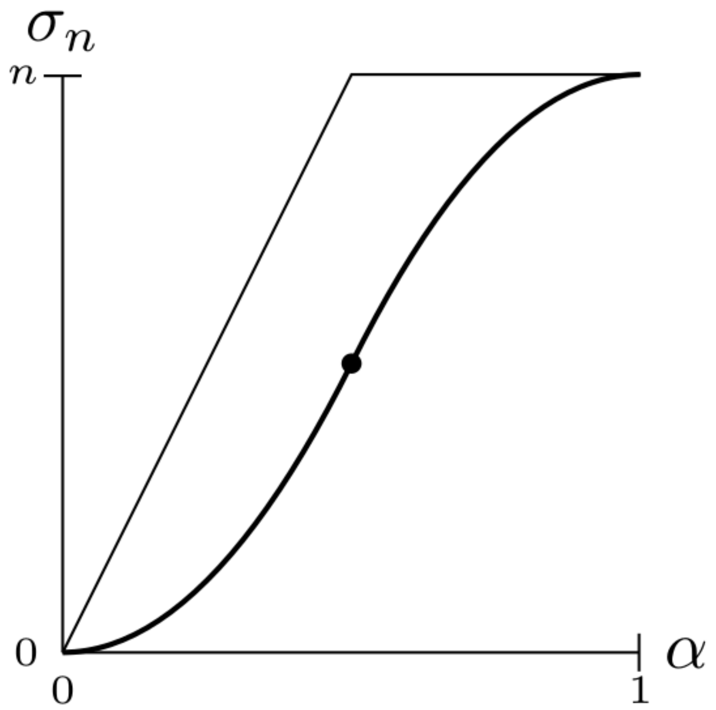


Theorem (Pegden, Procaccia, Yu, 2017)

In the geometry-free model, the conversion from vote share to seat share is somewhat close to proportionality, with a closed form as in the plots to the right.





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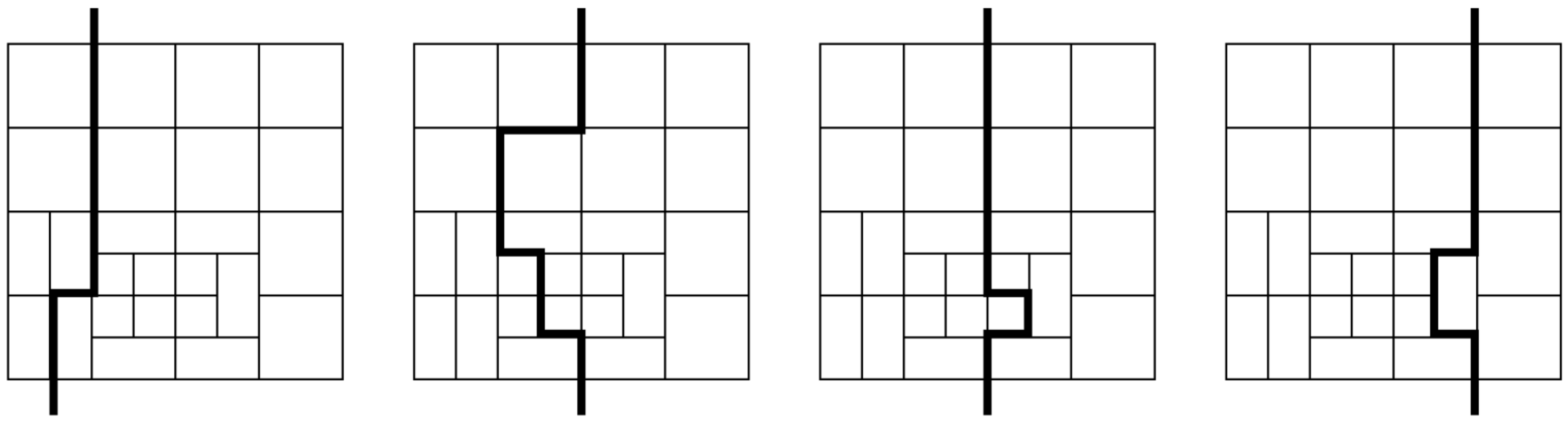

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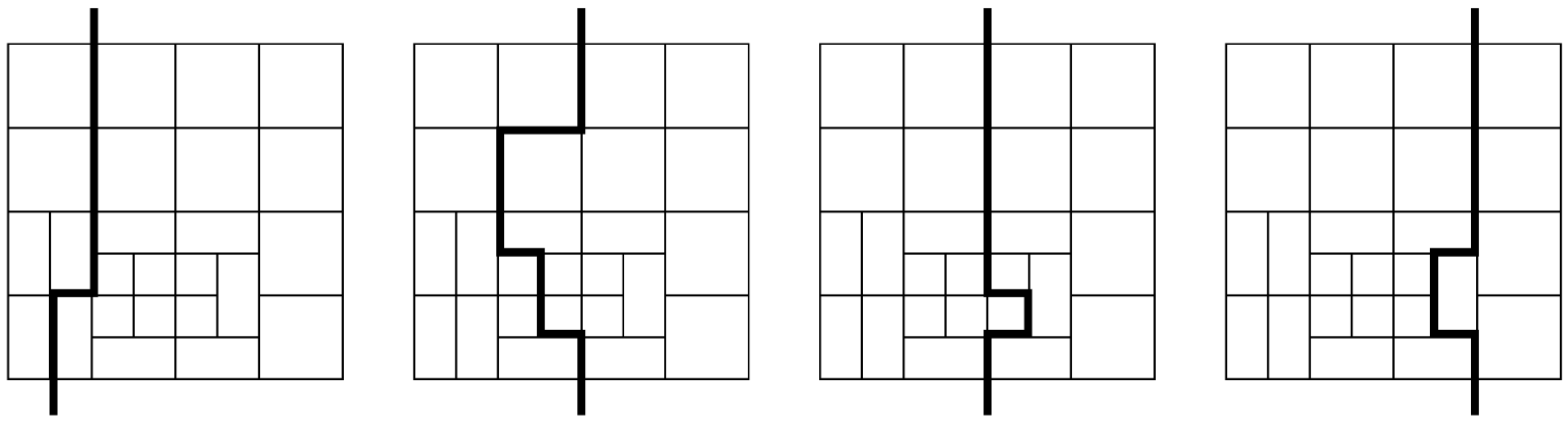

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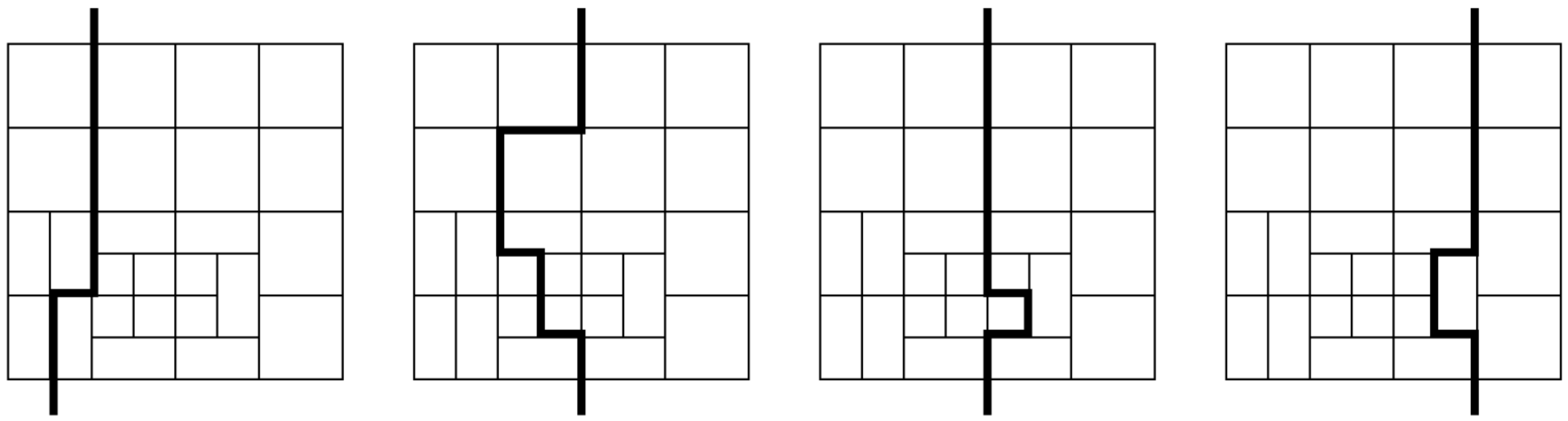

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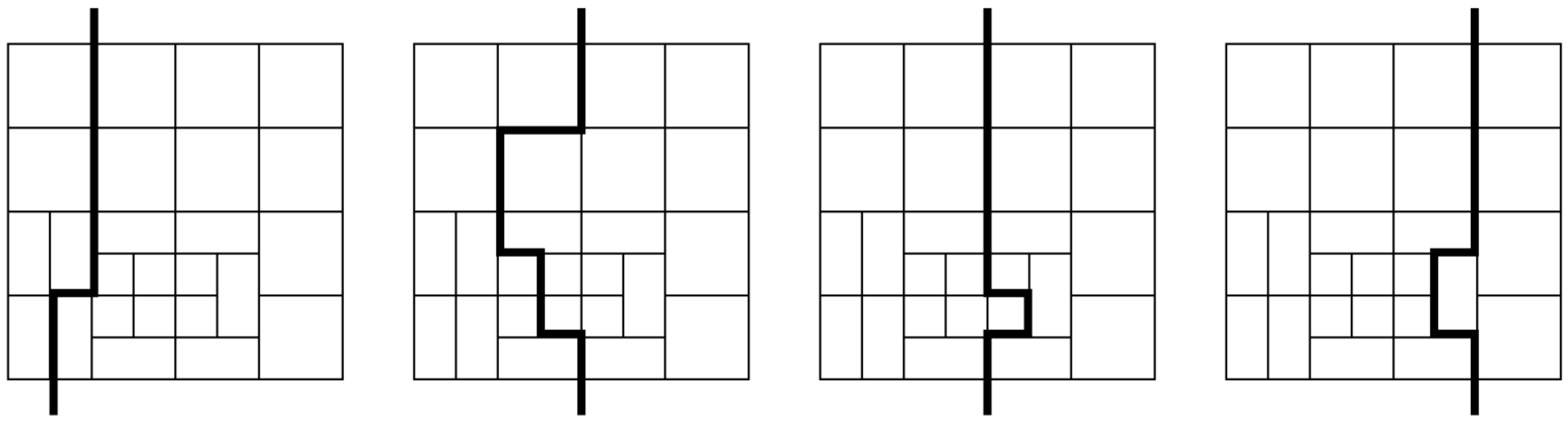

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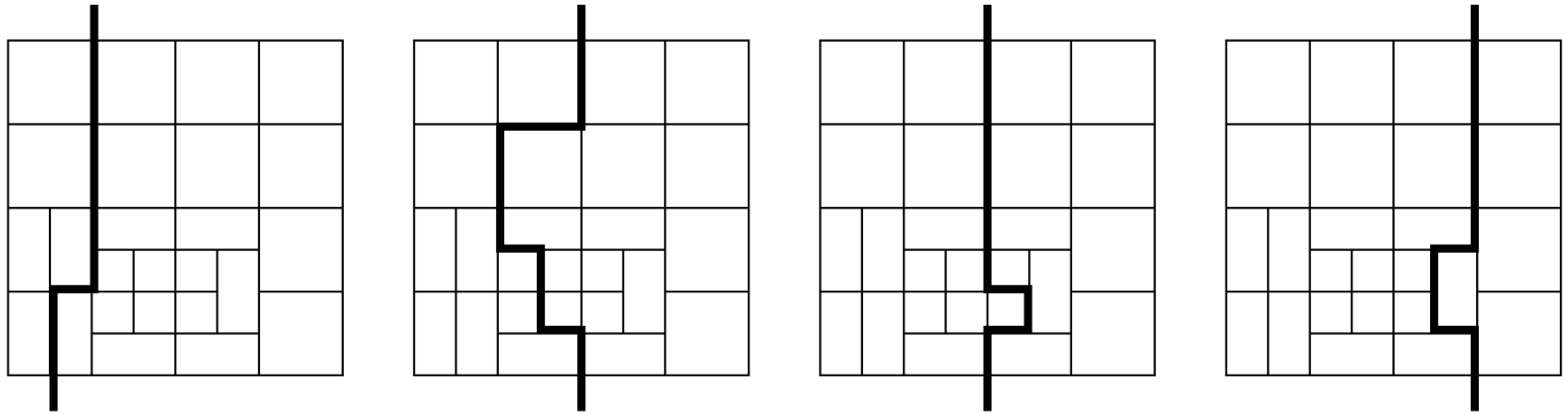

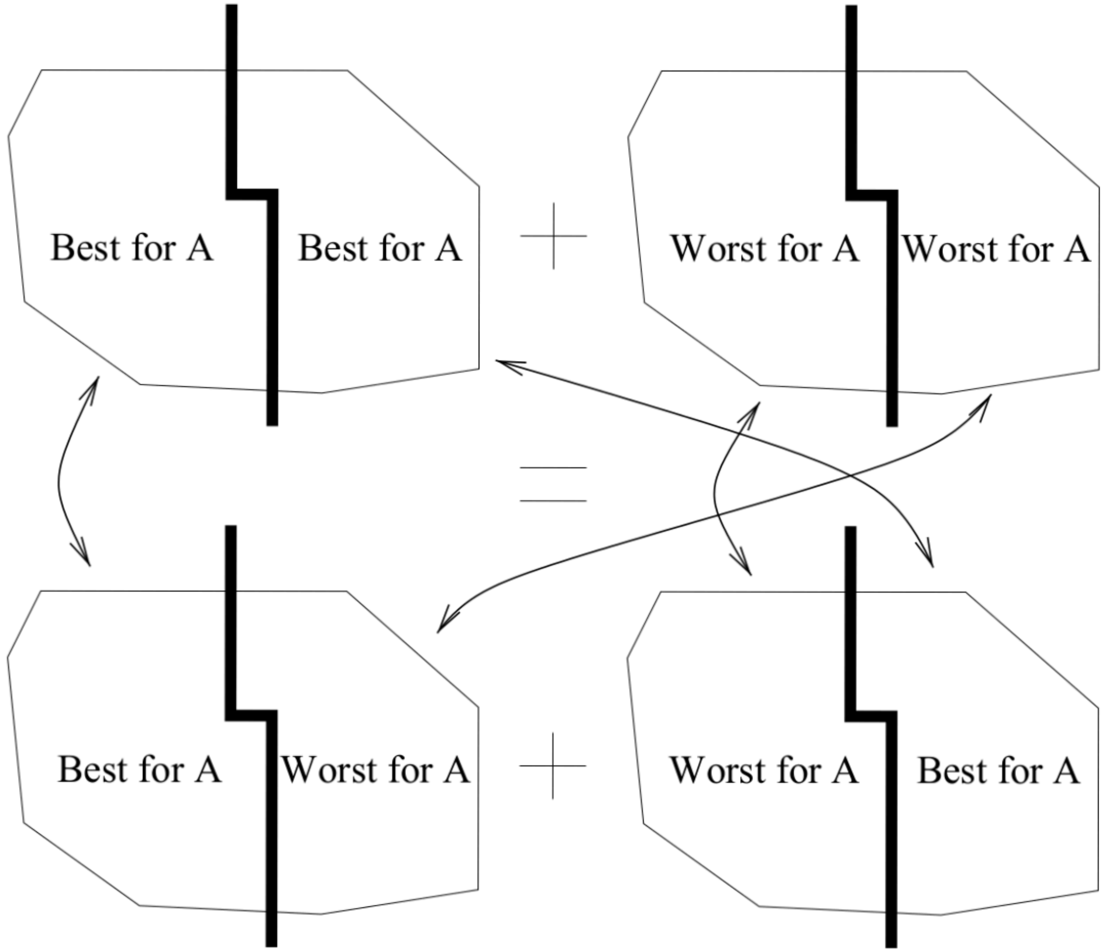

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Threshold election (T-F, 2018):

1. Party 1 makes the redistricting map
2. Party 2 chooses a threshold $m \in [\frac{1}{2}, 1)$
3. On election day, each party needs a threshold of $> m$ to win a district. Otherwise it is awarded randomly to each party with probability $\frac{1}{2}$

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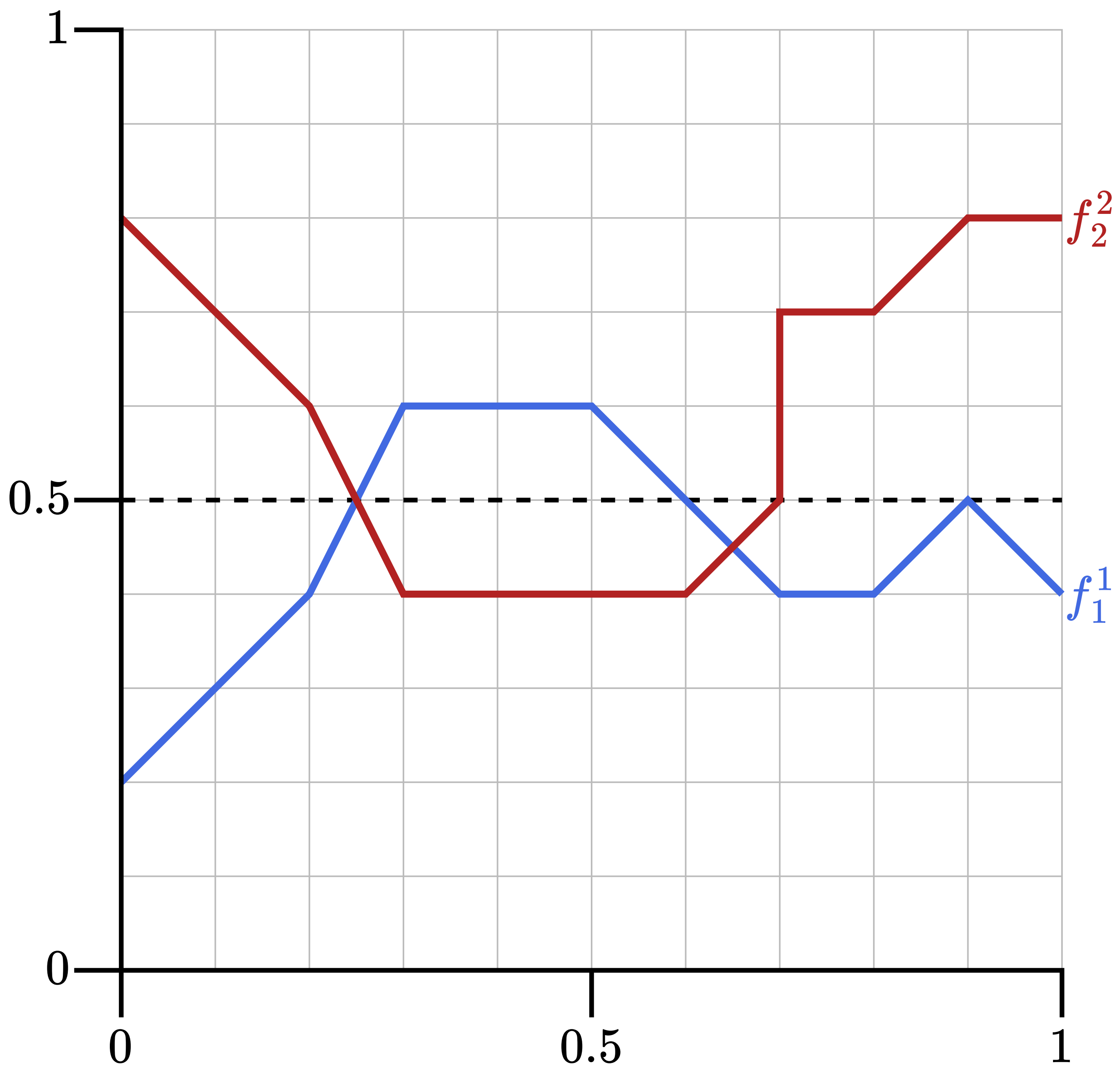

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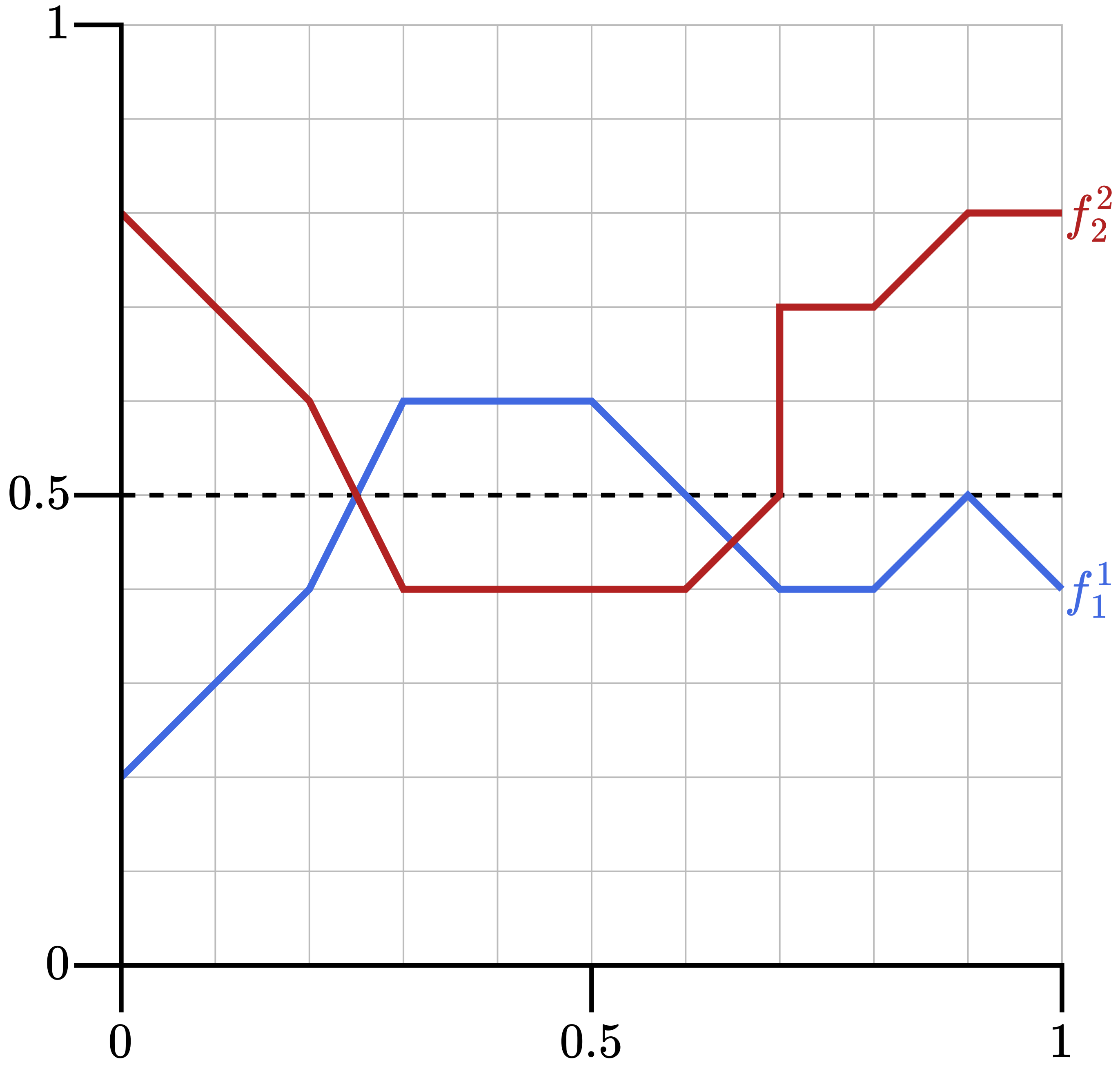

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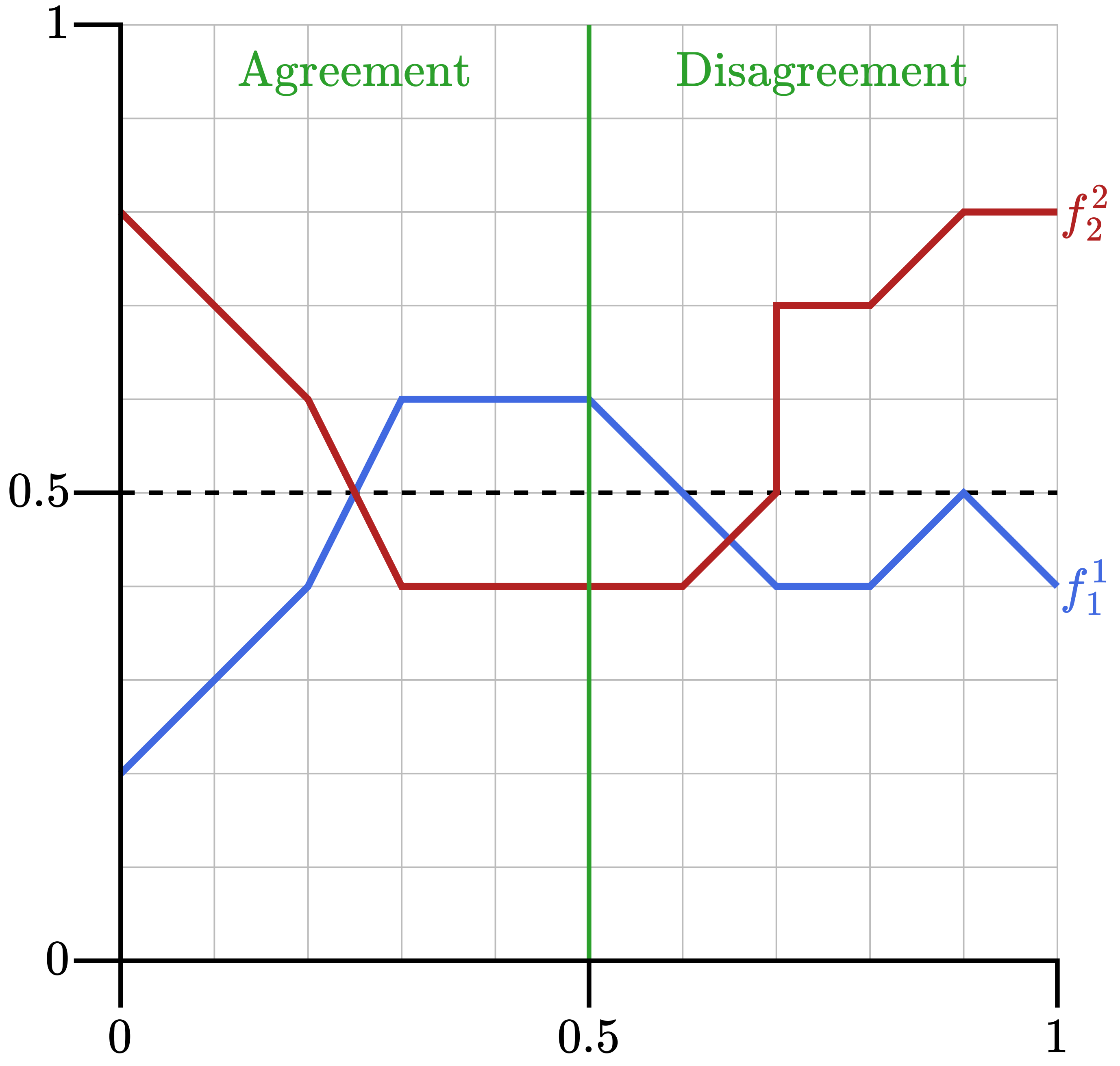

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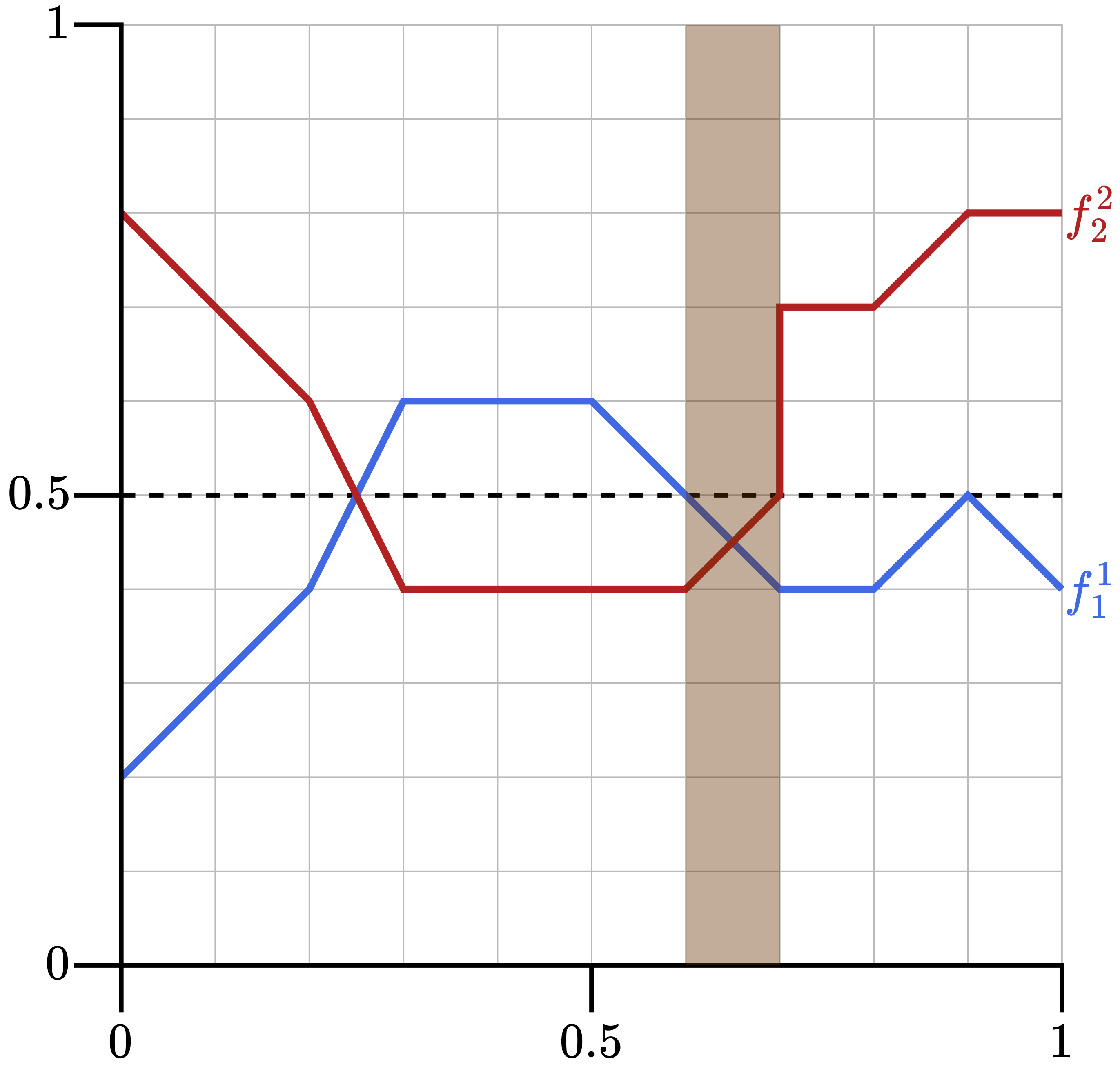

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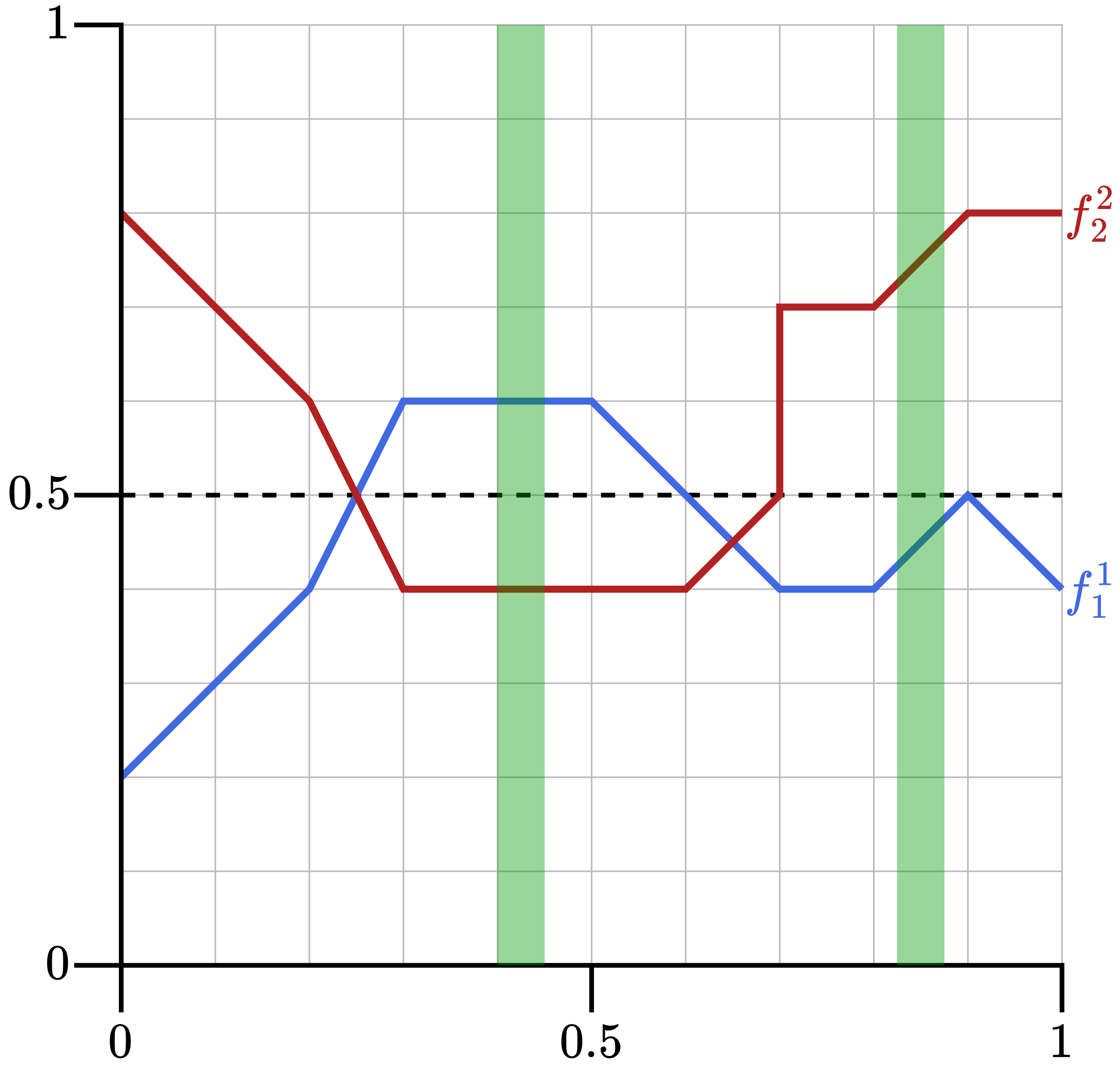

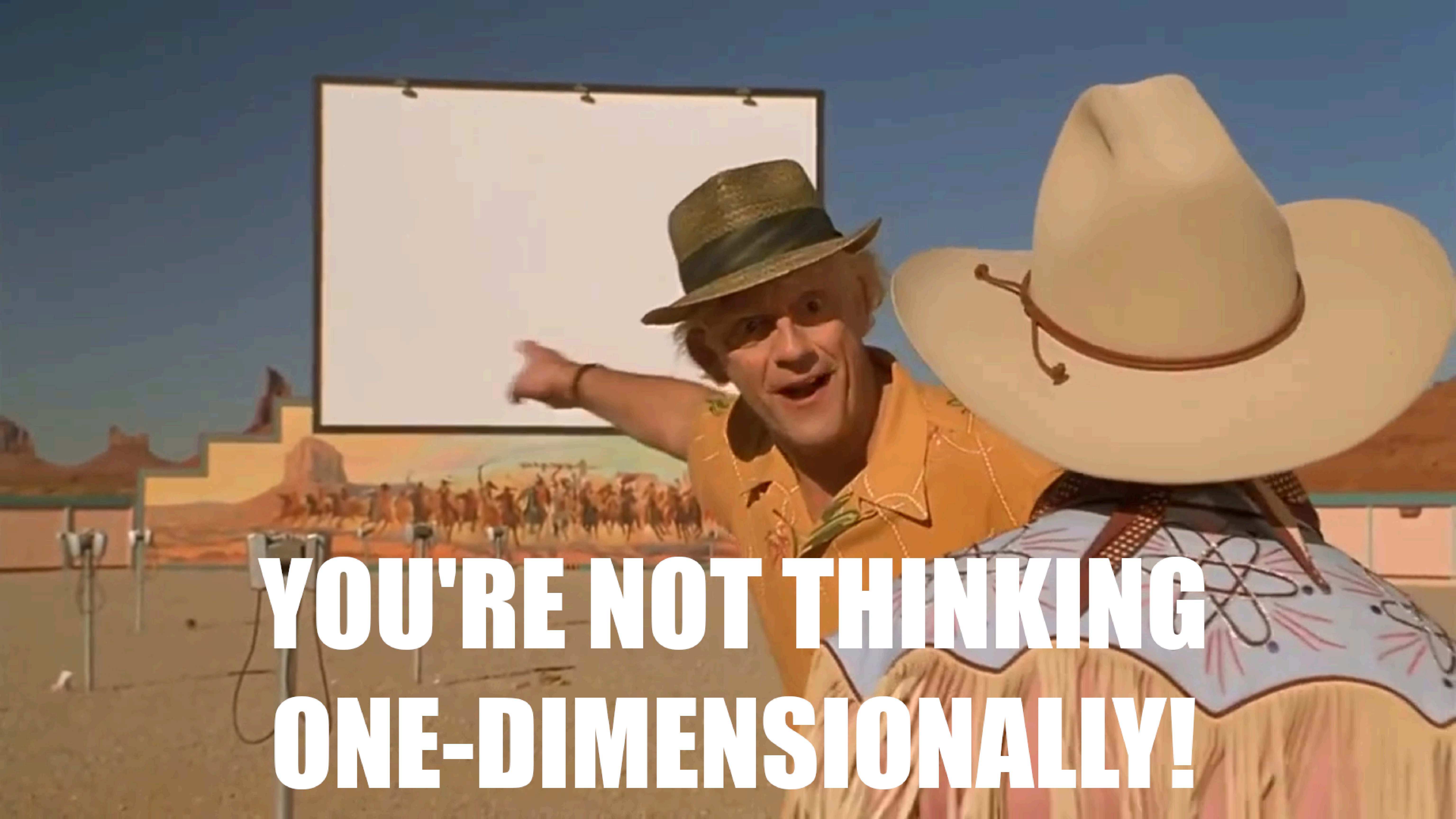
YOU'RE NOT THINKING
ONE-DIMENSIONALLY!

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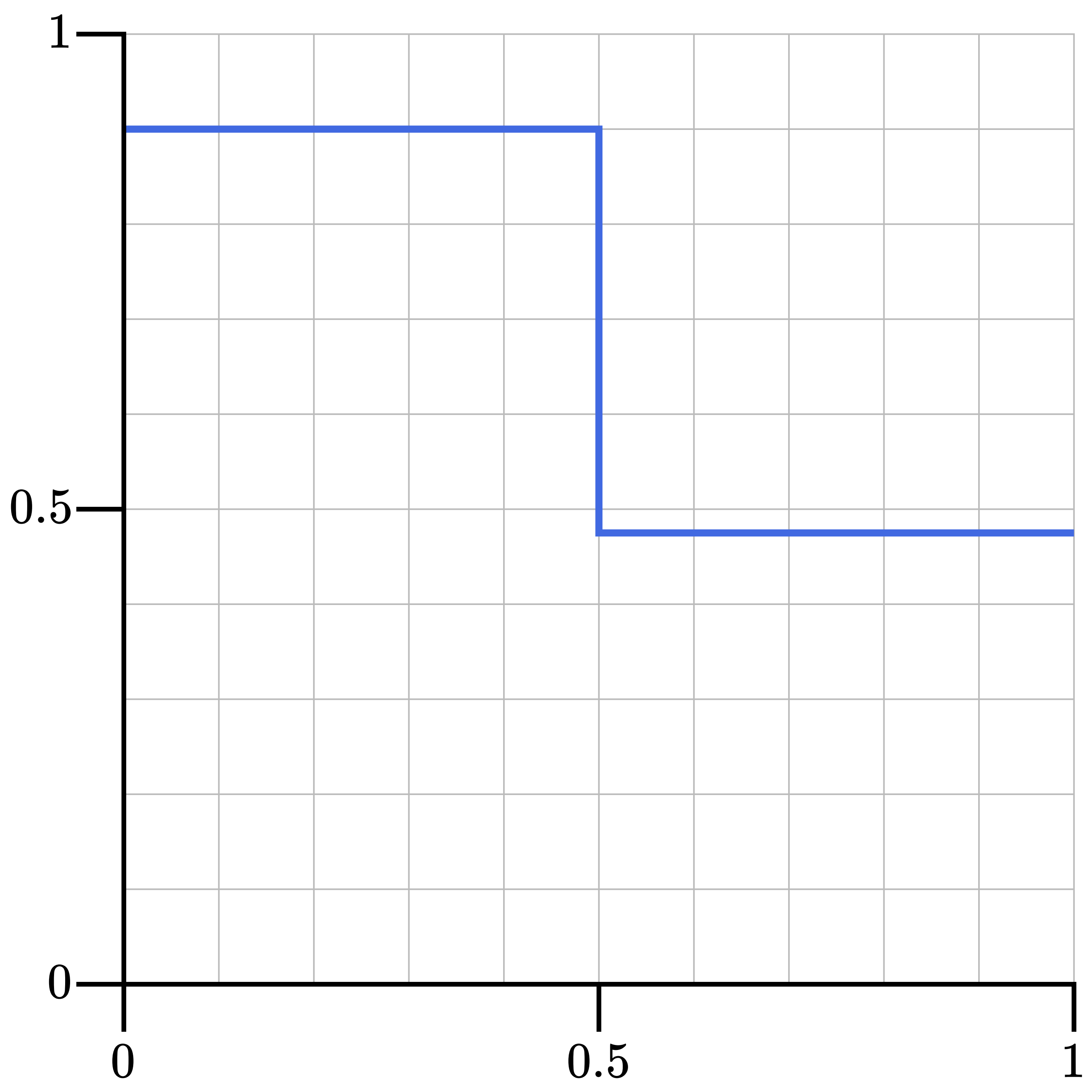

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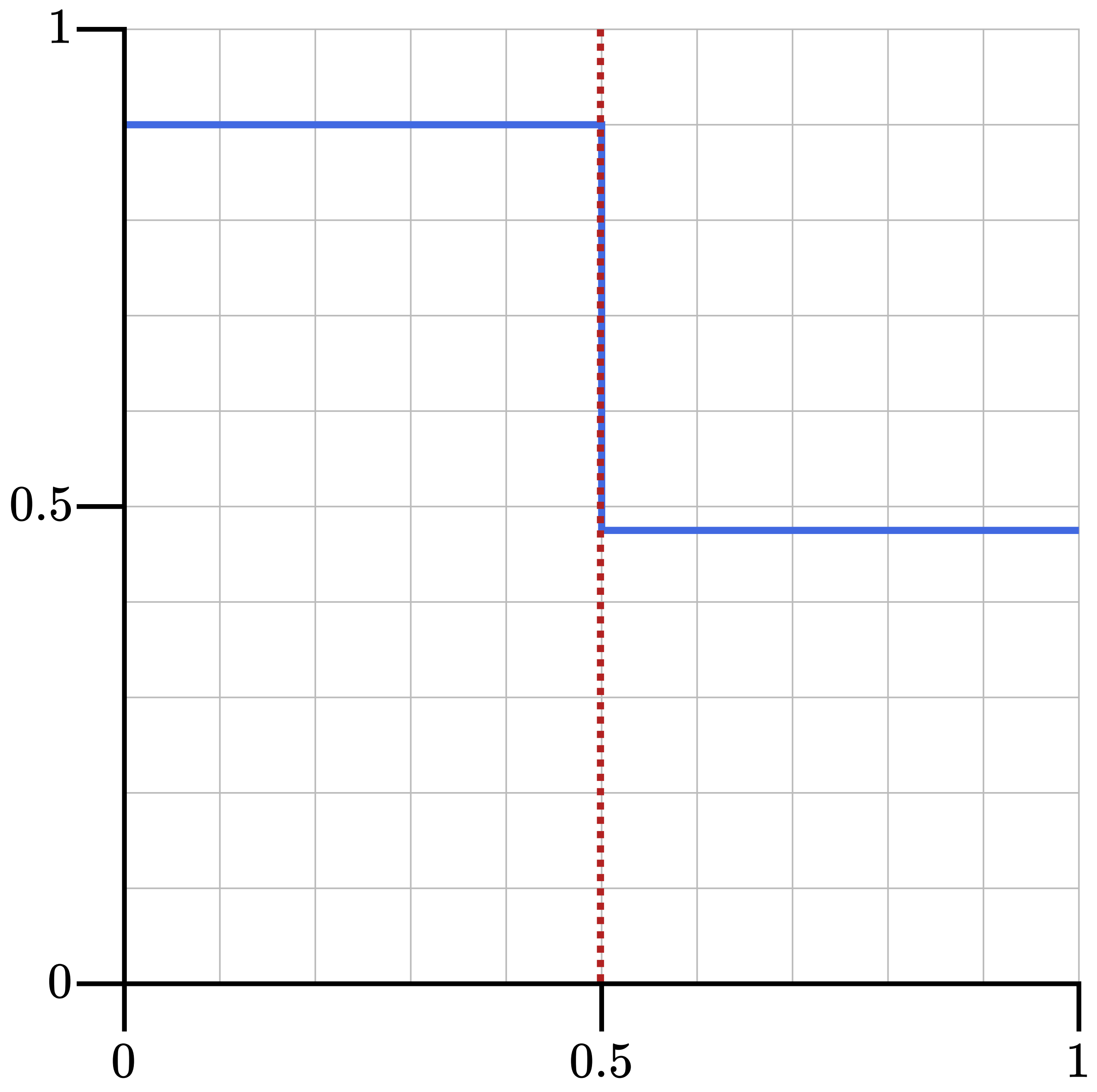

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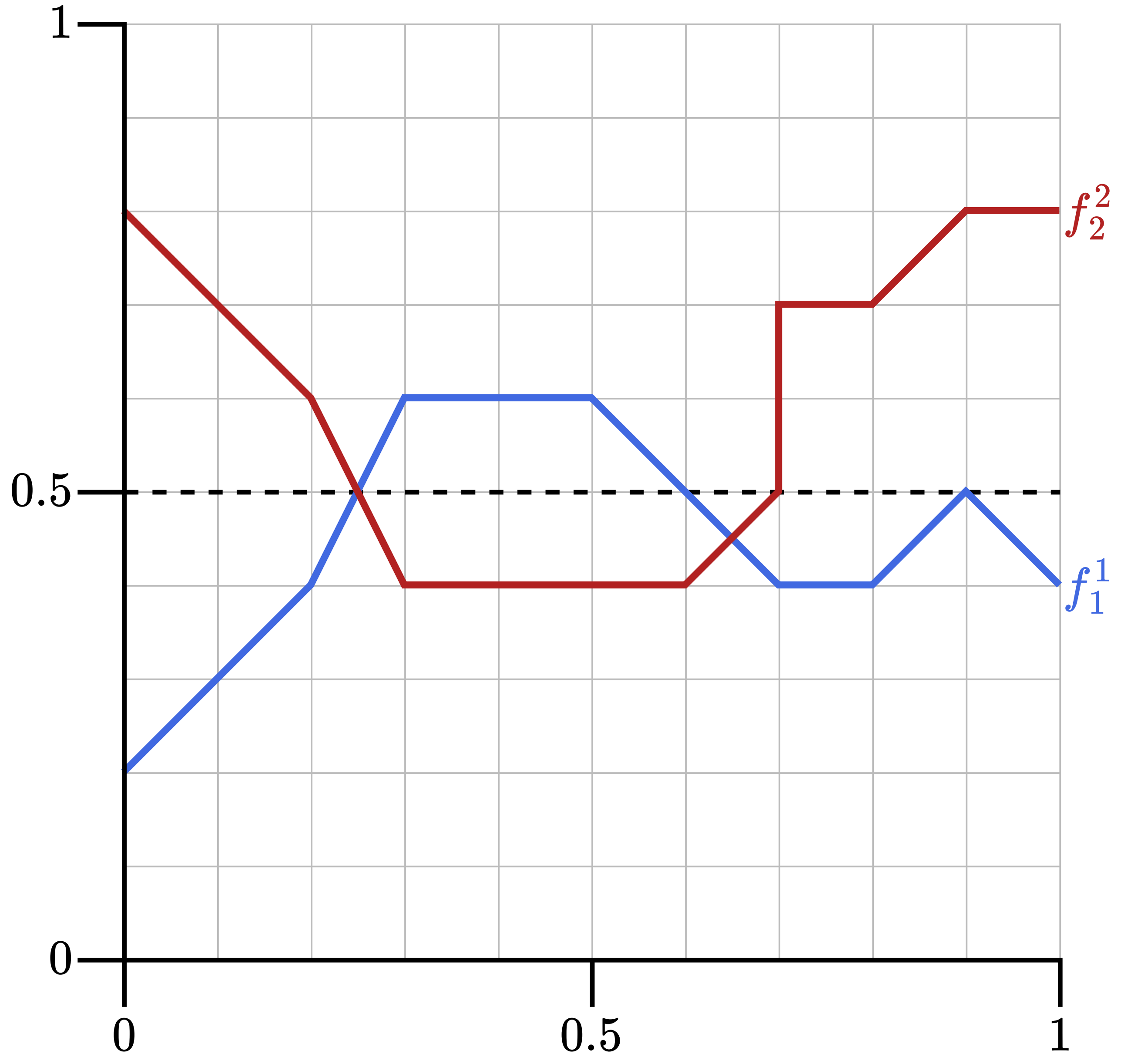

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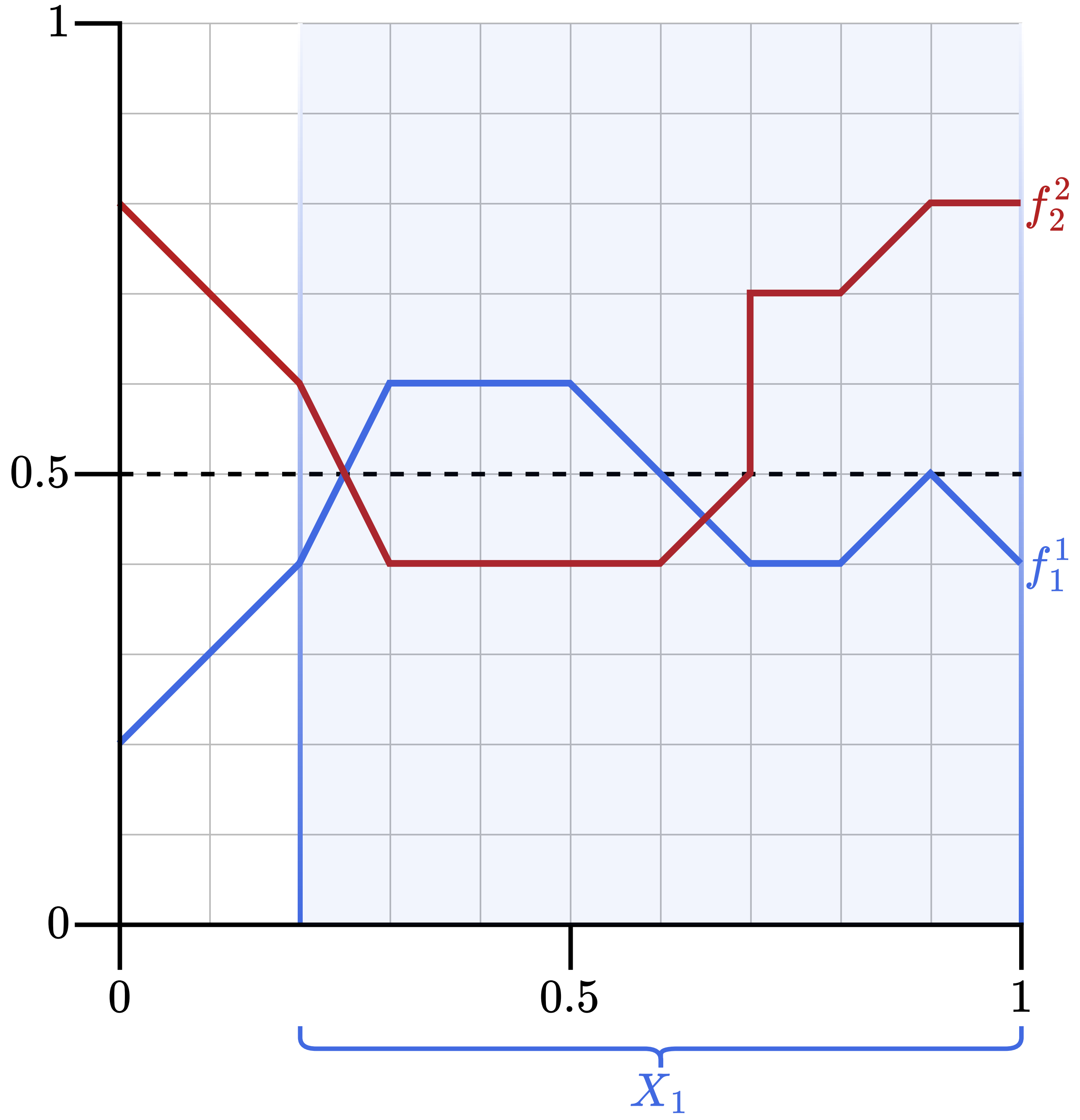

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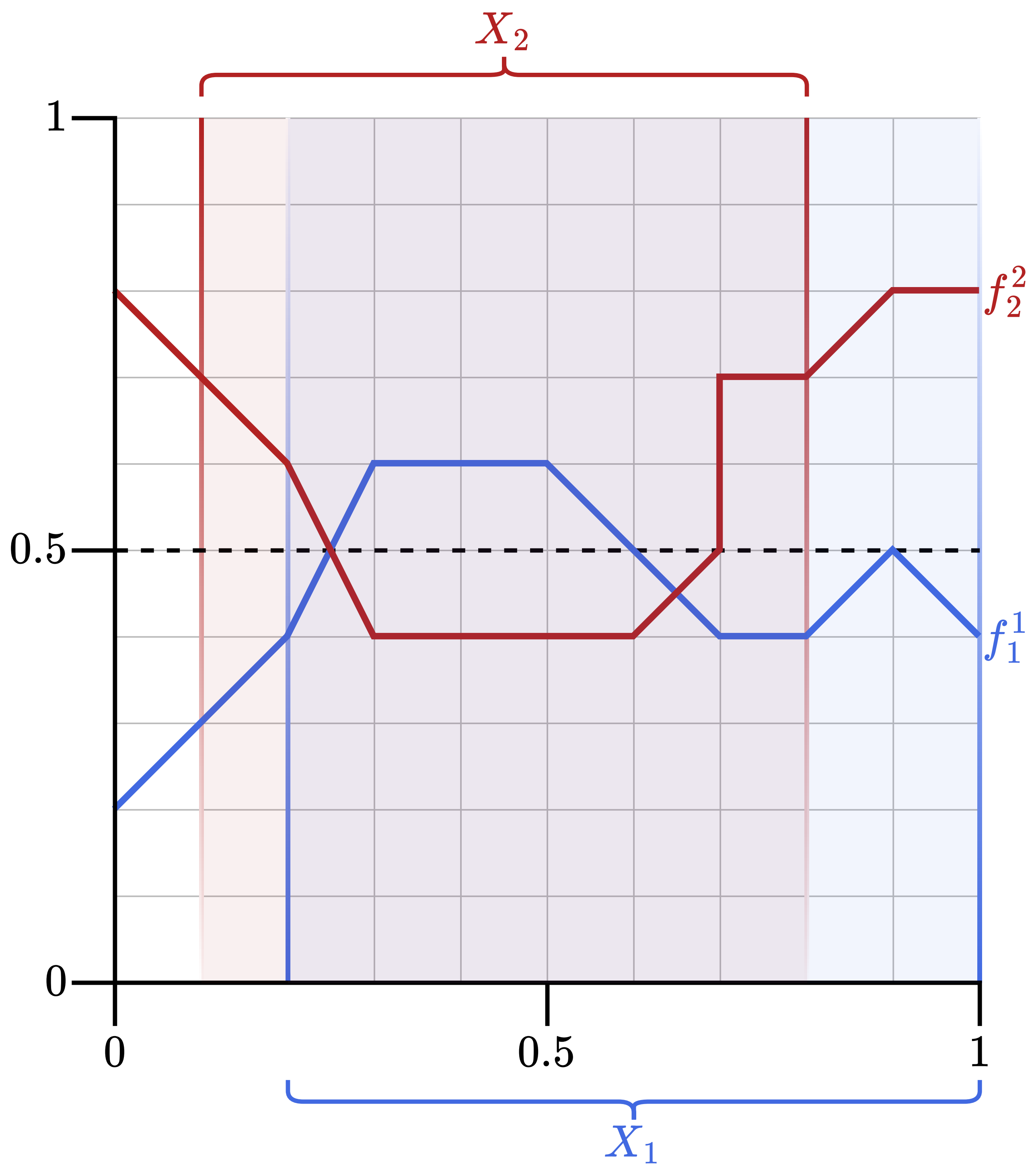

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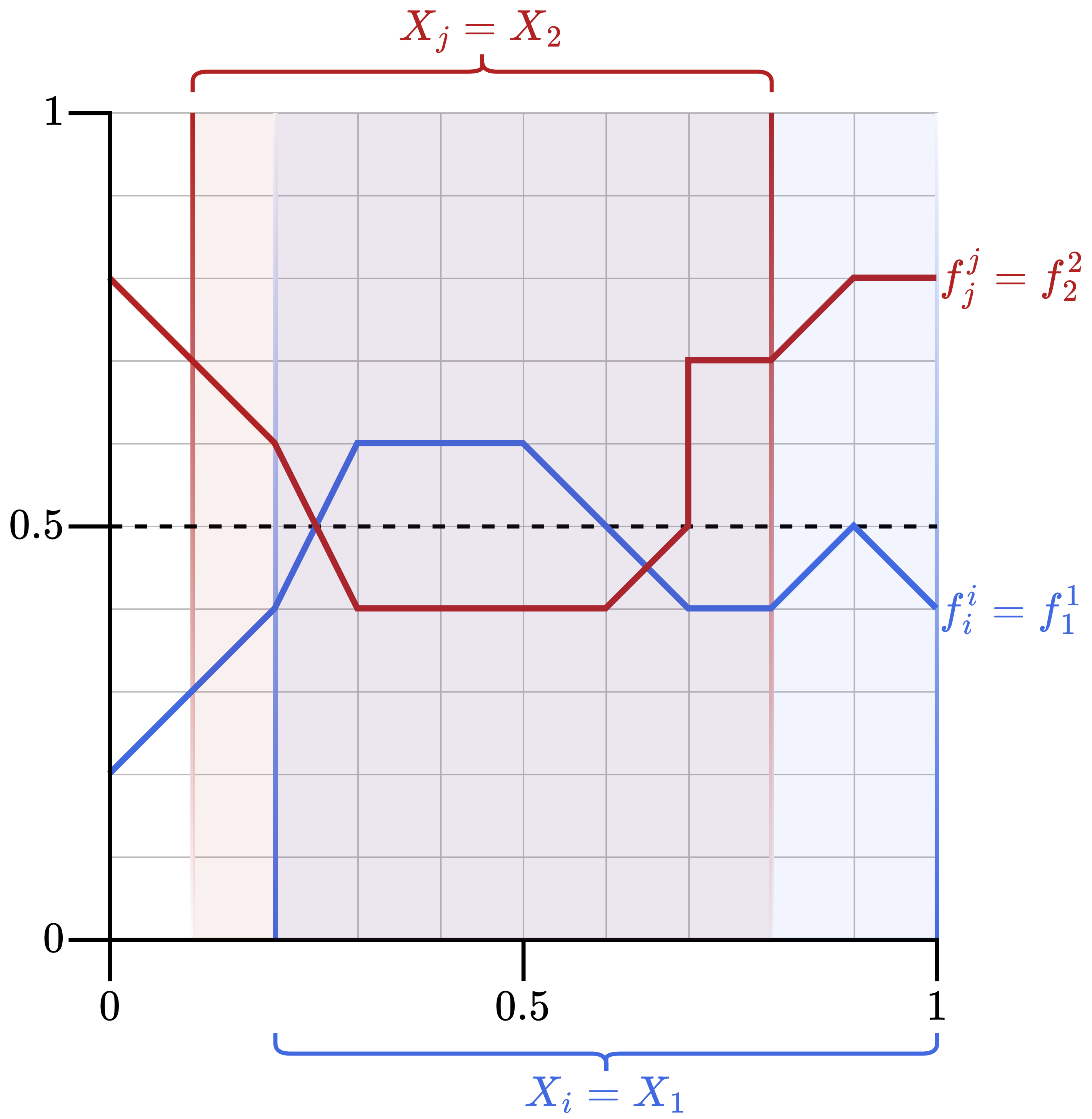

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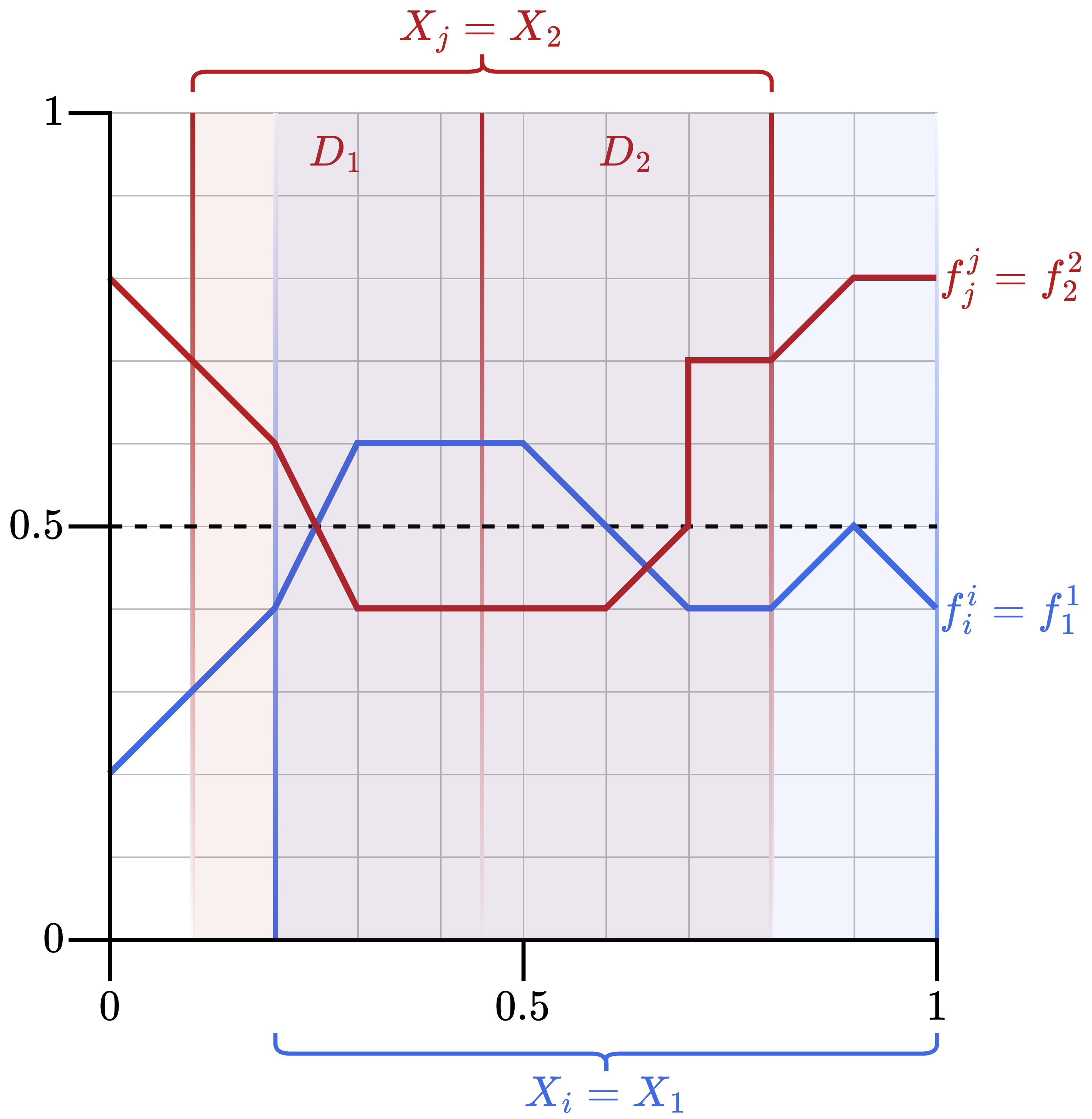

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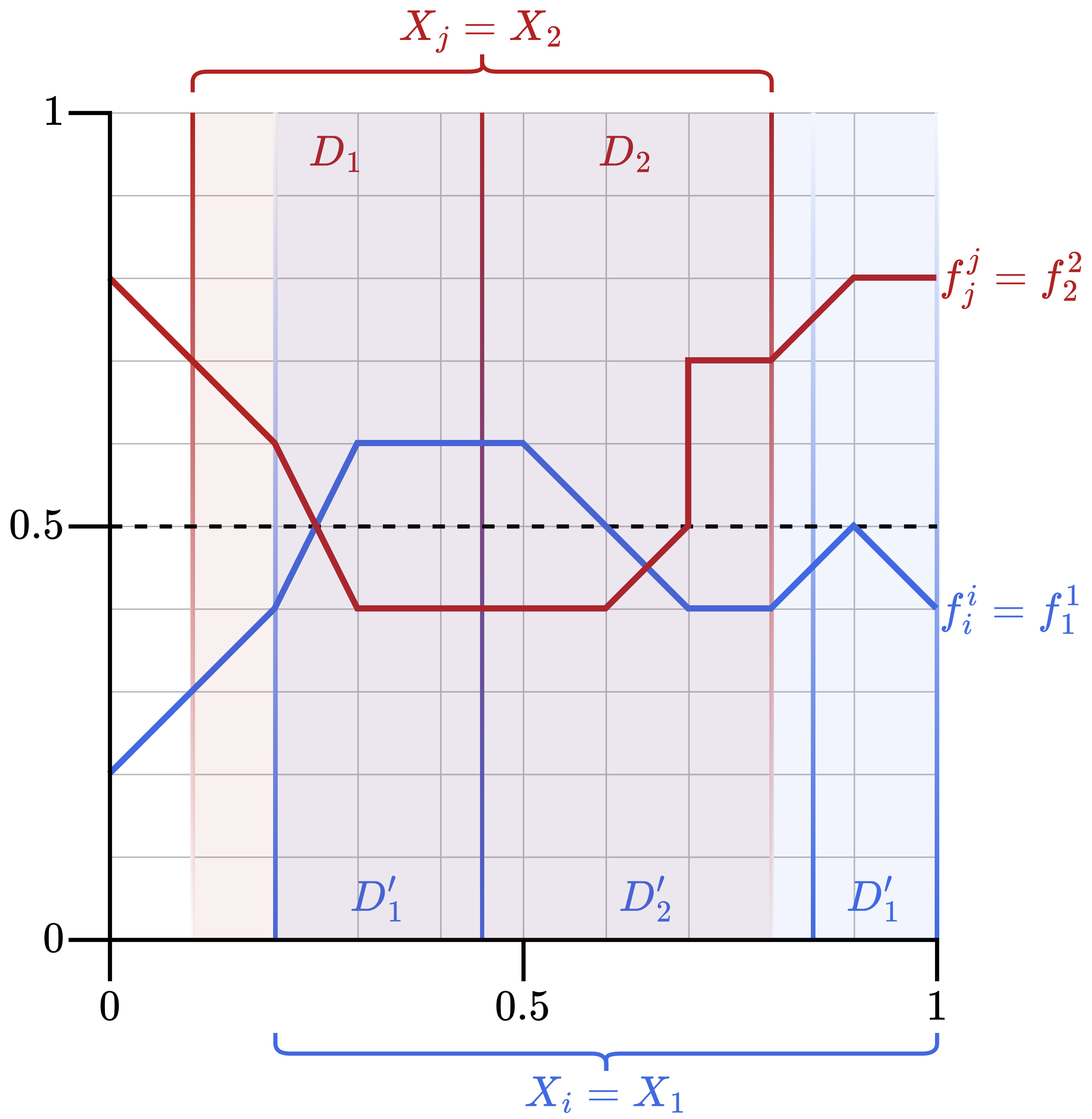

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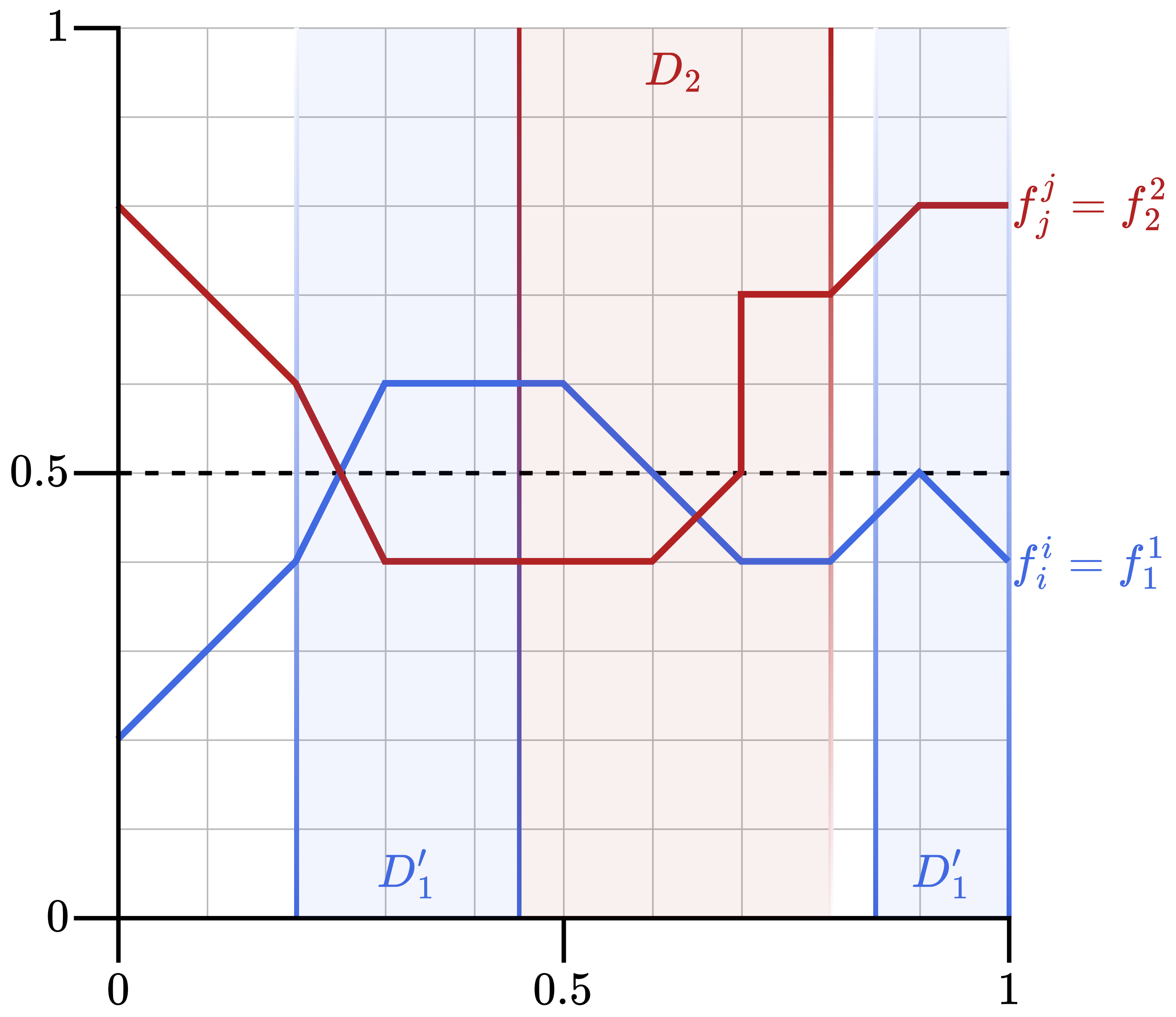

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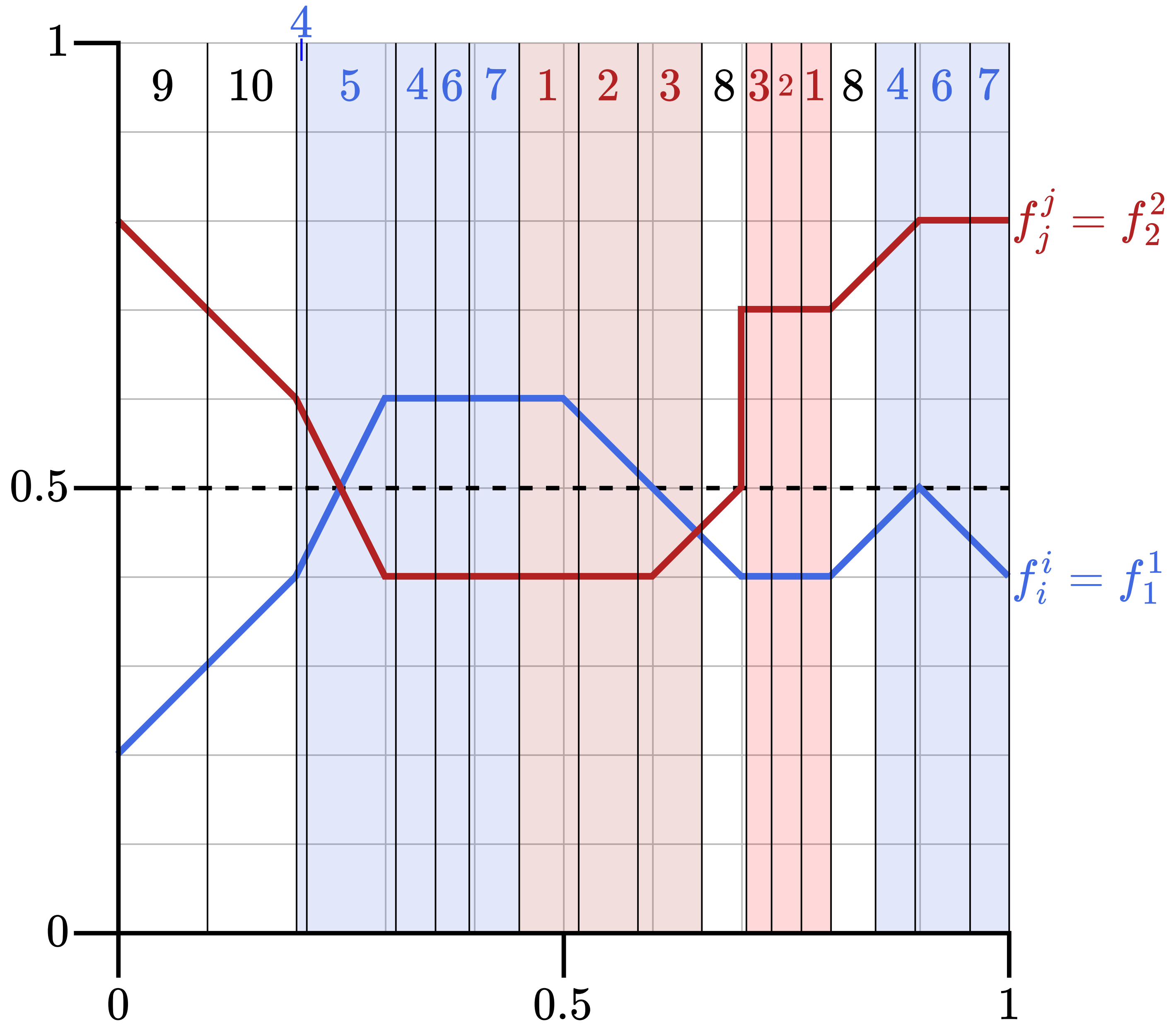

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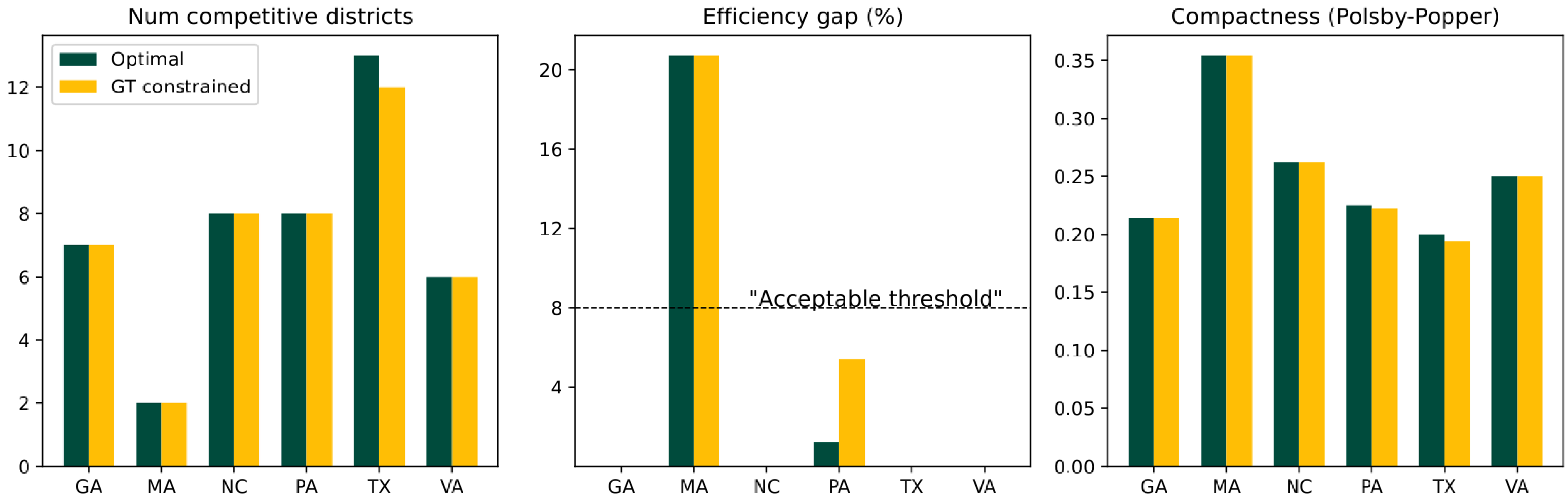

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